

Determination of the Measurement Uncertainty of the IAAT Projecting Mirror Test Stand and Improvement Thereof

Master Thesis

for the Acquisition of the Academic Degree
Master of Science (M.Sc.)

by **Joachim Zinßer**

High Energy Astrophysics Group
Institute for Astronomy and Astrophysics
Eberhard Karls University Tübingen

submitted on 14th August, 2019

Hiermit versichere ich, Joachim Zinker, dass die vorliegende Arbeit von mir selbstständig verfasst wurde und keine anderen als die angegebenen Hilfsmittel und Quellen benutzt wurden. Alle wörtlich oder sinngemäß aus anderen Werken übernommenen Aussagen sind als solche gekennzeichnet. Die vorliegende Arbeit war bisher weder vollständig noch in wesentlichen Teilen Gegenstand eines anderen Prüfungsverfahrens und die Arbeit wurde weder vollständig noch in wesentlichen Teilen bereits veröffentlicht.

Ort, Datum

Joachim Zinker

Abstract

It is the overarching goal of the present Master Thesis to develop an efficient and reliable system that examines optical parameters of spherical mirrors.

Special focus is placed on determining and improving the measurement uncertainty of an existing mirror test stand. This mirror test stand is designed to measure the reflectivity and the width of the *point spread function* (PSF) of spherical mirror facets that are used in *Imaging Air Cherenkov Telescopes* (IACTs). The central results of this thesis are predominantly based on experimental work. A secondary goal is the comprehensive automation of the mirror testing. This is achieved by a program that is not working completely autonomous; although it explains crucial steps to the experimentalist, whenever necessary.

The results of this work will be utilized to support the supply the *Cherenkov Telescope Array* (CTA) Consortium efficiently with affordable high quality mirror facets. As of now, it is possible for a technician to measure one mirror facet with an uncertainty of the reflectivity of 2% and an uncertainty of the PSF of 0.5 mm, within one hour.

Abstract

Das Ziel der vorliegenden Masterarbeit ist es ein effizientes doch zuverlässiges Verfahren für die Ermittlung optischer Qualitäten sphärischer Spiegel weiter zu entwickeln.

Insbesondere soll die Messgeneuigkeit eines bestehenden Teststandes, der im 2f-Verfahren, die Reflektivität und die Breite der Punktverteilungsfunktion von Spiegeln untersucht, ermittelt und gegebenenfalls verbessert werden. Die zentralen Ergebnisse sind fast ausschließlich experimentell ermittelt. Sekundäres Ziel dieser Arbeit ist eine weitestgehende Automatisierung des Testverfahrens. Dies wird durch ein Programm erreicht, das zwar nicht vollkommen autonom arbeitet, allerdings den Experimentator an entscheidenden Stellen konkret anleitet.

Die Erkenntnisse dieser Arbeit werden dabei helfen, das *Cherenkov Telescope Array* (CTA) Consortium effizient mit günstigen, qualitativ guten Spiegeln zu versorgen. So können nun Spiegelfacetten, selbst von angelerntem Personal, innerhalb einer Stunde mit einer Genauigkeit von 2% auf ihre Reflektivität und 0.3 mm auf die Breite ihrer Punktverteilungsfunktion hin untersucht werden.

Contents

1	Introduction	1
1.1	High-Energy Astrophysics	1
1.2	γ -Ray Induced Electromagnetic Showers	2
1.3	Imaging Air Cherenkov Technique	3
1.4	The Cherenkov Telescope Array	5
2	Mirrors for the Medium-Sized Telescope of CTA	9
2.1	Characteristics of a Spherical Mirror	9
2.2	Requirements on the Telescope Reflector	11
2.3	Requirements on the Mirror Facets	14
2.4	Different Mirror Designs	15
3	Mirror Testing Facility	17
3.1	The 2f-Setup	17
3.2	Requirements for the Mirror Testing Facility	17
3.3	The Rastering Mirror Test Stand	18
3.4	The Projecting Mirror Test Stand	20
3.4.1	The Light Source	21
3.4.2	The UV-Capable Camera	22
3.4.3	The Screen	24
3.4.4	Geometry of Light Sources, Screen and UV-capable Camera	25
3.5	Mirror Testing Procedure	30
3.6	Results of a Mirror Test	34
3.6.1	Data Analysis Software for the Projecting Mirror Test Stand	34
3.6.2	Uncertainties of the Analysis	37
3.6.3	Results of a Mirror Test Run	41
3.6.4	Comparison between Measurements	42
4	Investigations Concerning the Mirror Test Stand	45
4.1	CCD Camera Flat-Fielding	45
4.2	Image Scale	50
4.2.1	Determination via Testgrid	50
4.2.2	Determination via Distanzmeter	51
4.2.3	Comparison between the two Methods	52
4.2.4	Relevance of the Scale	52
4.3	Region of Interest around the Center of the Reflected Light Spot	55
4.4	Orientation between Camera, Screen and Light Source	57
4.5	Alignment of Light Source and Mirror	57
4.5.1	Isotropy of the Light Source	59

4.5.2	Center of the Reference Circle	60
4.6	Point-Like Characteristic of the Light Source	62
4.7	Stability of the Light Source Intensity	65
4.7.1	Simultaneous Measurements of Voltage Drop and Intensity	66
4.7.2	Theoretical Impact of a Difference in the Intensity	67
4.7.3	Measured Impact of a Difference in the Light Source Intensity	69
5	Outlook and Conclusions	71
5.1	Outlook on Possible Improvements of the PTS	71
5.1.1	Suggested Changes to the Setup of the PTS	71
5.1.2	Necessary Repetitions of Already Conducted Experiments	73
5.1.3	Suggested Means to Further Improve the Quality of the PTS	74
5.2	Conclusions	75
I	Outline of the Mirror Testing Procedure	77
II	Input for and Output from ACTAM	79
III	The Hand-Held Reflectometer	81
III.1	Description of the Reflectometer	81
III.2	Surface Reflectivity Measurement Procedure	82
III.3	Analysis and Results	84
III.4	Reliability of the Reflectometer	85
III.4.1	Uncertainty Caused by the Optical Fibers	85
III.4.2	Drift of the Lamp or Detector	85
IV	Features Visible on the CCD	89
V	Irregular Warm-Up Characteristic	91

List of Figures

1.1	Relation between the opacity of the Earth's atmosphere and the wavelength.	2
1.2	Different interaction processes between photons and medium.	3
1.3	The Cherenkov spectrum.	4
1.4	Photograph of H.E.S.S. telescopes CT5 and CT1.	5
1.5	Comparison between the planned sensitivity of CTA with other telescopes.	6
1.6	Comparison between the planned angular resolutions of CTA with other telescopes. . .	6
1.7	Spectral sensitivity of CTA.	7
1.8	Photograph of the prototype of a CTA MST in Berlin-Adlershof.	8
2.1	Sketch of light path geometry of a spherical mirror.	10
2.2	Sketch of rays reflected by a spherical mirror.	11
2.3	Sketch of a CTA MST mirror facet.	16
3.1	Photograph of the Rastering Mirror Test Stand.	19
3.2	3D-model of the Projecting Mirror Test Stand.	21
3.3	Photograph of the light source of the Projecting Mirror Test Stand.	22
3.4	Photograph of the UV-camera.	23
3.5	Spectral sensitivity of the UV-capable camera.	23
3.6	Reflectivity of Zenith Lite screen over the wavelength.	24
3.7	Sketch of an intersection between a light cone and a planar screen.	26
3.8	Overview of the mirror testing procedure with the <i>Projecting Mirror Test Stand</i>	30
3.9	Example of a picture of a reflected light spot.	32
3.10	Example of a reference picture.	33
3.11	Reflectivity over wavelength.	41
3.12	Θ_{80} -width over wavelength.	42
3.13	Reflectivity over wavelength for multiple mirror test runs.	43
4.1	Zero-exposure picture from the UV-capable camera.	46
4.2	Dark-frame from the UV-capable camera.	47
4.3	Histogram of a Dark-frame.	48
4.4	Flat-fielding correction matrix.	49
4.5	Grid used to determine the image scale.	50
4.6	Horizontal cut through a grid like in Figure 4.5.	51
4.7	Relation between the image scale and the distance between camera and screen.	53
4.8	Relation between the image scale and the reflectivity of a mirror.	54
4.9	Directed reflectivity over region of interest.	55
4.10	Photograph of reflected light spot.	56
4.11	Comparison of the reflectivity analyzed with and without geometric correction.	58
4.12	Histogram of the homogeneity of the reference light spot.	59
4.13	Calculated reflectivities using different reference-center positions.	61

4.14	Change of LED intensity with distance.	63
4.15	Change of all LEDs intensities with distance.	64
4.16	Relative intensity over ambient temperature for an LED.	65
4.17	Forward voltage over ambient temperature for an LED.	65
4.18	Development of the intensity over time, for all five LEDs.	66
4.19	Development of the temperature and the intensity of an LED over time.	67
4.20	Effect of the change in LED power output on the uncertainty of the reflectivity.	70
I.1	Sketch of the mirror testing procedure and the analysis.	77
III.1	Photograph of the hand-held reflectometer.	81
III.2	Raw data from a surface reflectivity test.	82
III.3	The nominal spectrum of the STAN-SSH mirror.	83
III.4	The patterns for the surface reflectivity tests.	84
III.5	Surface reflectivity measurement of the mirror MST MLT 124.	85
III.6	Comparison between the measurements with different optical fibers.	86
III.7	Change of the results from the hand held reflectometer with time.	87
III.8	Dark current measurement of the hand-held reflectometer.	87
III.9	Switch-on characteristic of the hand-held reflectometer.	88
IV.1	Crystal-like structure visible on some pictures.	89
IV.2	Droplet-like structure visible on some pictures.	89
V.1	Unexplained warm-up characteristic.	91

Preface

There is an image mostly lost for the modern human's eye. Although everybody, who had the chance to see the Milky Way, accompanied by the Magellanic Clouds, on a clear southern sky, understands instinctively what astronomy is all about. In a time with less light and less distraction, it is no surprise that *stargazing* is one of man's oldest pastimes and astronomy may as well be his first *science*.

The development of geometry and mathematics, the invention of clocks and calendars and the concept of an explainable natural world all followed. It is no coincidence that the *father of modern physics and the scientific method* was also the first man to use a telescope to look at the sky in 1610. Since then, the field of astronomy was always closely connected with technological advance.

In 1933, Karl Guthe Jansky did observations with radio waves, first detected by Heinrich Hertz in 1887, and thus, is the first astronomer to utilize another messenger than visible light. Observations using the energetic X-rays, detected by Wilhelm Conrad Röntgen in 1895, and the even more energetic γ -rays were just made possible by the developments in rocket technologies, from the 1940s onward, since the Earth's atmosphere is opaque in those energy bands.

Chapter 1

Introduction

1.1 High-Energy Astrophysics

High-energy astronomy is a broad term describing the study of the development of the cosmos and of celestial objects using messengers with energies significantly higher than visible light¹. The more energetic section of the electromagnetic spectrum is usually divided into ultra-violet light ($1\text{ eV} < E_{\text{UV}} < 100\text{ eV}$)², X-rays ($100\text{ eV} < E_X < 100\text{ keV}$) and γ -rays ($100\text{ keV} < E_\gamma$)³. Cosmic rays⁴, neutrinos and gravitational waves are also studied in high-energy astrophysics.

