EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-
Naturwissenschaftliche
Fakultät

Department of Mathematics

Module Handbook

Mathematics

Master of Education

Lehramt Gymnasium*

Winter Semester 2026

Last updated on 13th July 2026

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*This is a secondary school teaching degree with a major in mathematics. The module handbook is valid for the 2026 study and examination regulations.

Contents

1 Description of the Study Programme

1.1 Qualification Objectives

The M.Ed. Lehramt Gymnasium with Mathematics as a major is designed to equip future secondary-school teachers with the academic aptitude to teach Mathematics. The overarching and subject-specific qualification objectives, both with regard to the content of the degree programme and the skills to be acquired, are specified in the legal ordinance of the Ministry of Education and Cultural Affairs on framework requirements for the conversion of the general education teacher training degree programmes in Baden Württemberg.

As part of the M.Ed. Lehramt Gymnasium, graduates deepen their knowledge and skills in mathematics and mathematics didactics, which enable them to design targeted teaching, learning and educational processes in the subject of mathematics and to independently introduce new subject-related and interdisciplinary developments into teaching and school development.

Building on the fundamental issues in linear algebra, analysis, numerical mathematics, stochastics, geometry and algebra from the B.Ed. Lehramt Gymnasium, they expand their material and methodological skills in the areas of subject didactics and one or more further mathematical areas from the specialisations offerd in Tübingen: Algebra and Geometry, Analysis and Differential Geometry, Mathematical Physics, Numerical Mathematics and Optimisation or Stochastics. Graduates are proficient in the theoretical explanatory approaches, principles and methods of mathematics, are familiar with its cognitive and working methods and can apply them in the central areas of mathematics. They can present mathematical facts adequately in oral and written form using suitable media and explain key issues in mathematical fields and their relationship to school mathematics. They are able to solve mathematical problems systematically, strategically and using suitable tools to solve mathematical problems and to comprehend and develop mathematical proofs. Based on their general knowledge, graduates are able to access and critically scrutinise fundamental current issues in mathematics. They can use their in-depth knowledge to develop and solve their own approaches and can derive, analyse, prove and interpret specific questions from general mathematical concepts. Graduates are able to present, explain and discuss the results of their work in depth in front of a scientific audience, both in writing and orally. They have learnt to acquire new specialist knowledge independently and are therefore able to access new mathematical theories in their later professional life, which are incorporated into school mathematics.

Students combine their scientific knowledge with didactic methods, use suitable media and are able to use theoretical concepts and empirical findings from mathematics-related teaching-learning research to analyse thought processes and ideas of pupils in exercises and to guide individual learning processes. They know and evaluate the concepts for learning and teaching mathematics at school on the basis of subject-didactic theories and empirical findings. They are able to analyse, plan and exemplarily implement mathematics lessons with heterogeneous learning groups on the basis of subject-didactic concepts. Students can justify the educational value of mathematical content, convey the societal significance of mathematics and place it in the context of the objectives and content of mathematics lessons.

1.2 Structure of the Study Programme

The standard period of study for the Master of Education Lehramt Gymnasium is four semesters; 28 credit points must be earned in the subject of mathematics. Depending on whether you start in the winter or summer semester, the first or second semester is mainly filled by the school internship component, which is accompanied by a subject didactics course at the university. In Mathematics students will deepen their knowledge in one or more fields of mathematics via to specialisation modules. That way students have the opportunity to set their own focus in the study specialisations offered by the department by attending a lecture with exercise classes and a suitable seminar. The study programme is completed with the Master's thesis (15 credit points) in one of the two chosen subject areas (including their didactics) or in the educational sciences. With the Master's degree, students (if they fulfil any further requirements) can enter the preparatory service traineeship, professional life, a doctorate in the field of didactics of mathematics or switch to a further degree course.

Integrating a study component at a foreign university into this degree programme in a meaningful way is a challenge, as it involves coordinating two subjects and the educational sciences; whether you try to complete parts of the programme in all areas during your stay at the other university, or whether you try to structure your studies at the University of Tübingen in such a way that parts of the programme are shifted to other semesters in order to create free time so that you do not have to complete all three areas at the other university. Adding to the difficulty is the fact that one semester is largely occupied by the practical component. Therefore, it is essential to plan a suitable timeframe for a study component at a foreign university through a personal consultation with the Faculty Course Advisor. Essentially, from the mathematics perspective, any academic semester is suitable for this purpose. The decision will depend on the student's previous achievements and the courses offered at the chosen foreign university.

2 Study Plans

2.1 Overview by Modules

Here we provide an overview of the study plan as a table showing the modules to be taken.

ST	Module Number	Module Title	Type of Course	Type of Module	Course-work	Type of Exam	ECTS-Points
Section 1: Mathematics							
2	MAT-40-51	Specialisation 1	L+E	PMW	EC	wr. o. or.	9
3	MAT-40-52	Specialisation 2	L+E	PMW	EC	wr. o. or.	9
4	MAT-40-53	Seminar: Mathematical Specialisation	S	PMW	s.M.	Pr	4
Section 2: Didactics of Mathematics							
1-2	MAT-80-03	Subject Didactics Mathematics 3	S+SV	PMW	-	wr., or., E, Pr o. T	6
Section 3: Master Thesis							
4	MAT-40-54	Master Thesis	MT	PM	s.M.	MT	15

Abbreviations:

Type of Module	: PM=compulsory module, PMW=compulsory module with choice, WPM=elective module
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, S=seminar, IC=inverted classroom
Course Work	: EC=exercise certificate, PEC=practical exercise certificate, PC=practical certificate
Other	: h=hours, o.=or, s.M.=see module description, ST=suggested term

2.2 Overview by the Course of Studies

Firstly, we provide an overview of the possible course of study in the form of a table for both winter semester entry and summer semester entry.

Study Plan for Students Starting in the Winter Semester

FS	CP	Mathematics	Subject Didactics Mathematics	Master Thesis
1	3		Subject Didactics 3 (6 CP)	
2	12	of Special Areas of Mathematics (9 CP)		
3	9	Consolidation of Special Areas of Mathematics 2 (9 CP)		
4	4 + (15)	Seminar Mathematical Specialisation (4 CP)		Master Thesis (15 CP)

Explanation of the Abbreviations:
FS=semester, CP=credit points (ECTS points)

Figure 2.1: Study Plan for Students Starting in the Winter Semester

Study Plan for Students Starting in the Summer Semester

FS	CP	Mathematics	Subject Didactics Mathematics	Master Thesis
1	12	Consolidation of Special Areas of Mathematics 1 (9 CP)	Subject Didactics 3 (6 CP)	
2	3			
3	9	Consolidation of Special Areas of Mathematics 2 (9 CP)		
4	4 + (15)	Seminar Mathematical Specialisation (4 CP)		Master Thesis (15 CP)

Explanation of the Abbreviations:
FS=semester, CP=credit points (ECTS points)

Figure 2.2: Study Plan for Students Starting in the Summer Semester

2.3 Overview of Programme Structure with Semester Assignment

Overview of Programme Structure with Semester Assignment for Students Starting in the Winter Semester													
		Exam				**Teaching**				**Semester**			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points(CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
Section 1: Mathematics									**22**				
Consolidation of Special Areas of Mathematics 1								6	9				
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4			6		
2.	Exercise class					E	o	2			3		
Consolidation of Special Areas of Mathematics 2								6	9				
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4				6	
2.	Exercise class					E	o	2				3	
Seminar Mathematical Specialisation								2	4				
1.	Seminar	Pres		g	4	S	o	2					4
Section 2: Subject Didactics Mathematics									**6**				
Subject Didactics 3								4	6				
1.	Seminar	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	S	o	2		3			
2.	Seminar / Lecture	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	SL	o	2			3		
Section 3: Master Thesis									**15**				
Master Thesis									15				
1.	Master Thesis	MA		g		MA	o						15

Explanation of the Abbreviations:
- Marking system : g=graded, ng=non graded
- Form of examination : MA=master thesis, Or.=oral exam, Wr.=written exam, Pres=presentation, TP=term paper
- Form of teaching : L=lecture, SL=Seminar or lecture, E=exercise class, r=revision course, P=practical course, PS=introductory seminar, S=seminar
- Status : o=obligatory, f=facultative
- Other : o.=or, SWS=hours in class per week, CP=credit points=ECTS Points

Overview of Programme Structure with Semester Assignment for Students Starting in the Summer Semester													
		Exam				Teaching				Semester			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points(CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
Section 1: Mathematics									**22**				
Consolidation of Special Areas of Mathematics 1								6	9				
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4		6			
2.	Exercise class					E	o	2		3			
Consolidation of Special Areas of Mathematics 2								6	9				
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4				6	
2.	Exercise class					E	o	2				3	
Seminar Mathematical Specialisation								2	4				
1.	Seminar	Pres		g	4	S	o	2					4
Section 2: Subject Didactics Mathematics									**6**				
Subject Didactics 3								4	6				
1.	Seminar	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	S	o	2			3		
2.	Seminar / Lecture	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	SL	o	2		3			
Section 3: Master Thesis									**15**				
Master Thesis									15				
1.	Master Thesis	MA		g		MA	o						15

Explanation of the Abbreviations:

Marking system : g=graded, ng=non graded
Form of examination : MA=master thesis, Or.=oral exam, Wr.=written exam, Pres=presentation, TP=term paper
Form of teaching : L=lecture, SL=Seminar or lecture, E=exercise class, r=revision course, P=practical course, PS=introductory seminar, S=seminar
Status : o=obligatory, f=facultative
Other : o.=or, SWS=hours in class per week, CP=credit points=ECTS Points

