Returns to Scale in Electricity Supply

Practical Example: Nerlove (1963)

17th January 2007

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3. Task 2: Estimate the cost equation and interprete.
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8. Task 7: Testing constant returns to scale.
9. Task 8: Make a residual plot and interprete.

What's it all about?

Nerlove's paper

- classic study of returns to scale in a regulated industry (1963)

Features of the electricity industry

- monopolies supply power
- electricity prices are set by the utility commission
- factor prices are given
- Cobb-Douglas production and cost function
- cost minimization of firms

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The data

- cross-section data set of 145 firms in 44 states in the year 1955
- data on total costs, factor prices, output

Econometric approach

- OLS estimation of parameterized cost function
- assuming returns to scale to be constant (=r)

Subjects of interest

- characteristics of the cost function (e.g. elasticities)
- analysis of returns to scale

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Task 1:

Create the necessary variables to estimate the cost equation.

Linearized Cobb Douglas Cost Function

$$log(TC_i) = \beta_1 + \beta_2 log(Q_i) + \beta_3 log(p_{i1}) + \beta_4 log(p_{i2}) + \beta_5 log(p_{i3}) + \varepsilon_i$$

- TC_i: total costs for firm i
- Q_i: firm i's output
- p_{i1}: factor price for labor
- p_{i2}: factor price for capital
- p_{i3}: factor price for fuel

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- TC_i: total costs for firm i $\longrightarrow log(TC)$
- Q_i: firm i's output $\longrightarrow log(Q)$
- p_{i1}: factor price for labor $\longrightarrow log(PL)$
- p_{i2}: factor price for capital $\longrightarrow log(PK)$
- p_{i3}: factor price for fuel $\longrightarrow log(PF)$

Task 2:

Estimate the cost equation with OLS and interprete the results.

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Estimation in EViews

- Quick $\longrightarrow$ Estimate Equation (`UMOD`)
- Equation specification:
  $\boxed{log(TC)\ c\ log(Q)\ log(PL)\ log(PK)\ log(PF)}$

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Questions that should guide your interpretation

- Are the parameters significantly different from zero?
- What is the interpretation of the parameters?

Interpretation

- coefficients in **log-linear** equations are **elasticities**
- β_3: elasticity of total costs with respect to the wage rate, i.e. the percentage change in total costs when the wage rate changes by 1 percent.

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Task 3:

Are the Assumptions of the CLRM statisfied?

Assumptions

1. **Linearity**
   $y_i = log(TC_i)$, $\mathbf{x}_i = (1, log(Q_i), log(p_{i1}), log(p_{i2}), log(p_{i3}))'$
2. **Strict exogeneity**
   regressors are independent of the firm's efficiency ε_i because of some special features of the U.S. electricity industry in 1963
3. **No multicollinearity**
   in Nerlove's data set, $rank(\mathbf{X}) = 5$ and $n = 145$
4. **Spherical error variance**
   no correlation in the error term, but potential heteroskedasticity

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Task 4:

Which restriction is implied by the notion that the degree of returns to scale is constant $(\alpha_1 + \alpha_2 + \alpha_3 \equiv r)$?

Returns to Scale are constant

Remember that $\beta_3 = \frac{\alpha_1}{r}, \beta_4 = \frac{\alpha_2}{r}, \beta_5 = \frac{\alpha_3}{r}$. Then it follows that

$$\begin{aligned} \alpha_1 + \alpha_2 + \alpha_3 &\equiv r \quad | \quad \text{substitute the } \alpha \\ r\beta_3 + r\beta_4 + r\beta_5 &= r \quad | : r \\ \beta_3 + \beta_4 + \beta_5 &= 1 \end{aligned}$$

and thus the OLS equation is restricted by $\beta_3 + \beta_4 + \beta_5 = 1$.

- OLS equation is **overidentified**
- cost function is **linearly homogeneous** in factor prices

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Task 5:

Test with two alternative procedures if the above restriction ($\beta_3 + \beta_4 + \beta_5 = 1$) can be rejected.

Wald Coefficient Test *(alternative 1)*

- Estimate the unrestricted model (`UMOD`):

$$\boxed{log(TC_i) = \beta_1 + \beta_2 log(Q_i) + \beta_3 log(p_{i1}) + \beta_4 log(p_{i2}) + \beta_5 log(p_{i3}) + \varepsilon_i}$$

- View $\longrightarrow$ Coefficient Test $\longrightarrow$ Wald-Coefficient Restrictions

F-ratio $\frac{(SSR_R - SSR_U)/\#r}{SSR_U/(n-K)}$ *(alternative 2)*

- Estimate the restricted model (`RMOD`):

$$\boxed{log\left(\frac{TC_i}{p_{i3}}\right) = \beta_1 + \beta_2 log(Q_i) + \beta_3 log\left(\frac{p_{i1}}{p_{i3}}\right) + \beta_4 log\left(\frac{p_{i2}}{p_{i3}}\right) + \varepsilon_i}$$

- Calculate the F-ratio via the `SSR` of `UMOD` and `RMOD`

Task 6:

Reformulate the model to get the average costs as the dependent variable. What happens to the parameter β_2 in this new setting?

Reformulated model

$$\begin{aligned}
log(TC_i) &= \beta_1 + \beta_2 log(Q_i) + \beta_3 log(p_{i1}) + \beta_4 log(p_{i2}) + \beta_5 log(p_{i3}) + \varepsilon_i \mid - log(Q_i) \\
log\left(\frac{TC_i}{Q_i}\right) &= \beta_1 + (\beta_2 - 1) log(Q_i) + \beta_3 log(p_{i1}) + \beta_4 log(p_{i2}) + \beta_5 log(p_{i3}) + \varepsilon_i
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Discuss the difference in the two values for R^2 obtained in the total cost equation (1) and the average cost equation (2).

Does the higher R^2 make (1) preferable to (2)?

- (1) total cost equation: 0.926
- (2) average cost equation: 0.695

$\Rightarrow$ nonsense, because (1) and (2) represent the same model

$\Rightarrow$ high R^2 in (1) comes from the scale effect that total costs increase with firm size

$\Rightarrow$ equations must share the same dependent variable in order to compare their fit via R^2

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Task 7:

Test the hypothesis that there exist CRS ($H_0 : r = 1$).

Consider the restricted model

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then $r = 1$ if and only if the coefficient β_2 equals 1 since $\beta_2 = \frac{1}{r}$.

Application of the t-test

1. Calculate the t- ratio for the hypothesis ($H_0 : \beta_2 = 1$).
2. Look for the critical value in the t-distribution $t(n-K)$ ($K = 4$ and $n = 145$).
3. Conclude.

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Task 8:

Make a residual series from your restricted model. Plot the residuals against $\log(Q_i)$. Are you disturbed after looking at the plot?

Plot in EViews

- Object $\longrightarrow$ New Object $\longrightarrow$ Graph (Scatter diagram)

Notice from the Residual Plot

- as output increases, the residuals first tend to be positive, then negative and again positive $\longrightarrow$ degree of returns to scale is not constant as assumed in the log-linear specification
- residuals are more widely scattered for lower output $\longrightarrow$ failure of the homoskedasticity assumption

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Thank you for your attention and have a nice week!