Since Earth's atmosphere becomes opaque for electromagnetic radiation with wavelengths lower than 300 nm (see Figure 1.1), ground-based telescopes are not feasible to observe them directly. It was necessary to find a way to detect the radiation outside of the atmosphere. Beginning in 1949, X-ray detectors are mounted on rockets, on suborbital flights, and on high altitude balloons to gather some minutes to several hours of observation time. The first successful satellite, equipped with an X-ray detector, was *SOLRAD 1* [Newell66, p. 64]. It was launched in 1960 and flew for several months. In 1978 followed with *HEAO-2* the first space born X-ray telescope that was fully able to produce pictures [Giacconi79, p. 540]. Currently, space telescopes like *Chandra X-ray Observatory* and *XMM-Newton* are the most effective method to conduct X-ray astronomy.

The story of γ -ray astronomy begun similar, in 1961, with the *Explorer 11* satellite [Kraushaar65, p. 846], but it quickly took a different path. Since γ -rays can have much more energy⁵, than X-rays, two difficulties occur intensified.

- All materials are nearly transparent for γ -rays. Therefore, it is not yet possible to build optics to focus them. This yields to telescopes with bad angular resolutions and small collection areas.
- Even for very bright sources, the γ -ray photon flux is small compared with fluxes in lower energies⁶.

¹Electromagnetic waves with a wavelength λ between about 350 nm and 800 nm are considered visible light. The photon energy of ultra-violet light with $\lambda_{\text{UV}} = 300\text{ nm}$ is about $E_{\text{UV}} \approx 4.1\text{ eV}$.

²Ultra-violet light is usually not considered to be part of the high-energy field because its generation processes and the optics necessary to observe it are not conceptually different than those of visible light.

³The energies of what is usually considered X-rays and γ -rays, respectively, overlap in a regime between 10 keV [Demtröder17, p. 53] and 100 keV [Demtröder17, p. 110]. The difference between X-rays and γ -rays is better defined with the origin of the radiation. Photons in the high-energy regime that are generated by purely electromagnetic means are called x-rays and those photons, generated by nuclear reactions or annihilation are called γ -rays [Demtröder17, p. 12, 40, 50-52, 66].

⁴In this context, cosmic rays are either leptons like electrons and muons or hadronic matter like protons and nuclei. Electrons are indistinguishable from γ -rays if observed from the ground.

⁵The lower boundary between X-rays and γ -rays is commonly set at a photon energy of 100 keV and they have no defined upper limit. As an example, γ -rays with $E_\gamma \approx 80\text{ TeV}$ were already observed, in the Crab Nebula [Aharonian04].

⁶The Crab Nebula flux, for example, is a factor 1000 higher for photons with an energy of 1 TeV than for photons with an energy of 10 TeV [Aharonian04].

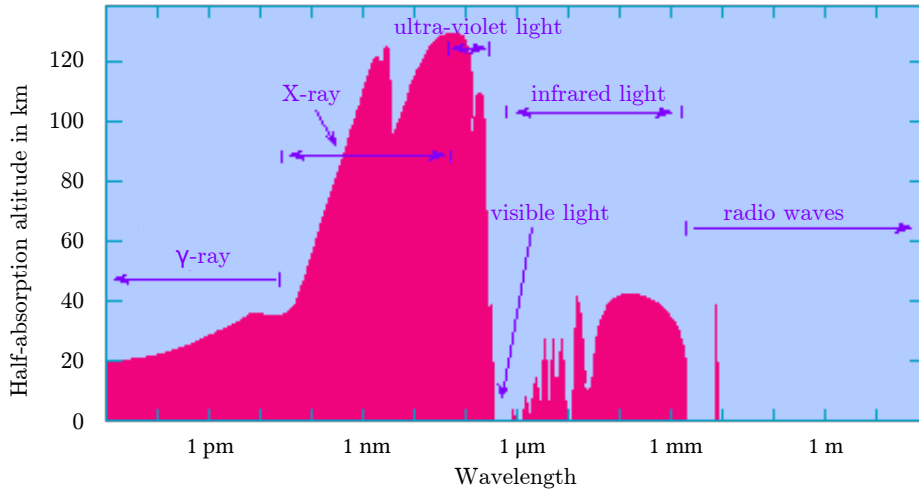


Figure 1.1: Half-absorption altitude of the Earth’s atmosphere over the different wavelengths of electromagnetic radiation. The half-absorption altitude states at which altitude half of all incoming intensity is absorbed. It correlates with the opacity. Modified by the author; originally obtained from the website of the *University of Rochester* www.pas.rochester.edu/~blackman/ast104/windows.html.

This necessitates either long exposure times or larger detectors. The size and therefore mass of a detector, mounted on a satellite, is limited by the current space faring technology.

The latter problem can be tackled with larger telescopes that are placed on Earth. Although the atmosphere is not transparent for γ -rays, they are still detectable from the ground because they emit secondary radiation when interacting with air.

1.2 γ -Ray Induced Electromagnetic Showers

When highly energetic radiation enters the atmosphere, it produces a cascade of secondary particles that are observable with a ground based optical telescope. This is true for γ -rays, electrons and hadrons. Since this thesis focuses on γ -ray astronomy, only γ -ray induced electromagnetic showers are explained in detail.

Several interaction processes can occur, when a photon enters a medium. Depending on the photon energy and the atomic composition of the surrounding medium, one process dominates the interaction. This is shown in Figure 1.2. In the high-energy regime of γ -rays, the interaction between photons and medium is dominated by pair production.

Pair production means that a single γ -photon produces an electron-positron-pair. This does not happen in vacuum because there must be a massive and charged entity, like a nucleus, to carry away momentum [Demtröder17, p. 87]. If the photon energy is larger than the rest mass energy of the electron-positron pair, the remaining energy is distributed as kinetic energy between the electron and the positron.

The electrons and positrons continue to travel through the atmosphere and their paths become diverted by the electric fields of nearby nuclei. This accelerates the electrons and positrons and causes them to emit bremsstrahlung [Demtröder17, p. 86]. Depending on the initial γ -photon energy, these bremsstrahlung-photons can be energetic enough to produce electron-positron pairs, in turn again. That yields to a cascade of electrons, positrons and γ -rays. To describe this particle shower, a simple model suffices [Heitler44, p. 234]:

- All interactions happen after a specific distance s , which is the average between the radiation

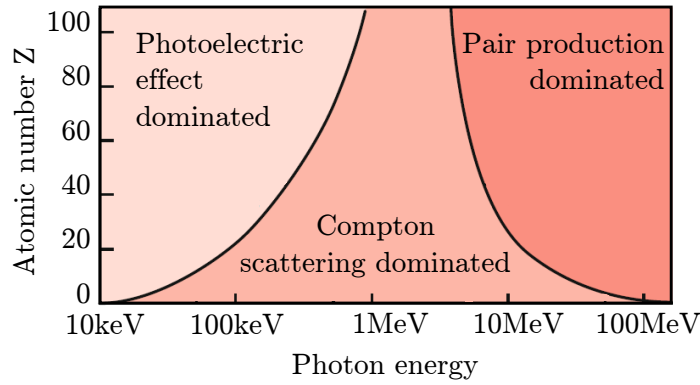


Figure 1.2: When electromagnetic radiation enters a medium, the dominant mode of interaction between photons and medium depends on the photons energy and the medium atomic number Z . Modified by the author; originally obtained from *Experimentalphysik 4* [Demtröder17, p. 88].

length of the electron⁷ x_{rad} and the mean free path of a γ -photon⁸ l .

- After an electron (or positron) travelled the distance s , it emits a photon with half its energy.
- After a photon travelled the distance s , it decays into an electron-positron-pair where both particles get half the photons energy.

A large fraction of the electrons or positrons have so much kinetic energy that they move faster than the speed of light in the respective medium. This is called superluminal motion and yields to a phenomenon similar to a sonic boom. The so-called *Cherenkov Effect* describes the light cone that follows the electron or positron. The superposition of Cherenkov light cones of the particles in a γ -ray induced shower generates an elliptical image from which the original direction and energy of the cosmic γ -photon can be reconstructed.

1.3 Imaging Air Cherenkov Technique

To observe γ -rays from the ground, it is necessary to observe the Cherenkov light of the electromagnetic showers, generated by them. As shown in Figure 1.3, this light is in the visible and near ultra-violet regime. Wavelengths between 300 nm and 550 nm are most relevant because shorter wavelengths are absorbed by the atmosphere and at longer wavelengths the background from the nightly sky dominates. Therefore, a Cherenkov telescope, bound to execute γ -ray astronomy, is actually an optical telescope.

The first *Imaging Air Cherenkov Telescope* (IACT) was the *Whipple 10 m Telescope* of the *Fred Lawrence Whipple Observatory*, built in 1968. It had a modified Davies-Cotton⁹ reflector with a diameter of 10 m and an angular resolution of about $15'$ [Weekes89, p. 393]. This is a fairly bad resolution for an

⁷The radiation length x_{rad} of an electron is the distance, after which the electron energy E_e has dropped to $\frac{E_e}{e}$ due to bremsstrahlung. In a medium with an atom density n_a and an atomic number Z , this distance becomes $x_{\text{rad}} = \left[\frac{4n_a Z^2 \alpha^3 \hbar^2}{m_e^2 c^2} \times \ln \frac{a(E_e)}{Z^{\frac{1}{3}}} \right]^{-1}$, where a is an experimental constant that describes the interaction radius of the nuclei [Demtröder17, p. 86].

⁸The mean free path l of a γ -photon with energy E_γ , in a medium with an atom density n_a and an atomic number Z , is $l = \left[n_a Z^2 r_e^2 a \left(3.11 \ln \left(183 Z^{-\frac{1}{2}} \right) - 0.074 \right) \right]^{-1}$ [Heitler44, p. 202].

⁹A Davies-Cotton telescope has a spherical reflector with a camera in the reflector focus. A modified Davies-Cotton reflector consists out of several spherical mirror facets with a radius of curvature r_{facet} that are mounted on a spherical support structure with a radius of curvature $r_{\text{structure}} = \frac{1}{2} r_{\text{facet}}$. The camera is then positioned in the center of the support structure sphere and therefore in the collective focus of the individual mirror facets [Weekes89, p. 380].