3 Module Descriptions

Section 1: Mathematics

Module Number: MAT-40-51	**Module Title:** Specialisation 1		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	every Semester		
Term	2		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Comment	A course must be selected from the catalogue of courses in Section 4.1 of the module handbook, comprising 4 hours of lectures and 2 hours of exercises per week. The approval of additional courses or alternative course formats (e.g., two courses with 2 hours of lectures and 1 hour of exercises each) is at the discretion of the head of the examination board, upon a written request by the student.		
Content	The content is determined by the choice of a course.		
Objectives	The students have acquired in-depth knowledge in a specific area of mathematics and gained further experience in presenting and communicating mathematical topics. They are capable of identifying the key statements of the lecture, reproducing the techniques used for their derivation and proof, and critically evaluating them. Additionally, they can integrate the methodological and theoretical foundations of the chosen mathematical subfield and place them within the broader mathematical context. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
see Comment	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Transfer	If applicable the module is a prerequisite for the modules Seminar Specialisation Mathematics and Master Thesis.
Prerequisites	-
Responsible Persons	The dean of studies at the Department of Mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-52	**Module Title:** Specialisation 2		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	every Semester		
Term	3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Comment	A course must be selected from the catalogue of courses in Section 4.1 of the module handbook, comprising 4 hours of lectures and 2 hours of exercises per week. The approval of additional courses or alternative course formats (e.g., two courses with 2 hours of lectures and 1 hour of exercises each) is at the discretion of the head of the examination board, upon a written request by the student.		
Content	The content is determined by the choice of a course.		
Objectives	The students have acquired in-depth knowledge in a specific area of mathematics and gained further experience in presenting and communicating mathematical topics. They are capable of identifying the key statements of the lecture, reproducing the techniques used for their derivation and proof, and critically evaluating them. Additionally, they can integrate the methodological and theoretical foundations of the chosen mathematical subfield and place them within the broader mathematical context. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
see Comment	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Transfer	If applicable the module is a prerequisite for the modules Seminar Specialisation Mathematics and Master Thesis.
Prerequisites	-
Responsible Persons	The dean of studies at the Department of Mathematics

Abbreviations:

Grading System	:	g=graded, ng=not graded
Examination Type	:	MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-53	**Module Title:** Seminar: Mathematical Specialisation		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4		
Workload **- Time in Class** **- Self-Study**	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	every Semester		
Term	4		
Language of Instruction	German		
Forms of Teaching and Learning	Seminar, talk, presentation, e-learning, blended learning		
Content	Various topics from the advanced fields of mathematics.		
Objectives	The students independently work on a coherent mathematical topic and prepare it in a didactical appealing fashion. They learn how to present their work to a group, how to be responsive to questions regarding the content and how to lead a professional discussion. The work and the presentation may be the foundation or a deepened study in the scope of a master thesis.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	(see table below)		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Seminar	S	o	2	4	yes	Pr	60-90	g	100

The acquisition of the credit points requires alongside with a successful presentation the regular active participation in the course, like by asking questions, contributing to a discussion or working on problem tasks. Additionally a written elaboration of the own talk or the issue of a handout for the participants may be required. These further efforts constitute the coursework of the module.

Transfer	-
Prerequisites	The participation in the module requires the successful completion of at least one of the modules Specialisation 1 or 2.
Responsible Persons	The dean of studies at the Department of Mathematics

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Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 2: Didactics of Mathematics

Module Number: **MAT-80-03**	**Module Title:** Subject Didactics Mathematics 3		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	2 Semester		
Frequency	every Semester		
Term	1-2		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture, exercise, seminar, talk, presentation, e-learning, blended learning, project work, case studies		
Content	In the first part varying topics are covered, who are especially related to the teaching profession and are also suited for a didactical refurbishment of the school internship semester. In the second part varying topics from didactics will be studied, which can lead up to recent research in didactics.		
Objectives	The students • know subject didactical principles and educational concepts and can evaluate and scrutinise them, • can compare and evaluate subject related approaches to central terms and theorems of the treated fields, • can plan, execute, analyse and evaluate competence oriented mathematical education on the basis of subject didactical concepts, • can motivate the general educating value of mathematical contents and methods and the social importance of mathematics and put it in context with the goals and contents of mathematical education, • can specifically use subject specific media, • can create a portfolio and document important experiences, findings and insights in a structured manner.		

<table>
<tr><td rowspan="4">Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td>Subject didactics 3: professional knowledge</td><td>S</td><td>o</td><td>2</td><td>3</td><td>yes</td><td>wr., or., E, Pr o. T</td><td>90-180 o. 20-30</td><td>g</td><td>50</td></tr>
<tr><td>Subject didactics 3: elective specialisation</td><td>SV</td><td>o</td><td>2</td><td>3</td><td>yes</td><td>wr., or., E, Pr o. T</td><td>90-180 o. 20-30</td><td>g</td><td>50</td></tr>
<tr><td colspan="10">The module consists of two parts (Professional Knowledge and Elective Specialisation), for which the teaching learning methods (lecture, exercise or seminar) as well as the type of examination (written or oral exam, presentation or paper, portfolio or tests) are usually different. This is taken into account by the fact that, the examination in this module consists of two equally weighted parts.</td></tr>
<tr><td>Transfer</td><td colspan="10">-</td></tr>
<tr><td>Prerequisites</td><td colspan="10">There are no prerequisites for participating in the module.</td></tr>
<tr><td>Responsible Persons</td><td colspan="10">Frank Loose, Walther Paravicini</td></tr>
</table>

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master’s thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 3: Master Thesis

Module Number: **MAT-40-54**	**Module Title:** Master Thesis		**Type of Module:** Compulsory Module
ECTS-Points	15		
Workload **- Time in Class** **- Self-Study**	Workload: 450 h	Time in Class: regularly, as required	Self-Study: 450 h
Duration	1 Semester		
Frequency	every Semester		
Term	4		
Language of Instruction	German		
Forms of Teaching and Learning	Master thesis		
Content	The master's thesis represents the culmination of the master's degree programme. Under the guidance of a supervisor, students are required to work on a well-defined problem in mathematics (including mathematics education), which may extend to the frontiers of current research. This involves using scientific methods and presenting the results in written form. Specifically, this includes: • formulating a scientific research question in consultation with the supervisor; • independently searching for and studying relevant scientific literature; • formulating appropriate research questions and methodological approaches for their solution; • independently carrying out the project, as well as presenting the project and its results in writing and, if applicable, orally, within the context of the current state of research. The results should contribute to scientific knowledge.		
Objectives	The students: • are capable of familiarising themselves with a problem, which may extend to the frontiers of current research, within a given timeframe and independently developing a solution approach, • can increasingly apply appropriate scientific methods independently and present the results in a scientifically appropriate manner, • are able to independently work on a scientific topic while applying their mathematical methodological knowledge, • deepen their problem-solving skills and can transfer their methodological knowledge, • are able to present the results of their project to a professional audience.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Master Thesis	MT	o	-	15	yes	MT	-	g	100
	The coursework consists of an examination interview on the contents of the master thesis. The coursework is graded as pass or fail. The module grade is the grade for the master thesis. The module is only passed if the examination and coursework have been successfully completed.									
Transfer	-									
Prerequisites	Academic admission requirements for the master's thesis module include, in addition to the general conditions specified in the general part of the study and examination regulations the successful completion of at least one of the modules Specialisation 1 or 2.									
Responsible Persons	The dean of studies at the Department of Mathematics									

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

4 Courses for the Module Specialisation

4.1 Catalogue of Courses

The following lists the courses that can be included in the modules Specialisation 1 and 2 in Section 3 Mathematics. Other courses may be approved upon written request to the head of the examination board.

<table>
<tr><th>Course Title:</th><th colspan="3">Algebraic Curves and Riemann Surfaces</th></tr>
<tr><td>Specialisation</td><td colspan="3">Geometry</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Exercise class 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Compact Riemann surfaces.
• Normalisation of plane curves.
• Topological genus.
• Coverings.
• Forms and integration.
• Sheaves and cohomology.
• Hodge theory.
• Arithmetic and geometric genus.
• Abel’s theorem.
• Riemann-Roch theorem.
• Serre duality.
• Jacobian and Abelian varieties.
• Riemann bilinear relations.
• Jacobi inverse problem.
• Elliptic curves and functions.
• j-Invariant.
• Uniformisation.
• Topology of non-compact Riemann surfaces.</td></tr>
<tr><td>Special Objectives</td><td colspan="3">Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. By example of the concept of sheaves, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor’s programme is required:
• MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.</td></tr>
</table>

Literature	**Possible References :** • Frederice Mangolte: Real Algebraic Varieties. Springer 2020. • Robert Silhol: Real Algebraic Surfaces. Springer 1989. • Riccardo Benedetti, Jean-Jacques Risler: Real Algebraic and Semi-algebraic Sets. Editions Herrmann 1990. • Alex Degtyarev, Viatcheslav Kharlamov: Topological properties of real algebraic varieties: du côté de chez Rokhlin. arXiv:math/0004134.		
Responsible Persons	Daniele Agostini, Ivo Radloff		

Course Title:	Algebraic Topology 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Set theoretical topology. • Basic concepts of category theory. • The fundamental group of a punctured topological space. • Theory of covering spaces. • Basic concepts of singular homology theory. • Applications.		
Special Objectives	The students learn how to realise ideas in topology, e.g. the detection of holes in topological spaces, into a precise theory, even with a sophisticated technique. In particular, they recognise how abstract concepts, e.g. from category theory and homological algebra, provide effective ways of speaking that enable the formation of ideas to be adequately implemented.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	**Possible References :** • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.		
Responsible Persons	Anton Deitmar, Frank Loose		

Course Title:	Algebraic Number Theory		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Rings of integers. • Class numbers. • Dirichlet's unit theorem. • Extension of Dedekind rings. • Valuation theory. • Local fields. • Adeles and ideles.		
Special Objectives	The students have learned the central terms, results and methods of algebraic number theory.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.		
Literature	**Possible References :** • Jürgen Neukirch: Algebraische Zahlentheorie. Springer 2007. • Alexander Schmidt: Einführung in die algebraische Zahlentheorie. Springer 2007. • Andre Weil: Basic number theory. Springer 1995.		
Responsible Persons	Daniele Agostini, Anton Deitmar		

Course Title:	Algorithms of Numerical Mathematics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		

Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS
Content	Advanced, important algorithms of numerics (without differential equations) such as: • Fast Fourier transformation; • QR algorithms for the calulation of eigenvalues; • Method of conjugated gradients and more general Krylov space methods as iterative methods in numeric linear algebra and in non-linear optimisation; • Simplex method and interior point methods in linear optimisation.
Special Objectives	The students have learned the key concepts, results, and methods of algorithmic numerical mathematics.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Literature	**Possible References :** • Peter Deuflhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008. • Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg 2009.
Responsible Persons	Christian Lubich, Andreas Prohl

Course Title:	Automorphic Forms		
Specialisation	Analysis		
Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Modular forms for the module group and its congruence subgroups. • Examples: Eisenstein series, Ramanujan delta function, theta series. • Modular curves. • Arithmetic applications and conjectures. • Hecke operators. • The L-function of a modular form and its connections with elliptic curves.		
Special Objectives	Students have familiarised themselves with the central concepts and methods of the theory of automorphic forms in examples and are able to use them. They have understood the connection between modular, real representation theory and adelic L-functions.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Literature	**Possible References :** • Deitmar, Anton: Automorphic Forms. Springer 2012. • Goldfeld, Dorian: Automorphic forms and L-functions for the group GL(n,R). Cambridge University Press 2015. • Serre, Jean-Pierre: A course in arithmetic. Springer 1973.
Responsible Persons	Anton Deitmar

Course Title:	Bayesian Networks and Causality		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	Uncertainty is a fact of life, and robust artificial intelligence for real life application has to be able to deal with it. Therefore, the development of mathematical representations that effectively incorporate probabilities was a key step in the development of artificial intelligence. However, human understanding goes further than that: We go beyond observing events, such as *Whenever the sprinkler is on in my greenhouse, the plants are wet* or *Whenever the plants are wet, the sprinkler is on*, to postulating a relationship of cause and effect: *If I turn on the sprinkler, the plants will be wet, but if I water the plants, that will certainly not activate the sprinkler*. In this course, Bayesian networks are studied, which are a widely used representation for probability distributions. In particular, commonly used inference and learning algorithms are discussed. Moreover, it will be shown that Bayesian networks do not only represent probability distributions, but are also able to express causal relationships. Finally the causal expressivity of Bayesian networks will be looked at, aiming to learn causal structure from observational data. • Part I: Bayesian Networks as an Efficient Representation of Probability Distributions: – Computing probabilities using Bayesian networks. – d-Separation: A graphical criterion for probabilistic independence. – Parameter and structure learning in Bayesian networks. • Part II: Bayesian Networks as a Representation for Causal Knowledge: – Functional causal models: A representation of causal knowledge. – Pearl's causal ladder: Predicting the effects of external interventions and reasoning with counterfactuals. – Causal Bayesian networks. – Causal structure discovery: Learning causal relationships from data. – Counterfactual identifiability: Answering counterfactual. questions using causal Bayesian networks.
Special Objectives	In the first part of the course, students have learned how Bayesian networks represent probability distributions and how this representation can be used to efficiently compute probabilities or determine whether two random variables are independent. In the second part, students have learned to distinguish inference tasks in artificial intelligence according to Pearl's causal ladder: probabilistic, interventional, and counterfactual reasoning, which generally require increasingly detailed knowledge. They are familiar with identifiability results, which provide assumptions under which certain queries on the causal ladder can be answered using only knowledge from lower levels. For instance, they know how to answer queries about the effects of external interventions using only knowledge of the correct probability distribution, which can often be estimated from observational data.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Literature	**Possible References :** • Judea Pearl: Causality: Models, Reasoning and Inference. Cambridge University Press 2009.
Responsible Persons	Stephan Eckstein

Course Title:	Representation Theory of Finite Groups		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h

Frequency	not regularly
Language of Instruction	German
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS
Content	• Groups and group actions. • Representations, irreducibility, Schursch's lemma. • Semisimplicity, Maschke's theorem. • Characters, orthogonality relations. • Isotypical decomposition, character tables. • Representations of the symmetric group. • Semi-simple Artinian algebras.
Special Objectives	In the lecture, students learn the basic principles of of representation theory and develop an understanding for the interaction of geometric and algebraic methods. methods.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	**Possible References :** • William Fulton, Joe Harris: Representation theory. Springer 1991. • Bertram Huppert: Character theory of finite groups. De Gruyter 1998. • Serge Lang: Algebra. Springer 2002. • Jean-Pierre Serre: Linear representations of finite groups. Springer 1977.
Responsible Persons	Jürgen Hausen, Thomas Markwig

Course Title:	Introduction to Analytic Number Theory		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		

Content	• Arithmetic functions and Dirichlet series, • Prime number theorem and Dirichlet's prime number theorem, • Zeros of the Riemannian zeta function, Riemann hypothesis and the explicit formula.
Special Objectives	The students understand the interplay between analysis and number theory. They can apply analytical methods to number theoretic problems. They understand the mechanism of analytical continuation through integral representation and have learned to independently transfer it to other cases, such as automorphic L-functions. They have gained an understanding of the Riemann hypothesis, which is considered the most difficult problem of all math, and understand its depth.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.
Literature	**Possible References :** • Komaravolu Chandrasekharan: Introduction to Analytic Number Theory. Springer 1968.
Responsible Persons	Anton Deitmar

Course Title:	Introduction to Berkovich Geometry		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Non-Archimedean fields, valuations, and absolute value functions. • Ultrametric triangle inequality and induced topology. • Affinoid domains. • Berkovich affine and projective line. • Analytification of algebraic varieties.		
Special Objectives	The students have become familiar with the most important examples of non-Archimedean fields and their induced topology. They have gained an understanding of the fundamental challenges in developing a theory of analytic geometry over these fields and have studied Berkovich's approach to addressing these issues. The students have examined the projective line in Berkovich's framework in detail, both set-theoretically and topologically. In doing so, they have encountered a type of geometric space fundamentally different from other examples encountered in their studies (such as vector spaces, varieties, or manifolds). They are also familiar with the connections to algebraic geometry through the analytification functor.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Literature	**Possible References :** • Annette Werner: Nichtarchimedische Geometrie. Vorlesungsskript.
Responsible Persons	Hannah Markwig

Course Title:	Introduction to Mathematical Logic		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Propositional logic. • Languages of the first order: – Completeness and compactness. • Theory of computations: – Register machines; – Gödelisation. • Incompleteness of arithmetic: – First and second incompleteness theorem. • Set theory: – Ordinal- and cardinal numbers; – Incompleteness of set theory.		
Special Objectives	Students are able to understand mathematical theorems and theories in the context of mathematical logic. They understand the limits of possible mathematical knowledge, recognise the difference between truth and provability and can apply basic theoretical model thinking to mathematical content.		
Prerequisites	There are no prerequisites.		
Literature	**Possible References :** • Rautenberg, Wolfgang: Einführung in die Mathematische Logik. Vieweg+Teubner 2008. • Ziegler, Martin: Mathematische Logik. Birkhäuser 2016.		

Responsible Persons	Anton Deitmar

Course Title:	Introduction to set theory		
Specialisation	Analysis		
Workload **- Time in Class** **- Self-Study**	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	**Content:** •		
Special Objectives	-		
Prerequisites	There are no further prerequisites.		
Literature	**Possible References :** •		
Responsible Persons	Frank Loose		

Course Title:	Introduction to Optimisation		
Specialisation	Scientific Computing		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 3 SWS + Exercise class 1 SWS		
Content	• Optimality theory for smooth, convex and linear optimisation problems optimisation problems with constraints. • Foundations of the theory of convex sets and functions. • Duality theory for convex and linear optimisation problems. • Solution methods for linear optimisation problems.		

Special Objectives	Students know and understand methods and algorithms for solving convex and linear optimisation problems. They have learnt to apply the methods to simple problems related to economics, technology or physics. They will be able to critically assess the possibilities and limitations of using the methods.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	**Possible References :** • Florian Jarre, Joseph Stoer: Optimierung: Einführung in mathematische Theorie und Methoden. Springer 2019. • Jorge Nocedal, Stephen J. Wright: Numerical optimization. Springer 2006.
Responsible Persons	Christian Lubich

Course Title:	Introduction to Dynamical Systems		
Specialisation	Analysis		
Workload **- Time in Class** **- Self-Study**	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Kepler's laws. • Equilibrium positions. • Stability. • Predator-prey model. • Poincaré-Bendixson theorem. • Limit sets. • Periodic orbits. • Celestial mechanics.		
Special Objectives	The students can ask and examine qualitative questions about the solutions of ordinary differential equations, like e.g.: How long do mathematical solutions exist? Are there equilibrium states or periodic orbits? When are trajectories stable?		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations.		

Literature	**Possible References :** • Morris W. Hirsch, Stephen Smale: Differential equations, dynamical systems, and linear algebra. Academic Press 1974. • Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 2010. • Carl Ludwig Siegel, Jürgen Moser: Lectures on celestial mechanics. Springer 1995.
Responsible Persons	Frank Loose

Course Title:	Introduction to Geometric Measure Theory		
Specialisation	Analysis		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem. • Surface- and cosurface formula. • Countable rectifiable sets, rectifiable varifolds.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.		
Responsible Persons	Reiner Schätzle		

Course Title:	Introduction to Geometric Measure Theory – Measure Theoretic Methods		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.		
Responsible Persons	Reiner Schätzle		

Course Title:	Introduction to Geometric Measure Theory – Varifolds		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	• Surface- and cosurface formula. • Countable rectifiable sets, rectifiable varifolds.
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Responsible Persons	Reiner Schätzle

Course Title:	Introduction to Commutative Algebra and Algebraic Geometry		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Rings and ideals. • Gröbner bases. • Localization. • Noetherian rings and modules. • Integral ring extensions. • Krull's principal ideal theorem and dimension theory. • Hilbert's Nullstellensatz and Noether normalisation. • Affine varieties, Zariski topology, morphisms.		