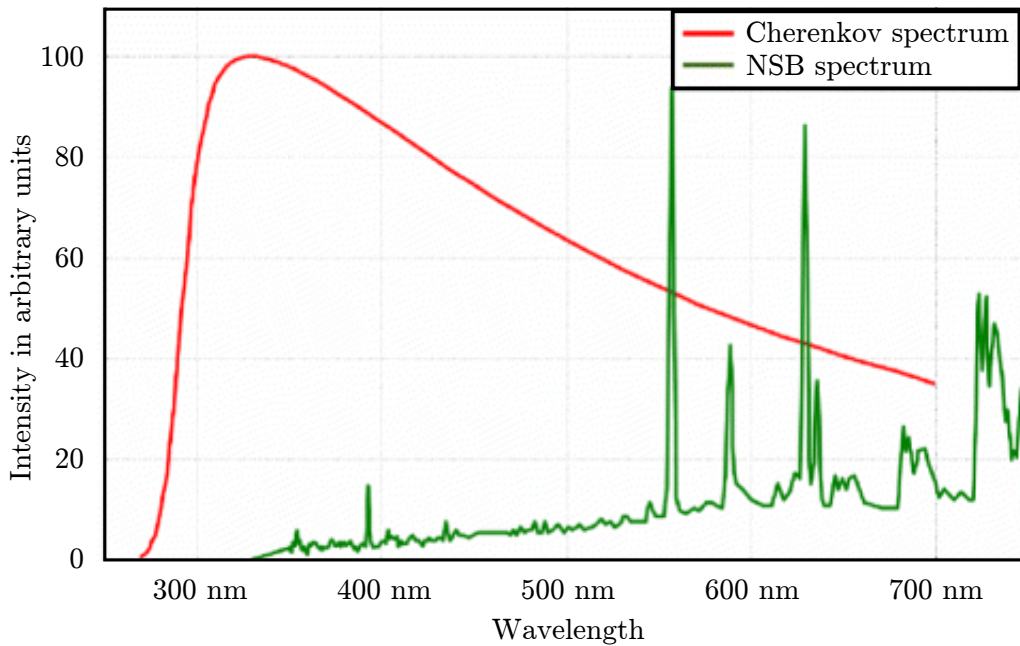


Figure 1.3: Cherenkov spectrum compared with typical night sky background (NSB) spectrum. Both in arbitrary units over the wavelength λ in nm. The Cherenkov spectrum is proportional to λ^{-2} with a cut-off at $\lambda = 300$ nm because of atmospheric absorption [Bouvier13, p. 2]. Modified by the author; originally obtained at [Bouvier13, p. 2].

optical telescope of its size ¹⁰, but in contrary to other optical telescopes that have to resolve point-like light sources, the diameter of particle cascades is in the order of 100 m and only a couple of kilometers above the telescope.

The time resolution is far more crucial for an IACT, since a typical Cherenkov shower only lasts for several 10 ns. The cameras for most IACTs are constructed as an array of *photomultiplier tubes* (PMTs). Those can be capable to resolve events on the nanosecond scale and detect even single photons.

This sensitivity comes in handy at detecting γ -rays with an energy of less than 1 TeV since the intensity of the Cherenkov shower depends on the photon energy of the γ -ray. The larger the reflector of an IACT, the lower is its photon energy threshold for detecting an event, because more intensity is collected and focused on the camera. At the *H.E.S.S.* experiment, for example, the addition of one 28 m telescope to the existing array of four 12 m telescopes lowered the threshold from about 1 TeV to some 10 GeV [Vincent05].

Since higher energy γ -photons produce brighter Cherenkov showers, they can easily be detected even with a small reflector. Unfortunately, the flux typically decreases with the increase in photon energy [Aharonian04]. The effective γ -ray detector area of an IACT is linked to the volume of atmosphere in its field of view. To detect more (bright) events at a low flux, it is beneficial to observe a larger volume of atmosphere. This is one of the reasons why IACTs are usually deployed in arrays of multiple telescopes. The other reason is an improvement in angular resolution if more than one telescope detects the same event; this makes it possible to reconstruct the three-dimensional shape of the particle cascade. This so-called *coincidence trigger* also helps to differentiate between Cherenkov showers from cosmic

¹⁰Compare: Even the resolution of the *Hale Telescope*, which was built in 1948, is mostly limited to $0.5''$ by the seeing of the atmosphere and not the optical quality of its 5.1 m reflector [Fienberg07].



Figure 1.4: Photograph of the telescopes of *H.E.S.S.*. On the left is the 28m telescope *CT5* and on the right is *CT1*, one of the four 12m telescopes. Picture taken at sunset, telescopes are in ready position.

γ -rays and those from other cosmic rays¹¹¹². Lastly, using multiple telescopes offers a degree of redundancy in the case of technical failures.

The first array of IACTs was built on La Palma in 1992. It was the *HEGRA* experiment, consisting, among others, of five 3.3m telescopes. The first experiment to expand its energy range by utilizing an array of IACTs of different sizes is the second phase of *H.E.S.S.*, beginning in 2012.

1.4 The Cherenkov Telescope Array

The *Cherenkov Telescope Array* (CTA) will be a γ -ray observatory operating the next generation of IACTs. It is predestined to surpass the sensitivity of all current IACT experiments by an order of magnitude (see Figure 1.5). In addition, the angular resolution of CTA will be significantly better than that of current generation IACTs (see Figure 1.6). To achieve those ambitious goals, scientists and engineers of 31 nations from all continents are planning to build up to 120 telescopes at two locations.

There will be at least three different types of telescopes to cover the planned energy range efficiently (see Figure 1.7). Currently, it is planned to build four 23m telescopes, so called *Large-Sized Telescopes* (LSTs), on the northern site on La Palma and four on the southern site in the Atacama desert at the *European Southern Observatory*. Those cover the lower part of the energy range. *Medium-Sized Telescopes* (MSTs) have an aperture of 12m. They cover the central part of the proposed energy range. Fifteen MSTs are planned for La Palma and 25 for Chile. For the highest energies, the 4m *Small-Sized Telescope* (SST) is in development. It is planned to build 70 SSTs in Chile but none on the northern site. This is, on the one hand side, due to space limitations but, on the other hand side, because all

¹¹The trajectories of charged cosmic rays are influenced by magnetic fields [Hillas96, p. 17]. Therefore, observing them does not produce useful information about the region on the sky that the telescope is aiming at. Since the fluxes of high-energy hadronic matter or muons are higher than that of γ -rays [Hillas96, p. 17] (which is in turn higher than the fluxes of high-energy electrons and positrons), it is important to remove from the data, at least the events, correlating with hadronic matter or muons. High-energy hadronic matter or muons that enter the atmosphere produce particle cascades of a different shape than high-energy electrons, positrons or γ -rays [Hillas96, 18-28].

¹²The website of the *Max-Planck-Institut für Kernphysik* has a simulation of different particle cascades and how different IACTs would observe them depending on their location: www.mpi-hd.mpg.de/hfm/CosmicRay/ChLight/ChLat.html.

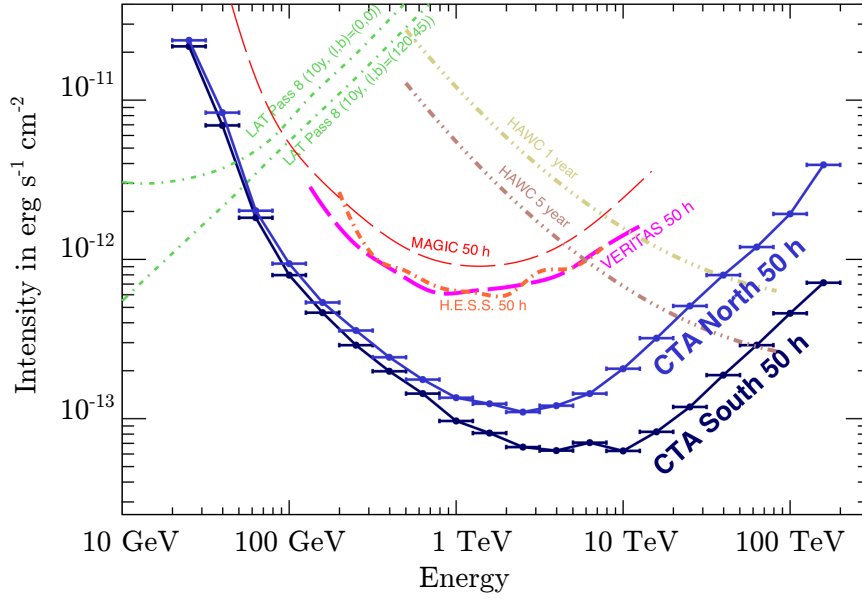


Figure 1.5: Differential sensitivity, interpreted as the minimal intensity of a source to be detected within 50 h and a significance of 5σ , over the γ -photon energy for different telescopes/arrays. It has to be noted that the significance curve of *H.E.S.S.* does not account for its 28 m telescope and that the significance curves of the two CTA sub-arrays are based on a simulation. Modified by the author; originally obtained from the official CTA-website www.cta-observatory.org/science/cta-performance/.

sources with spectra exceeding 10 TeV, solely visible with the SSTs, are galactic sources and therefore predominantly visible on the southern sky.

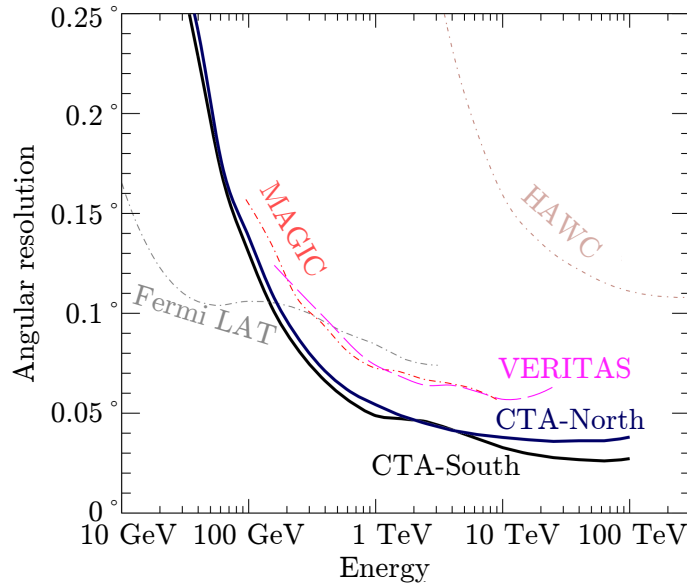


Figure 1.6: Angular resolution¹³ in degrees of different telescopes/arrays over the γ -photon energy. The resolution-curves of the two CTA sub-arrays are based on a simulation. Modified by the author; originally obtained from the official CTA-website www.cta-observatory.org/science/cta-performance/.

¹³In this case, the angular resolution is defined as the radius of a circle around the center of the reconstructed point that contains 68% of the intensity.

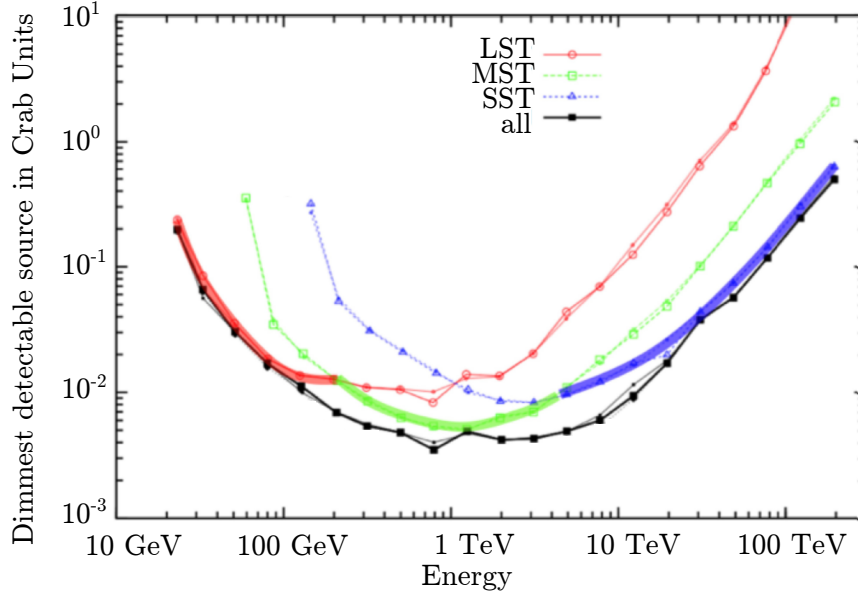


Figure 1.7: Spectral sensitivity of an exemplary array, comparable to CTA Southern Array and of its sub arrays. The sensitivity is indicated as the dimmest source, detectable in 50 h with a significance of 5σ , in Crab Nebula equivalent units¹⁴. Modified by the author; originally obtained from [Bigongiari16].