Special Objectives	The students have become familiar with the central concepts, results, and methods of commutative algebra and affine algebraic geometry. They have experienced the profound interplay between algebra and geometry through the example of affine varieties. Furthermore, the students understand how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Literature	**Possible References :** • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Responsible Persons	Daniele Agostini, Jürgen Hausen

Course Title:	Introduction to Partial Differential Equations		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces. • L^2 theory. • Important examples (Laplace equation, wave equation, heat equation). • Fundamental solutions (elliptic situation). • Weak solutions of elliptic equations.		

Special Objectives	The students got to know a central branch of analysis, whose terms and methods are fundamental for many fields, like numerics or stochastics. Also evolutionary equations, who have strong connections to geometry, are issue of the lecture. The students are acquainted with central terms, results and methods of linear partial differential equations and are able to use these methods in advanced courses.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Literature	**Possible References :** • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Course Title:	Introduction to Partial Differential Equations – Part 1		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces.		
Special Objectives	The students have familiarised themselves with the first basic features of a central area of analysis, the concepts and methods of which are fundamental for many other areas, such as numerics and stochastics. Students are familiar with the central concepts, results and methods of linear partial differential equations and can successfully apply these methods in the more advanced courses.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	**Possible References :** • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Course Title:	Introduction to Riemann Surfaces		
Specialisation	Geometry		
Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Coverings and fundamental groups. • Topological classification of the surfaces. • Theorem of Riemann-Hurwitz. • Differential forms and integration. • Sheaves and cohomology. • Theorem of Riemann-Roch. • Serre duality. • Kobayashi metric. • Theorem of Picard.		
Special Objectives	Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. Using the sheaf concept, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.		

Literature	**Possible References :** • Hershel M. Farkas, Irwin Kra: Riemann Surfaces. Springer 1992. • Otto Forster: Riemannsche Flächen. Springer 1977. • Klaus Lamotke: Riemannsche Flächen. Springer 2009. • Jürgen Jost: Compact Riemann surfaces. Springer 2006.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Course Title:	Introduction to Stochastic Differential Equations - Part 1		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Introduction to Brownian motion and stochastic integration. • Solution concepts for stochastic differential equations. • Stability of stochastic differential equations. • Numerical approximation of stochastic differential equations.		
Special Objectives	Students master the basic principles and techniques for constructing solutions of stochastic differential equations.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.		
Literature	**Possible References :** • Bernt Oksendal: Stochastic differential equations. Springer 2000.		
Responsible Persons	Andreas Prohl		

Course Title:	Elementary Number Theory		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h

Frequency	not regularly
Language of Instruction	German
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS
Content	• Divisibility in the integers. • Prime numbers. • Congruences. • Quadratic residues. • Arithmetic functions. • Multiplicative functions. • Classical theorems. • Applications.
Special Objectives	Students deepen their basic knowledge of integers and experience applying this knowledge to mathematical problems of various kinds.
Prerequisites	Knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	**Possible References :** • Friedhelm Padberg: Elementare Zahlentheorie. Spektrum Akademischer Verlag 2001. • Stefan Mueller-Stach, J. Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg 2006.
Responsible Persons	Thomas Markwig

Course Title:	Elliptic Curves and Cryptography		
Specialisation	Algebra		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Basic concepts of cryptography. • Symmetric cryptosystems, public key systems, discrete logarithm, RSA. • Factorisation into primes, attacks on cryptosystems. • Basic concepts of plane projective geometry. • Elliptic curves as Abelian groups. • Curves over finite fields, Frobenius morphism, endomorphism ring. • Counting points, Hasse bound, Weil conjectures, Schoof's algorithm. • Cryptosystems on elliptic curves, algorithms and attacks.
Special Objectives	Students are familiar with the basic tasks and concepts of cryptography. They understand the cryptographically motivated questions relating to elliptic curves and have an insight into the advanced algebraic and geometric techniques for answering them. They understand the challenges of algorithmic implementation and are familiar with standard algorithms.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory, • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.
Literature	**Possible References :** • Albrecht Beutelspacher, Jörg Schwenk, Klaus-Dieter Wolfenstetter: Moderne Verfahren in der Kryptographie. Springer 2015. • Joseph H. Silverman: The arithmetic of elliptic curves. Springer 2009. • Ian Blake, Gadiel Seroussi, Nigel Smart: Elliptic curves in cryptography. CUP 1999.
Responsible Persons	Thomas Markwig

Course Title:	Explicit Mathematics		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		

Content	• Applicative theories. • Explicit mathematics. • Universes in explicit mathematics. • Applications in proof theory.
Special Objectives	Students are familiar with an alternative logical theoretical framework for arithmetic and subsystems of analysis and are familiar with their function in proof-theoretical investigations.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	**Possible References :** • Solomon Feferman: A language and axioms for explicit mathematics, in Algebra and Logic. Lecture Notes in Mathematics, 450, pp. 87-139, Springer-Verlag, Berlin, 1975. • Solomon Feferman: Constructive theories of functions and classes. In Logic Colloquium ?78, (Proc. Mons Colloq.), pp. 159-224, North-Holland, Amsterdam, 1979. • Gerhard Jäger, Reinhard Kahle, Thomas Strahm: On applicative theories. In Andrea Cantini, Ettore Casari, and Pierluigi Minari, editors, Logic and Foundations of Mathematics, pages 83–92, Kluwer, 1999. • Reinhard Kahle: The applicative realm. Textos de Matematica, 40, Departamento de Matemática, Universidade de Coimbra, 2007. • Gerhard Jäger, Reinhard Kahle, Thomas Studer: Universes in explicit mathematics. Annals of Pure and Applied Logic, 109(3),141-162, 2001.
Responsible Persons	Reinhard Kahle

Course Title:	Financial Mathematics and Numerics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	The course is designed to provide a comprehensive introduction to the mathematical models and numerical techniques that are essential for understanding and solving problems in modern finance. The course will delve into stochastic processes, modeling random phenomena in financial markets, and the numerical methods necessary for approximating solutions to complex financial equations. Key topics include the mathematics behind derivative pricing models, such as the Black-Scholes framework, and the use of stochastic calculus (SDE theory) in risk management and portfolio optimisation. By integrating theory with numerical practice, this course bridges the gap between financial mathematics and real-world applications, equipping students with the quantitative skills required for careers in quantitative finance, risk analysis, and financial engineering.		
Special Objectives	The students know important mathematical models for the description of problems in financial mathematics and can apply numerical approaches to their solutions in a targeted manner.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-11 / MAT-20-13 Numerical Mathematics, • MAT-20-12 Stochastics.
Literature	**Possible References :** • Steven Shreve: Stochastic Calculus for Finance. Springer 2005.
Responsible Persons	Andreas Prohl

Course Title:	Functional Analysis		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Normed spaces, Banach spaces, dual spaces. • Hahn-Banach theorem, uniform boundedness principle. • Closed graph theorem, open mapping theorem, Banach-Alaoglu theorem. • Compact operators, normal operators, spectral theorems.		
Special Objectives	The students are aquainted with the basic principles and techniques of the theory of infinte dimensional spaces and can apply them to problems in analysis and geometry. They understand the complexity of problems of spectral theory and can use its results for the solution of analytical problems.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	**Possible References :** • Nicolas Bourbaki: Topological vector spaces. Springer 1987. • Adam Bowers, Nigel Dalton: An introductory course in functional analysis. Springer 2014. • Harro Heuser: Funktionalanalysis. Teubner 2006. • Markus Haase: Functional analysis. American Mathematical Society 2014. • Peter D. Lax: Functional analysis. Wiley 2002. • Gert Kjaergaard Pedersen: Analysis now. Springer 1995. • Walter Rudin: Functional analysis. McGraw-Hill 1991. • Dirk Werner: Funktionalanalysis. Springer 2011. • Kosaku Yosida: Functional analysis. Springer 1995. • Hans Wilhelm Alt: Lineare Funktionalanalysis. Springer 2012.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Course Title:	Geometry of Manifolds 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Manifolds and submanifolds. • Vector fields and flows. • Metrics, foundations of Riemannian geometry. • Complex structures. • Theorem of Gauß-Bonnet on surfaces.		
Special Objectives	The students know and understand the fundamental concepts of real and complex differential geometry and the basic techniques for handling them. Especially they have deepened their understanding of differential and integral calculus and have exemplarily experienced how mathematical concepts are used in a natural way in geometry.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	**Possible References :** • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011.
Responsible Persons	Christoph Bohle, Frank Loose

Course Title:	Geometric Group Theory		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Group actions on graphs, free groups. • Quasi isometries. • Growth types. • Hyperbolic groups. • Ends.		
Special Objectives	Students learn to explore properties of finitely generated groups using geometric tools, starting from the Cayley graph of the group. They are able to investigate the geometric properties of the Cayley graphs with the help of analytical methods and to work out their connections to the underlying group. Students understand how algebra and analysis can work together to develop a new theory at the interface of algebra and geometry that leads to interesting statements about groups.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		
Literature	**Possible References :** • Clara Löh: Geometric Group Theory - an Introduction. Springer 2017. • Thorsten Camps, Volkmar Große Rebel, Gerhard Rosenberger: Einführung in die kombinatorische und die geometrische Gruppentheorie. Heldermann Verlag 2008.		
Responsible Persons	Hannah Markwig		

Course Title:	Geometry in Physics		
Specialisation	Mathematical Physics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	The module provides an introduction to fundamental methods of differential geometry and their relevance for physics. Particular topics are manifolds, differential forms, Riemannian metrics and associated notions of curvature, Riemannian geometry of submanifolds, real vector bundles, and connections. Applications of these concepts in Physics are discussed.		
Special Objectives	Students obtain knowledge, understanding, and acquaintance with the use of the listed notions of differential geometry. They develop, in particular, a deeper understanding of differential and integral calculus and experience through examples how the mathematical notions are naturally applied within physical theories.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	**Possible References :** • John Lee: Introduction to smooth manifolds. Springer 2012. • John Lee: Riemannian manifolds: An introduction. Springer 1997. • Chris Isham: Modern differential geometry for physicists. World Scientific 1999. • Mikio Nakahara: Geometry, Topology and Physics. IOP Publishing 2003.		
Responsible Persons	Christoph Bohle, Carla Cederbaum, Stefan Teufel		

Course Title:	Ordinary Differential Equations - Analysis and Numerics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Non-linear ordinary differential equations: Theorems of Hartman-Grobman and Poincare-Bendixson, bifurcation theory. • Numerical approximation: linear multi-step processes, adaptive processes, geometric integration.
Special Objectives	Students are familiar with the basic methods for studying qualitative behavior and for simulating solutions of non-linear ordinary differential equations. They have learned constructive methods for solving them and are in principle able to implement these with the help of the computer.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics. Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.
Literature	**Possible References :** • Lawrence Perko: Differential equations and dynamical systems. Springer 1993. • David Griffiths, Desmond J. Higham: Numerical methods for ordinary differential equations. Springer 2010.
Responsible Persons	Andreas Prohl

Course Title:	Graph Theory		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Basic concepts in graph theory, • Basic graph theory algorithms, • Flows, cuts, connectedness, matchings, • Cycle and cut space (cohomology theory), • Spectral graph theory, matrix tree theorem, • Planar graphs, theorem of Kuratowski and Wagner, • Planar embeddings, • Graph colorings, • Theory of minors.		