The Medium-Sized Telescope

To date, two general designs and three different Cherenkov cameras are in development for the MSTs¹⁵ for CTA. There is a modified Davies-Cotton design and a Schwarzschild-Couder design. This work emits from an institute working on the Davies-Cotton MST and therefore focuses on this design. The prototype can be seen in Figure 1.8.

The *Institute for Astronomy and Astrophysics Tübingen* (IAAT) is involved in the development of one Cherenkov camera type and the quality-evaluation for the mirrors and the mirror mountings of the MSTs.

A single MST carries 86 hexagonally shaped mirror facets and has an effective collection area of 88 m^2 [Garczarczyk18a, p. 2]. The reflector and the Cherenkov camera are mounted in an altitude-azimuth drive. The azimuth motors are located inside a cylindrical tower and on its rotating top are the altitude motors that hold the telescope head. The goal of the MSTs is to point at any source on the sky, with a precision of $7''$, within 90 s [Garczarczyk18a].

Each mirror facet is mounted on three points. Two of them are actuators and one is a ball joint of fixed length. This mounting enables the mirror facets to overlay their reflected light spots at the Cherenkov camera [Garczarczyk18a, p. 2]. The mirror support unit, holding the actuators and the joint, is connected to a steel structure, forming a section of a sphere with 19.2 m radius, that is stiff enough so that the mirror facets do not require readjustments during normal operations [Garczarczyk18a, p. 2].

The Cherenkov cameras for the MST utilize PMTs with high quantum efficiency. With at least 1750 pixels, „arranged in a hexagonal matrix of 50 mm spacing“, the Cherenkov camera will have a field of view of more than 7.5° [Garczarczyk18a, p. 3]. One of the two Cherenkov cameras, co-developed by the IAAT, is the so-called *FlashCam* that was also tested in the MST-prototype in Berlin-Adlershof (Figure 1.8).

¹⁴ $2.83 \times 10^{-11} \text{ TeV}^{1.62} \text{ cm}^2 \text{ s}^{-1}$ is considered a Crab Nebula equivalent source.

¹⁵Two Cherenkov camera designs are in development for the Davies-Cotton MST design and one for the Schwarzschild-Couder design.

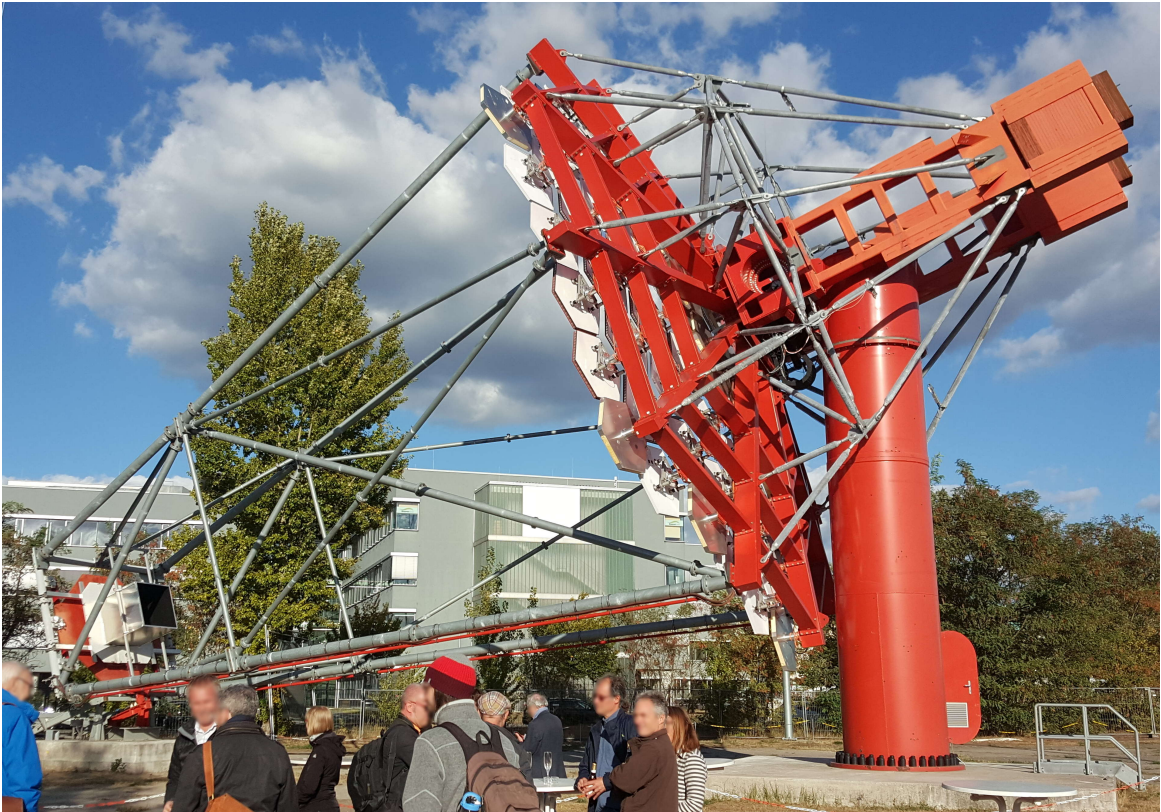


Figure 1.8: The prototype of an MST for CTA in Berlin-Adlershof. On this picture, it is not equipped with a Cherenkov camera, but prototypes of both Cherenkov cameras were already installed and tested with it. At the time of writing this document, some of its mirror facets are still dummies. The telescope is in its parking position.

Chapter 2

Mirrors for the Medium-Sized Telescope of CTA

This chapter focuses solely on the modified Davies-Cotton design of the MST. The Schwarzschild-Couder MST are not considered further, in the entire work. This chapter presents general information about spherical mirrors and specific information about the design of mirror facets that will be used in Davies-Cotton MSTs. Furthermore, the optical requirements for the Davies-Cotton MSTs and their mirror facets are explained here.

2.1 Characteristics of a Spherical Mirror

The MST utilizes spherical mirror facets. There are specific requirements, which have to be fulfilled by a mirror facet to be installed into a telescope. Some requirements refer to optical parameters like reflectivity or *Point Spread Function* (PSF) that are explained below.

Definitions Reflectivity and Point Spread Function

The reflectivity R is the ratio between the incoming intensity hitting the mirror I_{in} and the intensity reflected by the mirror I_{ref} .

$$R = \frac{I_{\text{ref}}}{I_{\text{in}}} \quad (2.1)$$

The PSF of an optical system describes the image resulting from an ideal point-like light source at infinity. Generally, the image of any object can be described as a convolution between the intensity-distribution of the object and the PSF of the optical system. Mathematically speaking, the PSF is a two-dimensional density-distribution that depends on the wavelength of the incoming light and the position of the image on the focal plane [Cornils06, p. 101, 102].

Focusing Properties of a Spherical Mirror

A spherical mirror reflects parallel light rays that enter the mirror not too far away from its principal axis¹ approximately into its focus (see the lime green arrow in Figure 2.2). The focus is defined as the equidistant point between the vertex² and the center of curvature that is on the principal axis. Parallel light that hits the surface of the mirror far away from the principal axis intersects this axis closer to

¹For a spherical mirror, the principal axis is not an intrinsic entity of the mirror but is rather defined by use. The principal axis becomes the line, connecting the center of curvature with the mirror surface that indicates the direction, in which the mirror is supposed to focus light.

²For spherical mirrors, the vertex is the point, where the principal axis intersects with the mirror surface. Often that is also the geometric center of that mirror.

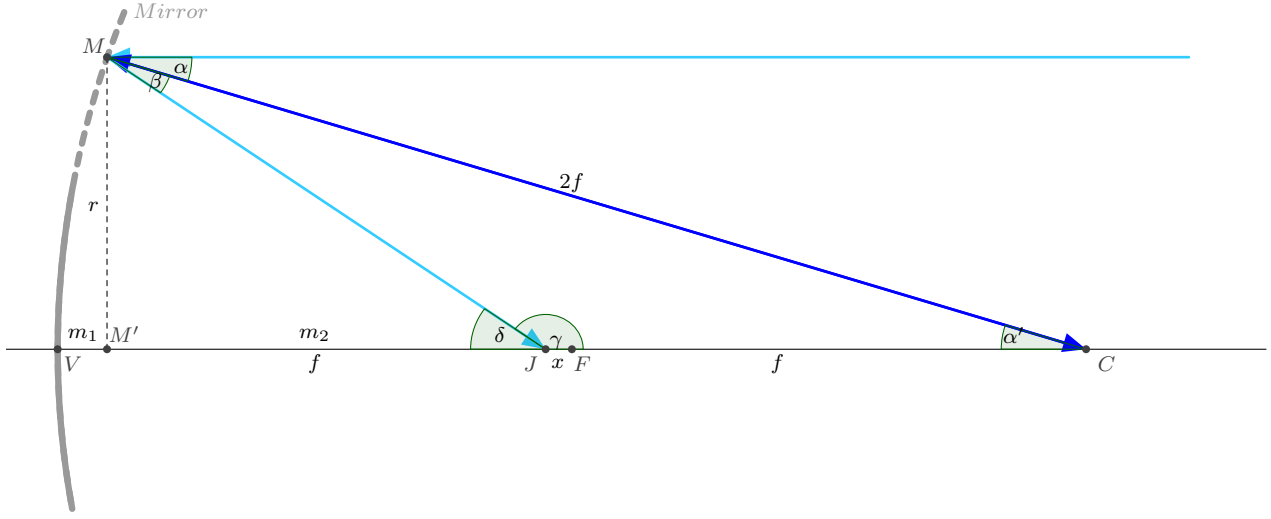


Figure 2.1: Light ray geometry of a spherical mirror with a focal length f . The dark blue arrows symbolize light rays from the center point C of the sphere. They hit all points M on the mirror surface orthogonally and are always reflected back into the same point C . The light blue arrows symbolize light rays that are parallel to the principal axis of the mirror. They hit the mirror surface under an angle α to the normal through the point M and are reflected under an angle β , with $\alpha = \beta$. The light rays are reflected into a junction point J on the principal axis. The junction has a distance x to the focus F of the mirror.

the mirror than the focus (see the light blue arrow in Figure 2.1 and Figure 2.2). The light ray that enters the mirror in a distance r to its principal axis intersects with this axis a distance x in front of the focus of the mirror. If the distance x is small, relative to the focal length f of the mirror, it can be neglected. Therefore, it can be approximated that the mirror reflects all parallel light from a circle with radius r around its principal axis into its focus.

To decide what radius r is acceptable for any given spherical mirror, it is necessary to calculate the distance x with regard to the radius r . The distance x can be obtained from the triangle $\triangle CMM'^3$ in Figure 2.1:

$$\begin{aligned}
 \cos \alpha &= \frac{m_2 + x + f}{2f} \\
 x &= 2f \cos \alpha - m_2 - f \\
 x &= 2f \cos \left(\arcsin \left(\frac{r}{2f} \right) \right) - r \tan \left(2 \arcsin \left(\frac{r}{2f} \right) \right) - f \\
 x &= \sqrt{4f^2 - r^2} - f - \frac{f - \frac{r^2}{2f}}{\sqrt{1 - \frac{r^2}{4f^2}}}
 \end{aligned} \tag{2.2}$$

The mirror facets used in a CTA-MST have a focal length of $f = 16.07$ m and a side-to-side diameter of 1.2 m [Garczarczyk18a]. The corners of the mirror facets are further from the optical axis but even with $r = 1.5$ m the distance x would just be 1.8 cm. Since this is about 0.1 % of the focal length of the mirror facet, the spread of their focal point can be neglected.

³ $\beta = \alpha$ because of the law of reflection and $\alpha' = \alpha$ because the incoming light blue arrow is parallel to the principal axis.

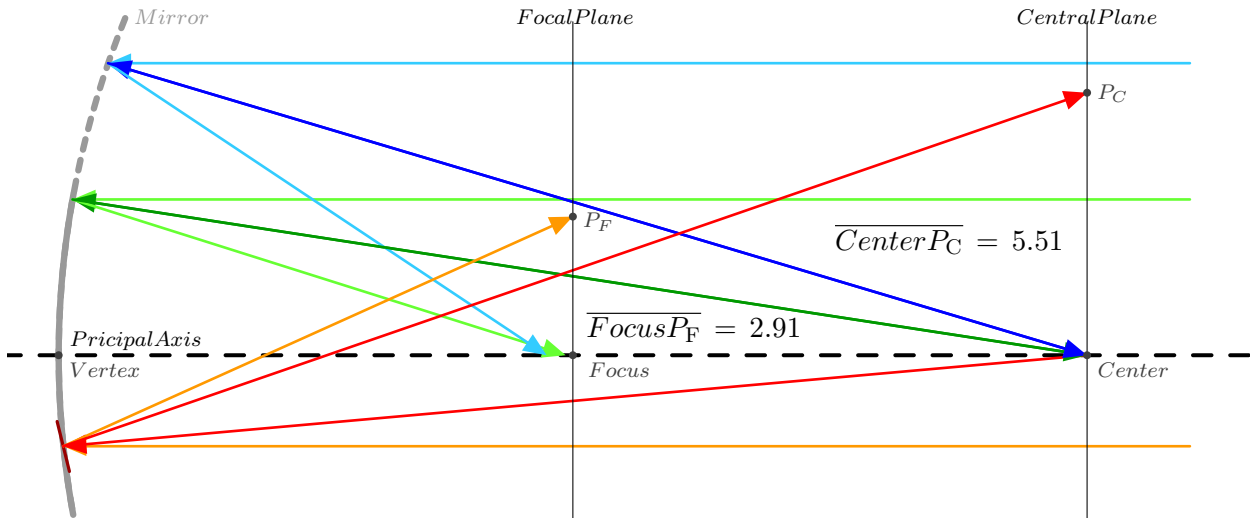


Figure 2.2: Drawing of rays reflected by a concave spherical mirror. The *Center* point is the center of curvature of the sphere and the *Focus* is precisely in the middle between the *Vertex* and the *Center*. The saturated colored arrows represent light rays from a point like light source in the *Center*. This light is reflected back into the *Center*. The pale colored arrows represent incoming parallel light rays that are reflected by the mirror. The lime green ray enters the mirror close to the *Principal Axis* and is reflected into the *Focus*. The light blue ray enters the mirror farer away from the *Principal Axis* and is not reflected into the *Focus*. Since the mirror facets of the MST are small enough to neglect this effect, this part of the mirror is only dashed, as a mirror facet would not be extended into that regime. The red and orange rays represent the effect of *micro-roughness*. Small sections of the mirrors surface (dark red area) are misaligned to its general shape. Light rays hitting these sections are not reflected into the *Focus* or *Center*. The same section of misaligned surface distorts central light rays about twice as far as parallel light rays (measured in the *Focal Plane* or *Central Plane*, respectively). In this sketch, the displacement of a central light ray from the *Principal Axis* in the *Focal Plane* is $\overline{FocusP_F} = 2.91$ ALU⁴ and the displacement of a central light ray from the *Principal Axis* on the *Central Plane* is with $\overline{CenterP_C} = 5.51$ ALU roughly twice as large. Their ratio depends on the distance of the reflection point to the principal axis. The limit of this ratio is exactly two.

Reasons for a Wide Point Spread Function

If the radius of curvature of the mirror facet differs from the nominal one of 32.14 m [Garczarczyk18a], it focuses the reflected intensity to its own focus. Therefore, the reflection at the nominal focus is spread out. This is one of the reasons why the PSF of the mirrors is not point-like. Further spread of the PSF can be traced back to the so-called *micro-roughness*. That describes the misalignment of small sections of the mirror facet surface (see the small dark red area in Figure 2.2). Intensity, hitting the mirror facet in these areas, is reflected according to their surface geometry (see the orange arrow in Figure 2.2). For a macroscopic observer, this intensity appears to be scattered randomly. Additionally, the mirror facet shape could differ from an ideal sphere.

2.2 Requirements on the Telescope Reflector

Because of the huge number of planned telescopes, the CTA Project Office has to find compromises between cost and performance. For the reflector of the MST, the following requirements were stated.

⁴ALU is an arbitrary length unit.

Shape and Size of the Reflector

After compensating for the shadowing effect of the Cherenkov camera and the holding structure, the MSTs are required to have a reflective surface of at least 88 m^2 [Garczarczyk18a, p. 2]. This is achieved with 86 identical⁵, hexagonally shaped, mirror facets, with a focal length of 16.07 m, that are mounted on a sphere with 19.2 m radius. The resulting reflector is similar to the design developed by Davies and Cotton. To achieve the original Davies-Cotton design, all mirror facets must have the same focal length, which is equal to the radius of curvature of the reflector. Such a reflector has spherical aberration. That means that even light, parallel to the principal axis of the telescope, is not focused exactly into the focus, if it is entering the reflector far away from its center. On the other hand, a spherical mirror produces less distortion than a parabolic reflector would do when light enters the telescope under an angle to the principal axis [CTA Consortium11, p. 268]. Lastly, the time that the light travels from the reflector surface to the Cherenkov camera is not the same for all points on a spherical reflector. This time dispersion is bearable if it does not exceed the intrinsic width of a Cherenkov shower of 3 ns [CTA Consortium11, p. 268] and therefore, only reflectors with diameters of up to 15 m are possible⁶ [Tayabaly15, p. 2].

Simulations convinced the CTA Project Office that a ratio F between focal length f and aperture d of $F = \frac{f}{d} \approx 1.4$ and a resulting field of view of 8° would work best for the MSTs [CTA Consortium11, p. 264]. The actual ratio of the CTA MST is with $F = \frac{f}{d} = 1.34$ close.

Reflectivity of the Reflector

On the internal website of the CTA Observatory jama.cta-observatory.org is a compendium of all requirements concerning CTA. They get updated regularly. At the time of writing this document, it was required that:

The initial average specular reflectivity of all Telescope Structure reflective surfaces must be $> 85\%$ at all wavelengths from 300 – 550 nm.
[JAM19, B-TEL-0120 Mirror Reflectivity - V3]

Only intensity with a wavelength between 300 nm and 550 nm is relevant for Cherenkov astronomy, because the atmosphere absorbs wavelengths below 300 nm and the night sky background begins to dominate at 550 nm [Bouvier13, p. 2] (see Figure 1.3 on Page 4).

In the case of the MST, all reflective surfaces of the telescope structure are mirror facets as described in the Sections 2.3 and 2.4. Hence, *average reflectivity* means that each individual mirror facet of an MST has to reflect 85 % of all the intensity that arrive anywhere on the mirror facet surface. The mirror facets have to fulfill this requirement only at the time of their installation into the reflector.

In this context, *Reflectivity* is defined as:

The fraction of photons incident on an optical element (facet) of a reflector dish, that are focussed [sic] into the required cFoV [camera field of view]. The mirror reflectivity does not include any effects from shadowing by, for example, the support structure of either incoming or outgoing light.
[JAM19, CTA-GLOS-204 Mirror Reflectivity - V1]

That means that it is possible to measure the reflectivity under ideal circumstances, since no effects from the mounting of the mirror facet are to be recognized. Furthermore, that means that not all reflected intensity has to be used to calculate the reflectivity of the mirror facets, but only the portion

⁵It is cheaper to mass-produce identical mirror facets. For parabolic reflectors, this is only possible to a lesser extend and therefore they are more expensive.

⁶Larger IACTs like *H.E.S.S. CT5* with a reflector with an effective aperture of 28 m, the 23 m LSTs or the 17 m *MAGIC* telescopes have to be realized as parabolic reflectors [Tayabaly15, p. 2].

of the intensity that is reflected into an area that is called *camera field of view* (cFoV). This area is defined as:

The Field of View of a CTA Camera is defined as twice the average angular distance of the centre of gravity of all Camera Pixel centres (i.e. the Camera centre) to the centre of all Edge Pixels.

[JAM19, CTA-GLOS-195 Camera Field of View (cFoV) - V2]

The center of gravity of all camera pixel centers is the center of the camera. The average distance of all pixels that sit on the edge of the camera to the center of the camera can be interpreted as the radius of the camera. The cFoV is the portion of the telescope reflector field of view that is occupied by the Cherenkov camera. For a *FlashCam* that is mounted in an MST, the cFoV is 7.5° [Garczarczyk18a, p. 3].

Conclusively, the telescope have to reflect at least 85 % of all incoming light (with wavelengths between 300 nm and 550 nm) into an area of 3.25° around the principal axis of the telescope reflector.

Point Spread Function of the Reflector

However, the direction of the reflected intensity is important: The PSF of any optics describes the image a point-like object would produce. The width of the PSF is the diameter of a circular region that contains a specific percentage of the point-like light source intensity.

The MST Telescope must focus light on to the focal surface with an optical PSF Θ_{80} of < 0.18 degrees, up to 2.8 degrees from the centre of the cFoV.

Note that this should be satisfied for all elevation angles above 30° .

[JAM19, B-MST-0130 Optical PSF Quality - V3]

The PSF of optics depend, among others, on the angle of the incoming light. For the MST reflectors, the PSF is defined for light entering the reflector under an angle of up to 2.8° to the center of the cFoV, which is on the principal axis of the reflector. The width of the PSF is defined for a circular region containing 80 % of the reflected intensity. The so-called Θ_{80} -width:

Theta₈₀ (Θ_{80}) is the standard parameter for characterising [sic] the optical PSF of a telescope. It is the opening angle (diameter) relative to the light centroid at a specific place within the focal plane, within which 80 percent of those photons that are reflected into a 1° diameter circle on the camera fall. Unless otherwise specified, the source of light should be assumed to be at infinity. Photons in the wavelength range 300 – 550 nm with the Cherenkov Reference Spectrum. Operating Illumination should be assumed.

[JAM19, CTA-GLOS-202 Theta₈₀ - V4]

Precisely, the Θ_{80} -width is the opening angle of a light cone from the reflector to a specific point on the focal plane. The specific point on the focal plane is the geometric center of the reflected intensity distribution on the focal plane. This light cone has to contain 80 % of the intensity that is reflected into any one circular area with a diameter corresponding to an angle of 1° that is completely inside the cFoV.

Therefore, the reflector is required to reflect 80 % of the intensity that are reflected into a region with an opening angle of 1° into a region with an opening angle of 0.18° ⁷.

⁷An opening angle of 0.18° correspond to 50 mm on the focal plane of the reflector. That is the pixel diameter of Cherenkov camera [Garczarczyk18a, p. 3].