Special Objectives	Students know the basic concepts of graph theory, can analyse the structure of graphs and use graph theory methods in practice. They will also recognise connections to geometry and algebra and be able to benefit from them.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	**Possible References :** • Bela Bollobas: Modern graph theory, Springer, 1998. • John Adrian Bondy, Uppaluri Siva Ramachandra Murty: Graph theory, Springer, 2008. • Reinhard Diestel: Graph theory, Springer, 2018. • Jonathan L. Gross, Jay Yellen, Mark Anderson: Graph theory and its applications, CRC Press, 2019.
Responsible Persons	Elmar Teufl

Course Title:	Groups and Representations		
Specialisation	Mathematical Physics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Groups: subgroups, homomorphisms, isomorphisms, group actions, orbits, stabilisers, equivalence classes, normal subgroups, cosets, factor groups. • Representations: faithful, unitary and irreducible representations, reducibility, characters, Schur's lemma(s), orthogonality of irreducible representations. • Applications: symmetries and degeneracies in quantum mechanics, selection rules. • Representations of finite groups: group algebra, regular representation, ideals, idempotents. • Symmetric groups: Young tableaux, Young operators, dimensions and characters. • Applications: identical particles in quantum theories. • Lie groups: Haar measure, representations, Lie algebras. • Tensor representations of classical groups: symmetry classes, Young tableaux. • Applications: SU(2) and SU(3) in particle physics (spin, isospin, flavour) • Moreover a selection of the following: – Irreducible representations of the Lorentz and Poincaré groups. – Applications: notion of particles in quantum theories. – Roots and weights; Killing-Cartan classification of semi-simple Lie algebras		

Special Objectives	The studens know the basic concepts of group and representation theory. They are able to apply these abstract algebraic concepts in the context of theoretical physics and have, thus, developed a deepend understanding for the connections between mathematics and physics. The students are familiar with a number of complex examples of applications of the representation theory of groups in physics.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	**Possible References :** • Irene Verona Schensted: A course on the Application of Group Theory to Quantum Mechanics. NEO Press 1976. • Barry Simon: Representations of Finite and Compact Groups. AMS 1996. • Wu-Ki Tung: Group Theory and Physics. World Scientific 1985.
Responsible Persons	Stefan Keppeler

Course Title:	Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	Starting from a system of axioms for plane absolute geometry with the basic concepts of incidence and congruence, the associated Bachmann reflection geometry is developed. After the introduction of the hyperbolic axiom, this is continued with reflection-geometric end theory. A Euclidean field is created from the rotations around an end and the translations along a straight line, with the help of which the hyperbolic plane under consideration is described algebraically.		
Special Objectives	The students have learnt to look at one and the same mathematical object (in this case absolute and hyperbolic planes) from completely different perspectives and to link them together. In particular, they have learnt about Bachmann's group-theoretically oriented reflection geometry, which rarely appears in the curriculum, and thus deepen their knowledge of groups. They also deepened their knowledge about the interweaving of geometry and algebra.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures. Knowledge from the following modules is helpful but not strictly required: • MAT-20-06 Introduction to Geometry.		

Literature	**Possible References :** • Friedrich Bachmann: Aufbau der Geometrie aus dem Spiegelungsbegriff. Springer 1959. • Robin Hartshorne: Geometry: Euclid and beyond. Springer 2000. • Helmut Karzel, Kay Sörensen, Dirk Windelberg: Einführung in die Geometrie. Vandenhoeck und Ruprecht 1973.
Responsible Persons	Hermann Hähl, Hannah Markwig

Course Title:	Information Geometry		
Specialisation	Geometry		
Workload **- Time in Class** **- Self-Study**	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Foundations of Information Geometry (e.g. Fisher information metrics and dual relationships for parametric statistical models, Kullback-Leibler divergence, natural gradient). • Application to neural data processing (in particular supervised learning in artificial neural networks).		
Special Objectives	Students have an elementary understanding of how to apply concepts of differential geometry to problems in information theory and statistics.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	**Possible References :** • Shun-Ichi Amari, Hiroshi Nagaoka: Methods of Information Geometry. AMS 2001. • Anthony C. C. Coolen, Reimer Kuehn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005. • Shun-Ichi Amari: Natural Gradient works Efficiently in Learning. Neural Computation 1998. • Yann Ollivier: Riemannian Metrics for Neural Networks I - Feedforward Networks. Information and Inference, IMA 2015.		
Responsible Persons	Christoph Bohle		

Course Title:	Information Theory, Pattern Recognition, and Neural Networks
Specialisation	Stochastics

Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Introduction to information theory: – The possibility of reliable communication over unreliable channels. The Hamming code and repetition codes. • Entropy and data compression: – Entropy, conditional entropy, mutual information, Shannon information content. The idea of typicality and the use of typical sets for source coding. Shannon's source coding theorem. Codes for data compression. Uniquely decodeable codes and the Kraft-MacMillan inequality. Completeness of a symbol code. Prefix codes. Huffman codes. Arithmetic coding. • Communication over noisy channels: – Definition of channel capacity. Capacity of binary symmetric channel; of binary erasure channel; of Z channel. Joint typicality, random codes, and Shannon's noisy channel coding theorem. Real channels and practical error-correcting codes. Hash codes. • Statistical inference, data modelling and pattern recognition: – The likelihood function and Bayes' theorem. Clustering as an example. • Approximation of probability distributions: – Laplace's method. (Approximation of probability distributions by Gaussian distributions.) Monte Carlo methods: Importance sampling, rejection sampling, Gibbs sampling, Metropolis method. (Slice sampling, Hybrid Monte Carlo, Overrelaxation, exact sampling.) Variational methods and mean field theory. Ising models. • Neural networks and content-addressable memories: – The Hopfield network.		
Special Objectives	Students are familiar with fundamental concepts of information theory – such as entropy and noisy channel coding – and how these are applied in computing and approximating probabilities.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.		
Literature	**Possible References :** • David J.C. MacKay: Information Theory, Inference, and Learning Algorithms. CUP 2003.		
Responsible Persons	Stephan Eckstein		

Course Title:	Interacting Many-Body Quantum Systems
Specialisation	Mathematical Physics

Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Bosons and fermions. • Bose-Einstein condensation. • Mean-field theory. • Nonlinear Schrödinger equation. • Hartree- and Hartree-Fock theory. • Bogoliubov approximations.		
Special Objectives	Students have learned about the difference between Fermions and Boson and its impact on many body systems. They understand that, at cold temperatures, the fluctuations of the particle number in a certain area is significantly suppressed for Fermions. They understand how the Bosonic nature leads to the famous effect of Bose Einstein condensation and that, in certain scaling regimes, interacting many body systems can in good approximation be described by nonlinear evolution equations. Finally they are able to give higher order corrections and to calculate the leading order correlations between the particles.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	**Possible References :** • Elliott H. Lieb, Robert Seiringer, Jan Philip Solovej, Jakob Yngvason: The Mathematics of the Bose Gas and its Condensation. Birkhäuser 2000. • Niels Benedikter , Marcello Porta , Benjamin Schlein: Effective Evolution Equations from Quantum Dynamics. Springer 2016.		
Responsible Persons	Peter Pickl		

Course Title:	Commutative Algebra		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Rings and Ideals. • Localisation and local rings. • Noetherian and Artinian rings and modules. • Integral ring extensions and Cohen-Seidenberg theorems. • Krull's principal ideal theorem and dimension theory. • Primary decomposition. • Normality, regularity and discrete valuation rings. • Hilbert's Nullstellensatz and Noether normalisation.
Special Objectives	The students are familiar with and understand the language and methods of commutative algebra, which are essential for studying the fields of algebra, geometry, and number theory. They recognise how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Literature	**Possible References :** • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Responsible Persons	Daniele Agostini, Thomas Markwig

Course Title:	Convex Geometry		
Specialisation	Geometry		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Cones, polytopes, polyhedra, fans, polyedral complexes. • Normal fans of polygons. • Triangulations, subdivisions, secondary fans, discriminants.
Special Objectives	In the lecture the students learn basic terms, results and methods of convex geometry. They develope a deepened understanding for the concept of duality of mathematical objects on the example of polytopes and fans. Furthermore they enhance their geometric view and their spatial sense.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	**Possible References :** • Günter M. Ziegler: Lectures on Polytopes. Springer 1998.
Responsible Persons	Hannah Markwig

Course Title:	Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Brief review of key concepts and results from algebra and number theory. • Historical ciphers and their cryptanalysis (Caesar, Vigenere, substitution); encryption schemes. • Diffie-Hellman protocol and fast exponentiation. • Discrete logarithms: Shanks' algorithm and Pollard's rho method. • RSA: correctness, security, and attacks. • Signature schemes.		