Spatial Resolution of the Reflector

The required spatial resolution, of an IACT depends on its energy threshold: An IACT has to be able to distinguish between γ -ray induced particle cascades and those from other sources. The widths of those particle cascades depend on their energy. A γ -ray with a photon energy of 1 TeV generates a particle cascades with a width of about 0.15° whereas the width of a hadron induced particle cascades is twice as large. At lower energies, 50 GeV for example, the widths of particle cascades from γ -rays or hadrons are much smaller, 0.05° and 0.07° , respectively [Garczarczyk16, p. 39].

Construction and Realization of the Reflector

The PSF of the reflector depends not only on the individual PSFs of the mirror facets and the accuracy of their alignment with one another, but also, on the relation between the real reflector shape and its ideal shape. It reduces the production complexity and cost if the spherical form of the reflector is approximated with straight steel beams. To ensure that the impact of this approximation on the PSF is small with regards to the overall spot size, it is required to place mirror facets not farther from their ideal position than 10 cm (measured in the direction of the principal axis) [CTA Consortium11, p. 271].

The mirror facets have to be so precisely aligned that they differ from their ideal orientation not more than $21''$ (that corresponds to one thirtieth of the required PSF) [CTA Consortium11, p. 276]. It is important to mount the mirror facets in a way that they, once aligned, keep their orientation to a fraction of the PSF, regardless of external influences like wind, temperature or the orientation of the telescope⁸. MSTs are constrained to stabilize the orientation of the mirror facets at least as good as this is done at *H.E.S.S.* that achieves a play of $30''$ within operational conditions [CTA Consortium11, p. 271].

2.3 Requirements on the Mirror Facets

Although the required optical performance of the whole mirror system is crude in comparison with contemporary optical telescopes [CTA Consortium11, p. 272] it is still important to produce the mirror facets according to specific standards. All mirror facets for (modified) Davies-Cotton MSTs are identical. This is a benefit, since they can be mass-produced and installed interchangeably into the reflector. That curbs the cost of the, in total, several 1000 m^2 of mirror-surface for CTA.

It is cheaper to build the mirror facets as large as possible because there would be fewer individually packed sensible mirrors, fewer actuators and a less complex support structure. Unfortunately, the optical performance of a Davies-Cotton reflector worsens with the size of the mirror facets [Bretz13, p. 5]. It was decided to use hexagonal mirror facets with a side-to-side diameter of 1.2 m [CTA Consortium11, p. 272].

The optical performance of the reflector depends on the performance of its mirror facets. Therefore, the mirror facets are required to reflect at least 85 % of the incoming intensity, between 300 nm and 550 nm into the cFoV [JAM19, B-TEL-0120 Mirror Reflectivity - V3]. It is required, that the width of the PSF of the mirror facets is $\Theta_{80} \leq 12\text{ mm}$, measured in the central plane of the mirror facet [Garczarczyk18b, p. 8]. A circle with a diameter of 12 mm in the central plane of the mirror facet is equivalent to a circle with a diameter of 6 mm in the focal plane or a cone with an opening angle of about $0.021^\circ = 1.28'$. The PSF has to be narrower than what is required of the whole reflector since the alignment of the mirror facets and optical aberrations of the reflectors form add uncertainty to the system.

Usually, IACTs do not have a dome to protect them. The reflective surface of the mirror facets needs to be resilient against several environmental threads. The reflective surface must be able to survive temperatures between -15°C and 35°C with an hourly gradient of up to 7.5°C [JAM19, B-ENV-0220

⁸The width of the PSF of the reflector is defined for all telescope elevations higher than 30° [JAM19, B-MST-0130 Optical PSF Quality].

and B-ENV-0230]. The mirror facets (and their mountings) have to be sturdy enough to withstand an average wind speed of up to 100 km h^{-1} , in dry condition [JAM19, B-ENV-0730 and B-ENV-0740]. Should there be precipitation, too, wind speeds of 90 km h^{-1} have to be survivable [JAM19, B-ENV-0430]. The mirror facets and their mountings need to be able to carry the additional weight from snow (up to 1 m for the telescopes in the northern array and up to 10 cm for the telescopes in the southern array) or ice (up to 2 cm) [JAM19, B-ENV-0520, B-ENV-0525, B-ENV-0620 and B-ENV-0625]. The reflective surfaces must not abrade from rain, hail or sand storms; hailstones can have 5 mm in diameter [JAM19, B-ENV-0530]. Under these conditions, the reflectivity of the mirror facets are only allowed to degrade 4 % per year [JAM19, B-TEL-0125 Mirror Degredation - V4]. In the span of six years, every mirror facet has to be re-coated [JAM19, A-GEN-2560 Optical Performance Restoration - V3].

Whilst according to all optical and environmental requirements, the mirror facets must also be lightweight and self-supporting. To keep the complexity of the mounting as low as possible the mass of a MST mirror facet (without its mechanical mountings) is limited to 18.7 kg [Garczarczyk18b, p. 12].

2.4 Different Mirror Designs

A normal mirror consists of a reflective layer of aluminium behind a glass sheet. The backside of the mirror is covered with a lacquer to protect the aluminium from oxidation. The glass adds structural integrity to the mirror and it protects the aluminium surface, too. There are two problems with this *consumer grade* design when it comes to astronomy: It is generally undesirable that both, the aluminium and the glass, are reflective surfaces. Although that enhances the total reflectivity of the mirror, it creates two images of every object, thus broadening the PSF of the mirror. In Cherenkov astronomy, especially, it is undesirable that the layer of glass absorbs ultra-violet light.

To tackle those issues, the classical mirror design is turned around. There is still a polished glass sheet in use but it has the reflective aluminium coating on the front side. Those mirrors are proven to work under the conditions⁹, an IACT encounters. The glass body shows no degradation over the lifetime of the stated experiments but the aluminium coating has to be renewed every couple of years [Garczarczyk16, p. 40]. Unfortunately, those mirrors are not only expensive to produce but also only a very few companies are able to produce them. This limits the amount of telescopes CTA can build each year [Garczarczyk16, p. 40].

Glass is quite a dense substrate and it is, unfortunately, not resistant against tensile forces. Only glass mirrors with an area of up to 1 m^2 can be expected to carry their own weight [CTA Consortium11, p. 272]. Aluminium is far superior in this regard. A mirror design that is both, cheaper and lighter, was developed for *MAGIC*. With this method, the desired shape of the mirror is milled into an aluminium plate that is mounted onto an aluminium honeycomb structure to give it stability. Finally, the reflective surface is protected by a thin layer of quartz. This results in cheap and lightweight but optically sub-par mirrors. The milling process generates surfaces that have a relatively high roughness and shapes that are not completely reproducible [Garczarczyk16, p. 40].

To achieve the CTA MST requirements but also to find a way to produce mirror facets quickly and cheaply, several designs were proposed that combine different elements from the existing designs with the technique of cold slumping. That describes the bending of a thin glass sheet over a precise mold. The bend sheet is glued to a suitable body that is structurally stable and therefore the glass sheet keeps its new form even after it is released from the mold. This procedure produces smooth and identical surfaces in a short amount of time [Garczarczyk16, p. 41].

There are subtle differences between the different proposed designs regarding their reflective coating or mechanical assembly but in general they follow the same plan¹⁰, see Figure 2.3:

⁹The conditions of the stated experiments are comparable to those of CTA because the northern site of CTA is the same location where *HEGRA* stood until its disassembly in 2002.

¹⁰Generalized design scheme is synthesized from the *INAF Design*, the *CEA Saclay Design* and the *IFJ-PAN Design*,

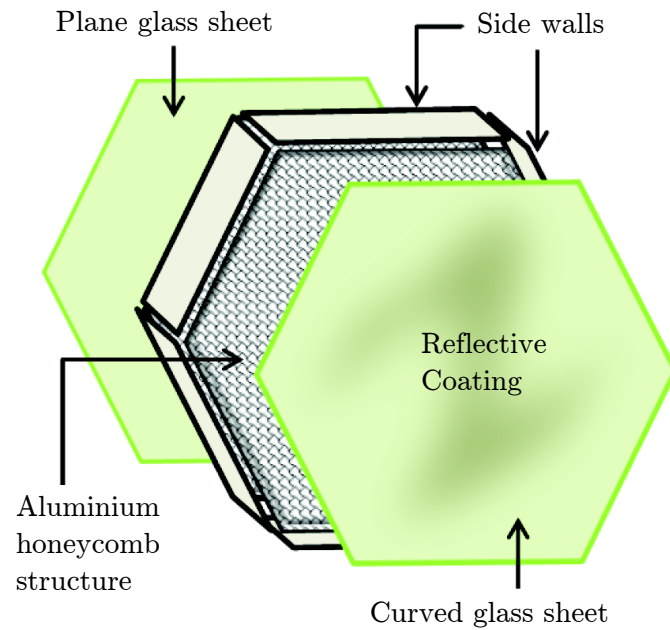


Figure 2.3: Sketch of an arbitrary mirror facet design, similar to the CEA Saclay Design. Modified by the author; originally obtained from [Garczarczyk16, p. 42].

- The shape of the mirror surface is obtained by cold slumping a glass sheet on a mold.
- This glass sheet is further kept in place by either an aluminium honeycomb structure or an array of aluminium tubes.
- The bend glass sheet is coated with a reflective layer of aluminium.
- The reflective surface is protected by a thin layer of, for example, quartz.
- The rear of the aluminium structure is closed by a second glass sheet that can be covered by a protective lacquer.
- The sides of the aluminium structure can be left open to allow humidity to leave the mirror or closed. In the latter case the mirror facets have to be watertight.

as described in [Garczarczyk16, p. 42]

Chapter 3

Mirror Testing Facility

In this chapter, the 2f-principle is discussed, followed by a detailed view on the mirror testing facility of the IAAT. Regarding her *Projecting Test Stand* (PTS), its various components are describes as well as an overview of the mirror testing procedure. Finally, the analysis of mirror test runs is examined and the chapter concludes with recent mirror test results.

3.1 The 2f-Setup

On an IACT, the mirror facets are bound to collect light from the sky. The reflector requirements are thus defined for parallel light [JAM19, CTA-GLOS-202 Theta_80 - V4]. The 2f-setup for Mirror Testing Facilities is designed to avoid the technical difficulties that come with the task to uniformly illuminate a mirror with an area¹ of about 1.25 m² with parallel light.

A point-like light source in the center of curvature² of the mirror facet illuminates the mirror uniformly and can be used to determine the properties of a mirror facet [Chadwick18, p. 16]. The green and dark blue arrows in Figure 2.2 on Page 11 show that light emitted from the center of curvature is reflected into the same spot. To set up a Mirror Testing Facility, following this design concept, a light source and a detector are positioned as close as possible to each other on the principal axis of the mirror facet in a distance of two times its focal length from the mirror facet surface.

The intercept theorem explains that the PSF measured with this method is twice as large as the PSF of the same mirror facet measured in the focus using parallel light (see the red arrows in Figure 2.2 on Page 11). Since the requirements for the PSF are defined for the usage on site, they refer to the PSF measured in the focus. The PSF measured with the 2f-setup needs to be corrected accordingly.

3.2 Requirements for the Mirror Testing Facility

A Mirror Testing Facility has to fulfill several requirements to conduct measurements that can be accepted by the *Cherenkov Telescope Array Observatory* (CTAO). The requirements for a 2f-setup Mirror Testing Facility that is aimed to do both, measure the reflectivity and the width of the PSF of a mirror facet of a CTA MST, are defined as the following:

- It has to be possible to measure with at least four different wavelengths between 300 nm and 550 nm. The shortest wavelength has to be 320 nm or less. This can be achieved via filters in front of the detector or monochromatic light sources with an FWHM of 20 nm or less [Chadwick18, p.16].

¹The area of a regular hexagon is $A_{\square} = 6 \times \frac{1}{2}sh$ with the side length s and the height h of one of the six regular triangles. The side-to-side diameter of the hexagonal mirror facets of a CTA MST equals 1.2 m = $2 \times h$, thus $A_{\square} = 6 \times \frac{1}{\sqrt{3}}h^2 = 1.247 \text{ m}^2$.

²The center of curvature is two times the mirror facets focal length away from its surface.

- A pixelated detector has to cover at least 80 % of the maximum required angular pixel size of the Cherenkov camera ³ [Chadwick18, p.16].
- It has to be possible to integrate the intensity over an area with a diameter 80 % of the maximum required angular pixel size of the Cherenkov camera [Chadwick18, p.16].
- The resolution of the detector has to be at least 250 pixels per axis over the length of $\frac{2}{3}$ of the pixel size of the Cherenkov camera for each of the four measurements with different wavelengths. Should this be not feasible, it is also allowed to take one wavelength-integrated measurement with this resolution and, for each wavelength, one measurement with a resolution of five pixels per axis over the length of $\frac{2}{3}$ of the pixel size of the Cherenkov camera [Chadwick18, p.17].
- The testing routine has to include a normalization measurement to determine the total amount of incident light (on the mirror) [Chadwick18, p.16].
- The surface of the mirror must be illuminated uniformly [Chadwick18, p.16].
- The intensity of the background light has to be reduced to less than 1 % of the intensity of the reflected light. This can be achieved by technical or analytical means [Chadwick18, p.16].
- The mirrors have to be mounted with the same mounting points in the testing facility as on the telescope [Chadwick18, p.16].

Additionally, the *Institute for Astronomy and Astrophysics Tübingen* (IAAT) set, after discussions with the *Mirror Testing Group*, the two following criteria for her own Mirror Testing Facility:

- It has to be possible to test a single mirror in 30 min.
- The results from the analysis of a test run should have an uncertainty not exceeding 1 %.

3.3 The Rastering Mirror Test Stand

The IAAT has two different mirror test stands, both working according to the 2f-principle. One uses a screen and a CCD camera as a pixelated detector and the other, older setup uses an additional photodiode. To examine the properties of the reflected light spot, it is necessary to measure the local variations of the intensity. A CCD measures a two-dimensional intensity distribution at once. A single photodiode can only measure the integrated intensity on its sensitive area, usually a few square millimeter. Therefore, it is necessary to move the diode through the reflected light spot and repeat the measurement at various locations.

The older mirror test stand utilizes this concept and is therefore named the *Rastering Mirror Test Stand* (RTS). The photodiode is mounted on the tray of a two-dimensional scanning table. The scanning table and its attachments are shown in Figure 3.1. According to the concept of the 2f-setup, the entire scanning table is considered to be in the center of curvature of the spherical mirror. Not pictured are the cart to hold the mirror, a CCD camera and the readout electronics of the RTS.

The light source of the RTS is a small halogen lamp⁴ behind an aperture that appears to be point-like. A chopper wheel is positioned directly in front of the light source. Thus, the intensity, used to probe the mirrors, flashes with a frequency⁵ f_{flash} depending on the angular velocity of the chopper wheel and the number of its windows.

³The pixels of a Cherenkov camera for a CTA MST are arranged in an array with a spacing of 50 mm. That is an angular pixel size of 0.18° in the telescopes focal plane [Garczarczyk18a, p. 3].

⁴Usually a 250 W, 24 V halogen display/optic lamp called *Xenophot* from *Osram* is used. It has a diameter of 12.5 mm but its filament is only $3.5 \text{ mm} \times 7 \text{ mm}$ large.

⁵The flash frequency is $f_{\text{flash}} \approx 263 \text{ Hz}$ in the current setup.

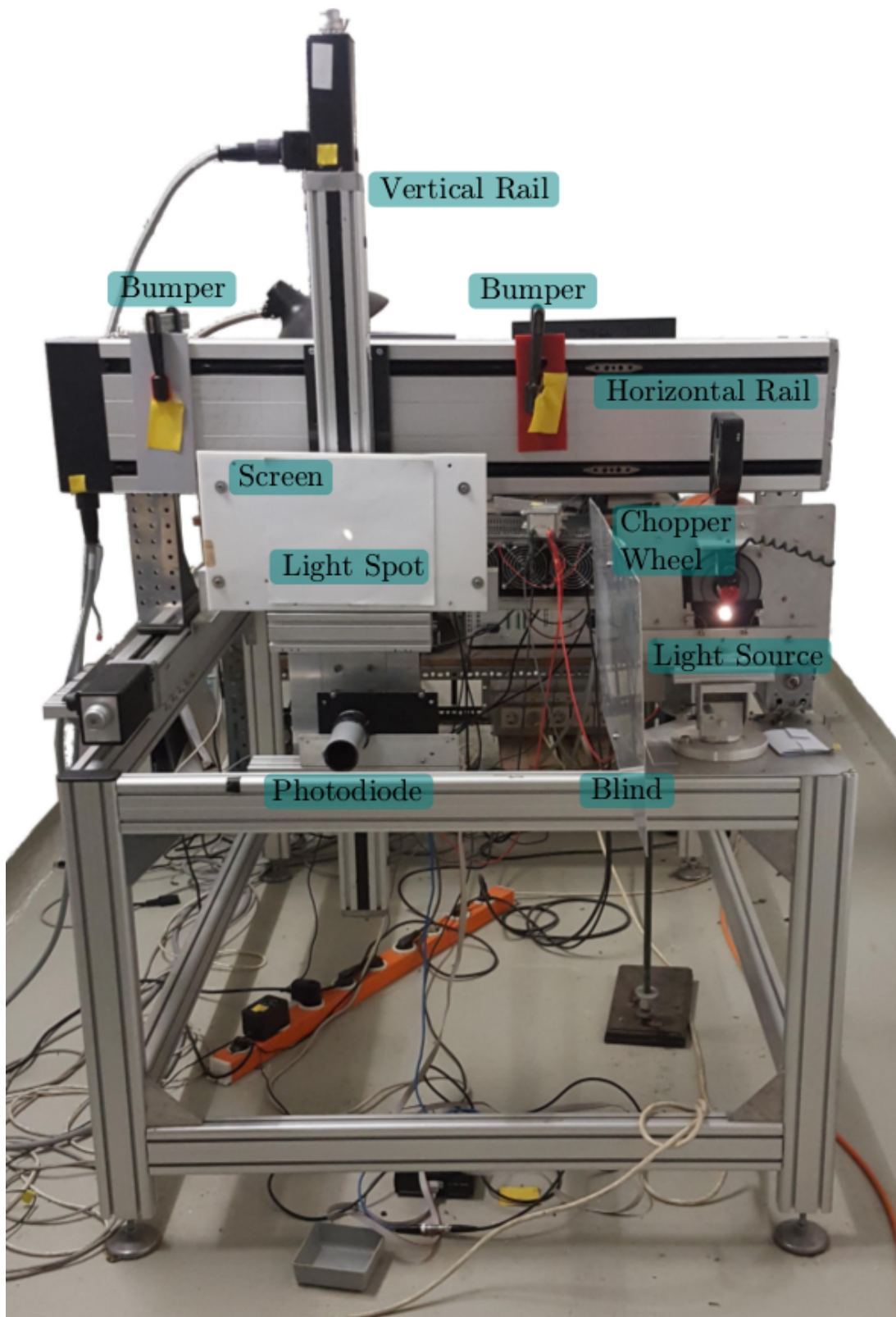


Figure 3.1: Photograph of the *Rastering Mirror Test Stand* (RTS). The screen and the photodiode can move on the vertical and horizontal axis. They are protected from the direct intensity of the light source by a blind. The light source, a 250 W halogen lamp, is placed behind a chopper wheel.

The photodiode is protected by a blind from direct illumination from the light source and by a baffle from scattered intensity in general. In front of the photodiode is a filter wheel. It has an open window and five filters⁶ to measure the reflected intensity and width of the PSF at different wavelengths.

Although possible, it is not feasible to take 250 measurements per axis over the length of $\frac{2}{3}$ of the pixel size of the Cherenkov camera with each filter, as required by [Chadwick18, p. 17] because a single mirror test run would take several weeks in this configuration. It is possible to take more than five measurements per axis with each filter but then another method to determine the PSF becomes necessary. This method has to achieve the higher resolution but can be wavelength insensitive [Chadwick18, p. 17]. To conduct a high-resolution PSF measurement, first, the screen of the RTS has to be moved in the reflected light spot. After that, pictures of the reflected light spot can be taken with the CCD camera.

The output current of a photodiode is assumed to be proportional to the incident light intensity. In the RTS, the reflected light from the mirror facet produces a fluctuating output current with a fixed phase correlation to the chopper frequency. Since the lock-in amplifier used to read-out the photodiode is working as a narrow band-pass filter, which is locked on the chopper frequency, the effect of background light at other frequencies is highly suppressed. This improves the signal-to-noise ratio for both reference and spot measurement.

The remaining chopped background originating from light that is scattered and reflected at the surroundings of the setup is reduced with a blind between light source and photodiode as well as by covering surfaces in the direct light cone with black velvet. Since the dark current of the photodiode is composed of a wide frequency spectrum, it is not completely suppressed by the lock-in amplifier, and therefore has to be subtracted.

The photodiode can be moved in front of the light source to take reference measurements that are used to estimate the intensity reaching the mirror. The advantage of the RTS is that both, the reflected intensity and the reference intensity, are directly measured. Another advantage lays in the lock-in amplifier that makes the RTS very insensitive to change in background illumination. Therefore, it is not necessary to deploy the RTS in a specialized environment. For the time being, the most concerning disadvantage is that one mirror test run takes about 8 hours with the RTS. This was the central motivation behind the construction of the Projecting Mirror Test Stand.

3.4 The Projecting Mirror Test Stand

The most time consuming part of any mirror test run with the RTS is the part where the photodiode is moved through the light spot to scan it. However, the RTS has the option to examine the PSF with a higher resolution by taking a photograph of the spot on the screen. The idea behind the *Projecting Mirror Test Stand* (PTS) was to measure both, the reflectivity and the PSF of a mirror, with this quick method.

The PTS consists of a light source and a screen, in twice the focal length of the mirror, as close to each other as possible. A UV-capable camera is used to take pictures of the reflected light spot on the screen. This is shown in Figure 3.2. To get the incoming intensity onto the mirror facet, reference pictures of the light spot of the light source have to be taken. Therefore, the screen has to be placed in front of the light source. The camera and screen are both mounted on a turnable arm to achieve this and to guarantee that their orientation between each other does not change between main and reference measurement.

⁶There are filters in the wavelengths of $\lambda_{\text{Filter } 1} = 310 \text{ nm}$, $\lambda_{\text{Filter } 2} = 340 \text{ nm}$, $\lambda_{\text{Filter } 3} = 400 \text{ nm}$, $\lambda_{\text{Filter } 4} = 470 \text{ nm}$ and $\lambda_{\text{Filter } 5} = 550 \text{ nm}$. The filters have an FWHM of 10 nm. The filters meet the requirements from [Chadwick18, p. 16].