Special Objectives	Students are familiar with the fundamental concepts and results of elementary number theory and algebra, as well as their application in cryptography. They can implement the methods covered in Python or SageMath in an exemplary manner and know what to pay attention to. Using classical ciphers, they understand typical strengths and weaknesses; they master the Diffie-Hellman protocol and are familiar with the man-in-the-middle attack. They can compute discrete logarithms in cyclic groups, understand the RSA scheme, and are able to interpret the recommendations of the Federal Office for Information Security (BSI). In various attack scenarios, they can identify weaknesses of RSA when the requisite conditions are not met. By engaging with numerous open problems in cryptography – whose solution approaches can, perhaps surprisingly, stem from very different areas of mathematics – students practise critical thinking. The exercises are central and support students in working independently and in a practice-oriented way, especially with CAS systems such as SageMath.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	**Possible References :** • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Christian Karpfinger, Hubert Kiechle: Kryptologie, Algebraische Methoden und Algorithmen, Vieweg 2010. • Dan Boneh, Victor Shoup: A Graduate Course in Applied Cryptography. 2023 (online Version: https://toc.cryptobook.us/). • Jonathan Katz, Yehuda Lindell: Introduction to Modern Cryptography. Chapman and Hall/CRC 2020.
Responsible Persons	Thomas Markwig

Course Title:	Lie Groups		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Manifolds and Lie groups, • Lie algebras and exponential map, • Covering spaces and classification of Lie groups by their Lie algebras, • Classical Lie groups, • Operations of Lie groups and homogeneous spaces.		

Special Objectives	Lie groups lie at the interface between geometry, algebra and analysis. They are suitable for describing the symmetries of geometric objects, but also algebraic equations or solutions of differential equations, in particular if these symmetries form a continuous set. The students learn from a prominent example how different disciplines of mathematics can work together very successfully and how a convincing formalism is developed that can precisely describe a variety of symmetry phenomena.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-01 Measure and Integration Theory, • MAT-20-04 Integration on Submanifolds of $\mathbb{R}^n$.
Literature	**Possible References :** • Joachim Hilgert, Karl-Hermann Neeb: Liegruppen und Lie-Algebren. Vieweg 1991. • Gerhard P. Hochschild: The structure of Lie groups. Holden-Day 1965. • Frank W. Warner: Foundations of differentiable manifolds and Lie groups. Springer 1983.
Responsible Persons	Anton Deitmar, Frank Loose

Course Title:	Linear Control Theory		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	Mathematical methods are indispensable for the management and control of complex systems and processes. The underlying theory is not only fascinating due to its diverse applications, but also, in its abstract form, due to the clarity and elegance of its methods and results. In this lecture, finite-dimensional systems are dealt with first, for which a good knowledge of analysis and linear algebra is sufficient. The aims are Kalman's controllability criterion and the resulting criteria for stabilisability. If there is enough time, we will extend the theory to infinite-dimensional systems. In the exercise classes we will apply the theory to concrete examples.		
Special Objectives	Students have learnt basic methods of linear control theory. At the same time, they have experienced and understood the interaction of various theoretical concepts from linear algebra and analysis and their benefits for specific applications.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		

Literature	**Possible References :** • Hans Wilhelm Knobloch, Huibert Kwakernaak: Lineare Kontrolltheorie. Springer 1985. • Jerzy Zabczyk: Mathematical Control Theory. Birkhäuser 1992. • Ruth F. Curtain, Hans Zwart: An Introduction to Infinite-Dimensional Systems Theory. Springer 1995.
Responsible Persons	Rainer Nagel

Course Title:	Mathematical Aspects of 3d Computer Vision		
Specialisation	Geometry		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	• Foundations of projective geometry. • Visual odometry. • Classical Simultaneous Localisation and Mapping (SLAM) methods and deep learning variants. • Computer-based experiments using the Robot Operating System (ROS2).		
Special Objectives	The students have acquired the mathematical foundations of 3D computer vision. They are familiar with modern architectures and the mathematical questions arising from them.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics. In addition, prior knowledge of deep learning is required, such as that provided in module MAT-50-14.		
Literature	**Possible References :** • Richard Hartley, Andrew Zisserman: Multiple View Geometry in Computer Vision. Cambridge University Press 2003. • Sebastian Thrun, Wolfram Burgard, Dieter Fox: Probabilistic Robotics. MIT Press 2005. • Luca Carlone, Ayoung Kim, Timothy D. Barfoot, Daniel Cremers, Frank Dellaert: SLAM Handbook: From Localization and Mapping to Spatial Intelligence. Cambridge University Press 2026.		
Responsible Persons	Christoph Bohle		

Course Title:	Mathematical Aspects of Deep Learning 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Artificial neural networks and their training using backpropagation/stochastic gradient descent. Probabilistic interpretation. Mathematical problems arising in neural network science. • Classical architectures for computer vision (CNNs) and sequential data (RNNs). (Self-)supervised learning methods. • Transformer models, a modern architecture based on the attention mechanism, and its applications in natural language processing (GPT) and computer vision. • Simple neuroscience models for neural networks. Biological plausibility of backpropagation and transformers.		
Special Objectives	The students have learned the fundamentals of information processing using artificial neural networks and biologically more plausible alternatives. They are familiar with contemporary architectures and mathematical questions arising in the framework.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.		
Literature	**Possible References :** • Ian Goodfellow, Yoshua Bengio, Aaron Courville: Deep Learning. MIT 2016. • Simon J. D. Price: Understanding Deep Learning. MIT Press 2023. • Christopher M. Bishop, Hugh Bishop: Deep Learning - Foundations and Concepts. Springer 2024.		
Responsible Persons	Christoph Bohle		

Course Title:	Theory of Mathematical Proofs		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		

Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS
Content	• Axiomatic theories, incompleteness. • Gentzen's proof of consistency for arithmetic. • Ordinal number analysis. • Provable recursive functions. • Predicative analysis. • Theories of inductive definitions.
Special Objectives	Students are familiar with the methods and results of modern proof theory, in particular with calculations for mathematical theories and their metamathematical properties.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	**Possible References :** • Wolfram Pohlers. Proof Theory. Springer 2009.
Responsible Persons	Reinhard Kahle

Course Title:	Mathematical Introduction to Data Science		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	The lecture provides a mathematically rigorous survey of core data-science techniques from regression and k-nearest neighbours to high-dimensional phenomena, dimensionality reduction, and modern classifiers (Perceptron, SVMs, Kernel Methods, Neural Networks). Emphasis is put on both theoretical foundations and hands-on Python implementation with real datasets.		
Special Objectives	The students can formulate and solve linear, polynomial and logistic regression models. They are able to implement k-nearest neighbours classification and build and evaluate classifiers (Perceptron, SVM (kernel methods), Neural Networks). Moreover, they know how to optimise convex objectives using gradient-descent techniques.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.		
Literature	**Possible References :** • Sven A. Wegner: Mathematical Introduction to Data Science. Springer 2024.		

Responsible Persons	Andreas Prohl

Course Title:	Mathematical Statistics		
Specialisation	Stochastics		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Statistical models, exponential families, sufficient statistics. • Rao-Blackwell theorem, Lehmann-Scheffe theorem, Cramer-Rao theorem. • Estimation methods, UMVU estimator, quality criteria, asymptotic behaviour of estimators. • Hypothesis testing, confidence interval, Neyman-Pearson lemma. • Testing methods, UMPU tests, 1- and 2-sample tests. • Models with growing density quotients, non parametric models. • Introduction in regression and variance analysis.		
Special Objectives	Students can model statistical relationships mathematically. They can mathematically construct, analyse, compare and apply statistical estimation and test methods and interpret their results.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics. Knowledge from the following modules is helpful but not strictly required: • MAT-75-01 Probability Theory.		
Literature	**Possible References :** • Peter J. Bickel, Kjell A. Doksum: Mathematical Statistics: Basic Ideas and Selected Topics. Chapman & Hall 2016. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Erich L. Lehmann, Joseph P. Romano: Testing statistical hypotheses. Springer 2005. • Erich L. Lehmann, George Casella: Theory of point estimation. Springer 1998. • Wiebe R. Pestman: Mathematical Statistics. De Gruyter 2009 • Helmut Pruscha: Vorlesungen über Mathematische Statistik. Springer Vieweg 2000. • Mark J. Schervish: Theory of Statistics. Springer 1995.		

Responsible Persons	Martin Möhle, Elmar Teufl

Course Title:	Nonparametric Estimation of Distributions		
Specialisation	Stochastics		
Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	This course explores a fundamental question in statistics: Given n data points from an unknown random vector, how well can we infer the true characteristics of the random vector, i.e., its distribution? In particular, we will focus on the role of the dimension of the vector, leading to a treatment of the curse of dimensionality. Further, we will explore how prior knowledge (e.g., smoothness or structural understanding of the random vector) can help overcome the curse of dimensionality. In contrast to parametric statistics, where the distribution is assumed to belong to a finite-dimensional family (e.g., estimating mean and covariance matrix of a normal distribution), the focus is purely on nonparametric settings, where no such parametric forms are given a priori. These settings require suitably chosen evaluation criteria based on distances between probability distributions, and we will treat both density estimation with L^p-type losses and estimation in Wasserstein distance.		
Special Objectives	The statistical learning problem for probability distributions can be formulated precisely. The importance of the chosen metric and of the dimension of the distribution is well understood. Students are familiar with the fundamental proof techniques for establishing both upper bounds, using explicit estimators, and lower bounds, using information-theoretic methods. They develop a structural understanding of why$(G!7(Bandwhich$(G!7(Bassumptions are necessary to achieve convergence rates that are better than those dictated by the curse of dimensionality. Through the exercise classes, students demonstrate their ability to work independently on topics covered in the lectures.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.		
Literature	**Possible References :** • Alexandre B. Tsybakov: Introduction to Nonparametric Estimation. Springer 2009. • Jonathan Niles-Weed, Quentin Berthet: Minimax estimation of smooth densities in Wasserstein distance. The Annals of Statistics 50.3 (2022): 1519-1540. • Nicolas Fournier, Arnaud Guillin: On the rate of convergence in Wasserstein distance of the empirical measure. Probability Theory and Related Fields 162.3 (2015): 707-738. • Daniel Bartl, Stephan Eckstein: Fast Wasserstein rates for estimating probability distributions of probabilistic graphical models. arXiv:2510.09270 (2025).		
Responsible Persons	Stephan Eckstein		

Course Title:	Numerics of Stationary Differential Equations		
Specialisation	Scientific Computing		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	Numerical covering of boundary value problems of stationary (i.e. time independent) ordinary and elliptic partial differential equations, with emphasis to the methods of finite elements.		
Special Objectives	The students have learned the central terms, results and methods of the numerical treatment of boundary value problems of stationary differential equations.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations, • MAT-20-11 / MAT-20-13 Numerical Mathematics. Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.		
Literature	**Possible References :** • Dietrich Braess: Finite Elemente. Springer Spektrum 2013. • Wolfgang Hackbusch: Theorie und Numerik elliptischer Differentialgleichungen. Teubner 1986.		
Responsible Persons	Christian Lubich		

Course Title:	Numerics of Stochastic Differential Equations		
Specialisation	Scientific Computing		
Workload **- Time in Class** **- Self-Study**	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		

Content	• Random number generator, Ito-Taylor expansion. • Strong and weak approximation, consistency. • Euler-Maruyama method, Milstein method, stochastic Runge-Kutta method. • Approximation of stopped diffusion processes.
Special Objectives	Students master the basic principles and techniques for the numerical approximation of solutions of stochastic differential equations.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Literature	**Possible References :** • Peter E. Kloeden, Eckhard Platen: Numerical solution of stochastic differential equations. Springer 1999.
Responsible Persons	Andreas Prohl

Course Title:	Optimal Control Theory with Ordinary Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Brief overview of existence and uniqueness theory for ODEs. • Numerical solutions to ODEs. • Introduction to optimal control problems with ODEs. • Existence and uniqueness theory for linear quadratic optimal control problems (LQ problems). • Pontryagin's maximum principle. • Numerical approximation of LQ problems.		
Special Objectives	Students are familiar with the problems of optimal control using ordinary differential equations and various approaches to solving the problem. They are also familiar with qualitative statements on unambiguous solvability.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations.
Literature	**Possible References :** • Matthias Gerdts: Optimal Control of ODEs and DAEs. De Gruyter 2012.
Responsible Persons	Andreas Prohl

Course Title:	Optimal Transport		
Specialisation	Scientific Computing		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• The course introduces the theory of optimal transport at the interface of analysis, probability, and optimisation. Starting from the classical problems of Monge and Kantorovich, the fundamental measure-theoretic and analytical tools are developed. • Building on this, central results are presented, including the existence and duality of optimal transport plans, c-cyclical monotonicity, Brenier's theorem, and the geometric structure of Wasserstein spaces. • In the second part of the course, the algorithmic perspective of optimal transport is explored. Topics include entropic regularisation, the Sinkhorn algorithm, and selected applications, for example in image processing, statistics, and machine learning.		
Special Objectives	The students are familiar with the fundamental concepts, methods, and key results of the theory of optimal transport and are able to situate these within a broader mathematical context spanning analysis, probability, and optimisation. They can understand the classical formulations due to Monge and Kantorovich, as well as essential structural concepts such as duality, c-cyclical monotonicity, and the geometry of Wasserstein spaces, and apply them in simple settings. Furthermore, they are acquainted with basic algorithmic approaches to optimal transport and can assess their relevance for selected applications. In the exercise classes, they deepen their understanding of the lecture material, further develop their ability to construct precise mathematical arguments, and practise the written and oral presentation of their own solution methods.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 Submodules Measure and Integration Theory, and Integration on Submanifolds of $\mathbb{R}^n$. • MAT-20-12 Stochastics.		

Literature	**Possible References :** • Alessio Figalli, Federico Glaudo: An Invitation to Optimal Transport, Wasserstein Distances, and Gradient Flows. EMS 2023. • Gabriel Peyré, Marco Cuturi: Computational Optimal Transport. Now Publishers 2019.
Responsible Persons	Nicolai Jork, Andreas Prohl

Course Title:	Optimisation with Differential Equations		
Specialisation	Scientific Computing		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Direct method in the calculus of variations, Euler-Lagrange equation. • Brouwer-Minty theorem, non-linear evolution equations. • Gateaux and Frechet differentiability. • Proof of existence of optimal controls, necessary optimality conditions. • Adjoint, convergent optimisation methods in Banach spaces. • Variational discretisation concepts.		
Special Objectives	Students master the basic principles and techniques for deriving optimality conditions for prototypical control problems with constraints in the form of partial differential equations.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics.		
Literature	**Possible References :** • Michael Hinze, Rene Pinnau, Michael Ulbrich, Stefan Ullrich: Optimization with PDE constraints. Springer 2009.		
Responsible Persons	Andreas Prohl		

Course Title:	Problem Solving Strategies
Specialisation	Algebra

Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Basic Techniques and Problem Reduction. • Proofs and Logic. • Sets and Functions. • Elementary Combinatorics. • Counting Techniques and Cardinality. • The Pigeonhole Principle (Box Principle). • Bounds, Estimation, and Inequalities. • The Extremal Principle. • The Invariance Principle: Invariants and Monovariants. • Proofs by Colouring. • Graph-Theoretic Modelling.		
Special Objectives	Students acquire a versatile toolkit of mathematical and heuristic problem-solving strategies that they can draw upon both in their subsequent studies and, where appropriate, in competition-oriented mathematical problem solving. This includes fundamental results and proof techniques, as well as the ability to recognise characteristic structural features of a problem that indicate suitable solution approaches or allow unsuitable strategies to be ruled out at an early stage.		
Prerequisites	Knowlegde from Linear Algebra and the submodule Algebraic Structures are assumed.		
Literature	**Possible References :** • George Pólya: How to Solve It: A New Aspect of Mathematical Method. Princeton University Press 1973. • Arthur Engel: Problem-Solving Strategies. Springer 1998.		
Responsible Persons	Thomas Markwig		

Course Title:	Propagation of Chaos		
Specialisation	Mathematical Physics		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		

Forms of Teaching and Learning	Lecture 4 SWS
Content	• Interacting many body systems (quantum and classical), importance of correlations. • Mean-field situations (e.g., Vlasov) and collisions (Boltzmann). • Explicit treatment of correlations. • Large deviations from the expected value.
Special Objectives	Students learn how different kinds of many-body systems can be described by effective, non-linear equations. They are able to distinguish and compare different types of convergence of microscopic many-body systems against the effective theory, both in classical and quantum mechanical situations. Based on an argument similar to the law of large numbers, they understand how the independence of particles leads to the effective equation. They learn to prove that independence is indeed preserved - at least approximately - under the evolution of time (propagation of chaos). Building on this, they understand various proof strategies adapted to the respective situation.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Literature	**Possible References :** • Louis-Pierre Chaintron, Antoine Diez: Propagation of chaos: a review of models, methods and applications. arXiv:2203.00446. • Francois Golse: Mean-Field Limits in Statistical Dynamics. arXiv:2201.02005.
Responsible Persons	Peter Pickl

Course Title:	Software Obfuscation: Mixed Boolean-Arithmetic Expressions		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	• What is software obfuscation and what is it used for? What techniques are there, and why is deobfuscation so difficult? What does a reverse engineer do? Introductory examples. • Mathematical foundations: ideals in $\mathbb{Z}$, greatest common divisor, Euclidean algorithm, time complexity estimates, rings and fields, computations in residue class rings, permutations, matrices and determinants, linear algebra over rings. • Boolean algebras and Boolean functions. • Fundamentals of mixed Boolean-arithmetic expressions (MBA). • Obfuscation using MBAs and permutation polynomials. • Deobfuscation using MBAs; analysis and comparison of state-of-the-art tools and their weaknesses. • Deobfuscation using program synthesis.
Special Objectives	The students are familiar with initial techniques in the field of software obfuscation and deobfuscation and understand the mathematical results required for this. They are able to apply these confidently.
Prerequisites	Knowlegde from Linear Algebra and the submodule Algebraic Structures are assumed.
Literature	**Possible References :** • Yongxin Zhou, Andrew Main, Yuxin Gu, Harrison Johnson. Information Hiding in Software with Mixed Boolean-Arithmetic Transforms. In: International Workshop on Information Security Applications (WISA). Springer, 2007, pp. 6175. • Bernhard Reichenwallner, Peter Meerwald-Stadler. Ecient Deobfuscation of Linear Mixed Boolean-Arithmetic Expressions. In: Proceedings of the 15th ACM Workshop on Articial Intelligence and Security (AISec 2022). ACM, 2022, pp. 119130.
Responsible Persons	Thomas Markwig

Course Title:	Game Theory		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The focus is on Nash and generalised Nash equilibrium problems and their numerical solution.		
Special Objectives	Students are familiar with the fundamental issues of game theory. They are familiar with analytical and numerical approaches to analysing them.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	**Possible References :** • Christian Kanzow, Alexandra Schwartz: Spieltheorie. Birkhaeuser 2018.		

Responsible Persons	Andreas Prohl

Course Title:	Statistical Learning 1		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Non-parametric regression, regression estimator. • (Universal) consistency. • Rate convergence. • Stone's theorem. • Kernel estimator, k-NN estimator. • Slow rate convergence, minimax convergence rates.		
Special Objectives	Students are familiar with basic non-parametric regression estimators, in particular with their universal consistency and rate convergence. They are familiar with the basic principles and methods of stochastic learning as required for machine learning applications.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.		
Literature	**Possible References :** • Laslo Györfi, Michael Kohler, Adam Krzyzak, Harro Walk: A distribution-free theory of nonparametric regression. Springer 2002.		
Responsible Persons	Andreas Prohl		

Course Title:	Stochastic Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		

Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS
Content	• Stochastic processes, filtrations, martingales. • Wiener process, random walk, Donsker's theorem. • Diffusion semigroup, Ito's integral. • Solution of a stochastic differential equation. • Markov property, Malliavin calculus, rough path theory.
Special Objectives	Students master the basic principles and techniques for constructing solutions of stochastic differential equations.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Literature	**Possible References :** • Bernt Oksendal: Stochastic differtial equations. Springer 2000.
Responsible Persons	Andreas Prohl

Course Title:	Theoretical Aspects of Machine Learning		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	The lecture covers some recent aspects of theoretical machine learning such as: • The theory of Reproducing Kernel Hilbert Spaces (RKHS). • Applications of RKHS theory such as SVMs, kernel regression, kernel PCA, and kernel mean embeddings. • Approximation capabilities of neural networks. • Dynamics of neural networks and the neural tangent kernel. • Recent advances in high dimensional statistics, in particular overparametrisation and generalisation.		

Special Objectives	The students learn the mathematical foundations of supervised learning theory, neural networks, support vector machines and kernel methods. They are familiar with fundamental modern topics in machine learning and with their theoretical basis, mathematical approach and conceptual tools as needed for the discussion and justification of algorithms.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Literature	**Possible References :** • Mehryar Mohri, Afshin Rostamizadeh, Ameet Talwalkar: Foundations of Machine Learning. MIT Press 2012. • Shai Shalev-Shwartz, Shai Ben-David: Understanding Machine Learning: From Theory to Algorithms. CUP 2014. • Peter L. Bartlett, Andrea Montanari, Alexander Rakhlin: Deep learning: a statistical viewpoint. Acta Numerica 2021. • Daniel A. Roberts, Sho Yaida, Boris Hanin: The Principles of Deep Learning Theory: An Effective Theory Approach to Understanding Neural Networks. Cambridge University Press 2022.
Responsible Persons	Andreas Prohl

Course Title:	Theory and Numerics for Constrained Optimisation Problems		
Specialisation	Scientific Computing		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	We start with the unconstrained convex minimisation problem (on spaces), and the gradient method with step size control according to Armeijo for the approximate calculation of a minimum, as well as its variants. The simplex method solves linear programmes on polyhedra. Central to this is the convex (non-linear) minimisation task on sets, and the characterisation of a minimum with (necessary) optimality conditions (tangent cone, linearised tangent cone, Abadie condition, Karush-Kuhn-Tucker conditions). In addition, numerical solution methods based on these theoretical concepts (interior points method, penalty methods, SQP method) are presented and analysed.		
Special Objectives	The participants have become familiar with relevant current algorithms for the optimisation of constrained optimisation problems: these include gradient methods with step size control, the simplex method, interior point methods, penalisation methods and the SQP method. You will be able to analyse the algorithms and compare their complexity.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-11 / MAT-20-13 Numerical Mathematics.		

Literature	**Possible References :** • Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.
Responsible Persons	Andreas Prohl

Course Title:	Topology		
Specialisation	Geometry		
Workload **- Time in Class** **- Self-Study**	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Review of metric spaces: closed sets, environment, continuity, complete metric spaces, compactness in metric spaces metric spaces. • Set-theoretic topology: topological spaces, continuity convergence, compactness, separation axioms. • Spaces of continuous functions: Urysohn's lemma and applications, Stone-Cech compactification, the theorem of Stone-Weierstraß, notions of convergence in functions, compactness in spaces of functions. • Baire's spaces and application of Baire's theory: Baire's function classes, existence theorems. • Outlook on algebraic topology.		
Special Objectives	Students have familiarised themselves with the central concepts, results and methods of set-theoretical topology and have understood that this theory can be used to describe many phenomena in different areas of mathematics. In this way, they link their knowledge of very different areas of mathematics.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	**Possible References :** • Felix Hausdorff: Grundzüge der Mengenlehre. Von Veit & Comp. 1914. • Boto von Querenburg: Mengentheoretische Topologie. Springer 2001. • Volker Runde: A Taste of Topology. Springer 2005.		
Responsible Persons	Ulrich Groh, Rainer Nagel		

Course Title:	Tropical Enumerative Geometry
Specialisation	Geometry

Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Enumerative geometry of algebraic curves, in particular in the plane. • Tropical enumerative problems and multiplicities. • Combinatorial methods, floor diagrams and lattice paths. • Correspondence theorems for curves in the plane through given points. • Tropical and classic Gromov-Witten theory in genus 0. • Real counts, Welschinger invariants and polynomial invariants. • Hurwitz numbers. • Tropical correspondences for Hurwitz numbers.		
Special Objectives	The students know basic terms, results and methods of enumerative geometry in the context of tropical geometry methods. They develop a deeper understanding of the possibilities and limitations of the tropical access in connection with more complex issues. Furthermore, they deepen their knowledge in the field of algebraic geometry towards modular spaces and Gromov-Witten theory.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		
Literature	**Possible References :** • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.		
Responsible Persons	Hannah Markwig		

Course Title:	Tropical Enumerative Geometry - Part 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	• Enumerative geometry of algebraic curves, in particular in the plane. • Tropical enumerative problems and multiplicities. • Combinatorial methods, floor diagrams and lattice paths. • Correspondence theorems for curves in the plane through given points. • Real counts, Welschinger invariants and polynomial invariants.
Special Objectives	The students know basic terms, results and methods of enumerative geometry in the context of tropical geometry methods. They develop a deeper understanding of the possibilities and limitations of the tropical access in connection with more complex issues. Furthermore, they deepen their knowledge in the field of algebraic geometry towards moduli spaces.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	**Possible References :** • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.
Responsible Persons	Hannah Markwig

Course Title:	Calculus of Variations		
Specialisation	Analysis		
Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Direct method of calculus of variations. • Euler-Lagrange equations. • Palais-Smale condition. • Mountain-Pass Lemma according to Ambrosetti-Rabinowitz.		
Special Objectives	In the first part of the course, students have learnt the direct method of calculus of variations, which is primarily used to prove the existence of weak solutions of partial differential equations, but also has applications in e.g. differential geometry. They have also acquired the necessary basics from functional analysis and partial differential equations and can also use these in a different context, e.g. geometric analysis. In the second part of the course, students learnt about a so-called mountain-pass lemma. With its help, they can analyse non-uniqueness in the existence of solutions of partial differential equations.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Literature	**Possible References :** • Michael Struwe: Variational Methods, Springer 2008. • David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order, Springer 1998. • Walter Rudin: Functional Analysis, Mc Graw Hill Education 1991.
Responsible Persons	Reiner Schätzle

Course Title:	Probability Theory		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Characteristic functions and additions to the central limit theorem. • Conditional expectations and further measure-theoretic foundations. • Markov chains and martingales in discrete time, classification, asymptotic behaviour, stopping times, stationarity, ergodicity. • Introduction to processes in contiuous time like Poisson processes and Brownian motion.		
Special Objectives	The students got to know the central terms results and methods of probability theory. They can model, analyse and interprete stochastic dependency structures of random quantities in a measure theoretically founded manner. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.		

Literature	**Possible References :** • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Richard Durrett: Probability, Theory and Examples. Cambridge University Press 2019. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Jean Jacod, Philip E. Protter: Probability essentials. Springer 2004. • Olav Kallenberg. Foundations of Modern Probability. Springer 2021. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2020. • David Meintrup, Stefan Schäffler: Stochastik. Springer 2005. • Albert N. Shiryaev: Probability-1. Springer 2016.
Responsible Persons	Martin Möhle, Elmar Teufl

Course Title:	Consistency Proofs		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	• Historical examples of the question of consistency (limits; parallel axiom; set-theoretic paradoxes). • Philosophical foundational programs (logicism; formalism; intuitionism). • The Hilbert program and Gödel's theorems. • Gentzen's transfinite consistency proof for number theory. • Alternative approaches to consistency (including Gödel's T). • Current situation of consistency proofs.		
Special Objectives	Students learn about the historical context from which the question of the non-contradiction of formal mathematical theories arose, as well as the relevant modern techniques for investigating this question mathematically. They are able to categorise the problem of non-contradiction in mathematics both historically and philosophically. In addition, they have acquired the mathematical tools to be able to comprehend the corresponding proofs of non-contradiction and, to a certain extent, to carry them out themselves.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		

Literature	**Possible References :** • Kurt Gödel: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatsh. f. Mathematik und Physik 38, 173-198 (1931). • Gerhard Gentzen: Die Widerspruchsfreiheit der reinen Zahlentheorie. Math. Ann. 112, 493-565 (1936). • Reinhard Kahle, Michael Rathjen (Hrsg.): Gentzen's Centenary: The quest for consistency. Springer 2015. • Reinhard Kahle, Michael Rathjen (Hrsg.): The Legacy of Kurt Schütte. Springer 2020.
Responsible Persons	Reinhard Kahle

Course Title:	Number Theory and Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• RSA cryptosystem, primality tests, AKS algorithm. • Factorisation methods, number field sieve. • Quadratic reciprocity in cryptography. • Evaluation of the discrete logarithm. • Dynamical systems and Pollard's rho algorithm. • Elliptic curve cryptography. • Lattices and post-quantum cryptography. • Zero-knowledge proofs, digital signatures and hash functions.		
Special Objectives	The students know the basic concepts of elementary number theory and their applications in cryptography. They have deepened and extended their knowledge about neighbouring disciplines: They encounter methods of the theory of dynamical systems and become acquainted with elliptic curves over finite fields. They understand how fundamental cryptographic protocolls are working. Through studying many open problems of crytography, whose solutions may suprisingly come from most distinct branches of mathematics, the students learn to think critically.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		

Literature	**Possible References :** • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Stefan Müller-Stach, Jens Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg+Teubner 2011. • Joseph H. Silverman, John T. Tate: Rational points on elliptic curves. Springer 1992. • Nigel Smart: Cryptography: An introduction. McGraw-Hill 2003. (online version: https://www.cs.bris.ac.uk/~nigel/Crypto_Book/). • Lawrence C. Washington: Elliptic curves: Number theory and cryptography. Chaman & Hall/CRC 2008.
Responsible Persons	Thomas Markwig