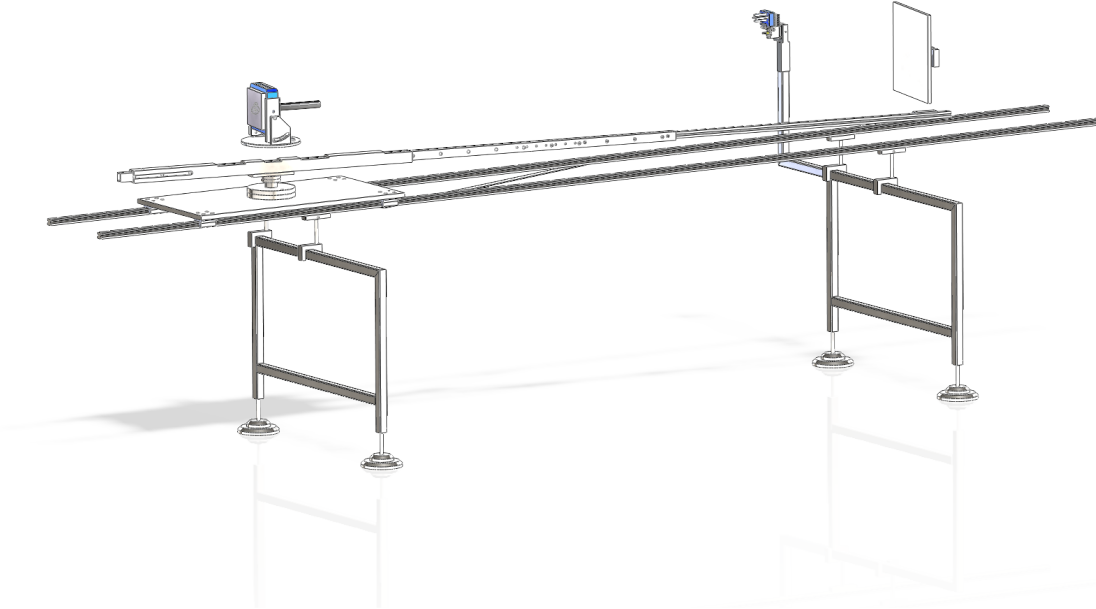


Figure 3.2: 3D-model of the *Projecting Mirror Test Stand* (PTS) in the measurement position.

3.4.1 The Light Source

The light source of the PTS is an array of five monochromatic high power LEDs (see Table 3.1). They are soldered on the same circuit board, as close to each other as possible, to reduce the influence of different light path geometries [Spangenberg16, p. 30].

Colored LEDs were chosen, over a halogen lamp with color filters, because of their vastly improved service life and temporal stability. The halogen lamp of the RTS has to be replaced after 200 hours of operation and needs to be burned in for 12 hours before using it for the first time. On the other hand, only one of the five LEDs needed to be replaced over the course of two years of operation with the PTS. Before each measurement, the halogen lamp has to warm up for two hours. An examination of the optimal warm up times for each LED is conducted in Section 4.7. They are usually about one minute.

Further requirements for the light source of the PTS are that the light cone has to illuminate the

Table 3.1: Specifications of the five LEDs used as light source in the *Projecting Mirror Test Stand* (PTS). With these LEDs, the PTS meets the requirement stated on Page 17 for at least four different wavelengths between 300 nm and 550 nm. Obtained from the data sheets [Roi14d,Roi14c,Roi14b,Roi14a,Roi16].

Name	Semiconductor	Peak Wavelength	FWHM of the Peak	Maximal Power	Opening Angle
SMB1N-525V	InGaN	525 nm	35 nm	1300 mW	126°
SMB1N-490	InGaN	490 nm	23 nm	320 mW	130°
SMB1N-405V-02	AlInGaN	405 nm	12 nm	2100 mW	126°
SMB1N-365V	AlInGaN	365 nm	10 nm	2300 mW	22°
DUV310-SD356	AlGaIn	308 nm	14 nm	60 mW	120°

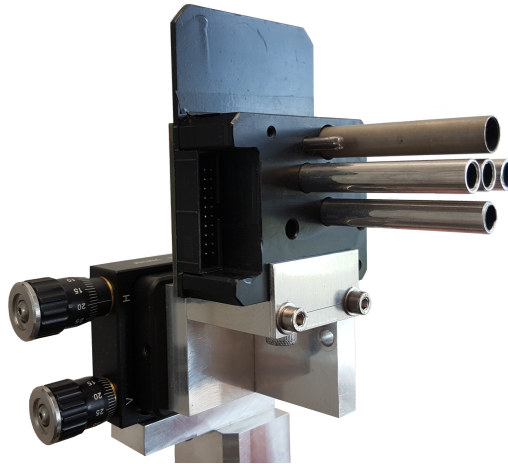


Figure 3.3: Photograph of the light source of the *Projecting Test Stand*. The aluminium collimator tubes and the polyoxymethylene (POM) block are visible but the circuit board with the LEDs is covered by the block. On the left, the stand of the light source is visible, with gauges to set their orientation.

mirror evenly so no part of it gets over- or underrepresented when analyzing the reflectivity. Since the PTS is quite sensitive for background illumination, the light cone should be just wide enough to illuminate the mirror.

Most of the LEDs emit light in a cone with an opening of more than 120° ; this is far too wide. Therefore, their board is equipped with a block of black polyoxymethylene (POM) with five holes in it that serve as collimators for the LEDs. To reduce the opening of the LEDs even further, aluminium tubes are installed as additional collimators. In total, the collimators have a length of 67.1 mm and a caliber of about 6 mm⁷. In a distance of 32.14 m, this results in a light spot about 3 m in diameter. This has been found to be a suitable trade-off between illuminating the whole reflective surface of an MST or LST mirror facet and the amount of back-scattered light from surfaces surrounding the mirror facet⁸.

The inner surface of these tubes is covered with a light absorbing velvet to prevent internal reflections. Reflections on the inside of a collimator are particularly undesirable because they would not only increase the amount of stray light but they would also change the emission geometry of the light source. When reflected intensity leaves the collimator it may not follow the inverse square law of a point-like light source and can even create features on the screen [Spangenberg16, p. 34].

3.4.2 The UV-Capable Camera

The camera used in the PTS is an *Alta U47-UV*⁹ from *Apogee*. It applies an *E2V CCD47-10* CCD sensor featuring 1024×1024 square pixels with $13 \mu\text{m}$ side length [Apo]. Its response is linear up to 100 000 photoelectrons and the read-out ADC has 16 bit resolution. [Apo]. The camera is optimized for observations in the ultra-violet regime and has its peak sensitivity at a wavelength of 240 nm (see Figure 3.5). The difference in sensitivity for different wavelengths does not affect the quality of mirror tests since pictures of the reflected light spot are only compared to reference pictures taken with light of the same wavelength.

The camera has an active cooling system that consists of four fans and an electric cooling unit. This

⁷The calibers of the collimators vary. The collimator of the UV-LED has a caliber of 6.7 mm the others have a caliber of 5.7 mm.

⁸Reminder: MST mirror facets have a side-to-side diameter of 1.2 m [Garczarczyk18a].

⁹The camera underwent repair and refurbishment work and got its readout-electronics replaced with them from the model *Alta F47* because the model *Alta U47* was discontinued.

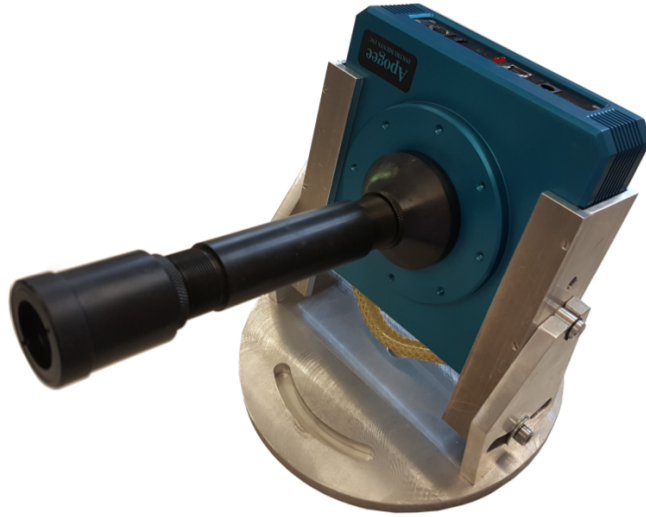


Figure 3.4: Photograph of the *Alta U47-UV* camera in its aluminum mount with the fixed focal length objective.

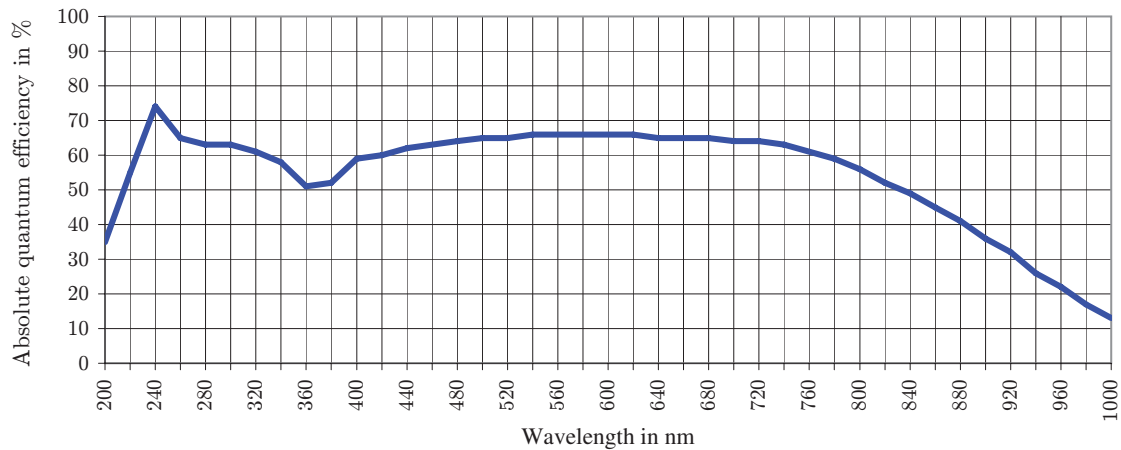


Figure 3.5: Sensitivity as absolute quantum efficiency in percent over different wavelengths in nanometer. The camera *Alta U47-UV* has its highest sensitivity at a wavelength of 240 nm. Modified by the author; originally obtained from the data sheet of the camera [Apo].

system can be programmed to cool the CCD down to 55 °C below ambient and hold its temperature with a precision of 0.1 °C [Apo]. The camera remains operational with a CCD temperature between −22 °C and 27 °C [Apo]. To reduce thermal noise of the CCD as much as possible, without the risk of letting it get too cold, the camera is usually operated at a CCD temperature of −20 °C.

The camera is securely mounted between two aluminium guidances on an aluminium disk. The mounting mechanic was designed by J. Spangenberg [Spangenberg16, p. 37]. Protective elements, made out of rubber, were added later. The camera can rotate about 30° upwards from the mounting position and the mount itself can be rotated approximately 75° to the left and right. The camera is mounted on the rotationary arm near its pivot.

The camera is equipped with a fixed focal length objective that is also transparent for ultra-violet light. With an object-width of 275 cm, it has a field of view of about 15 cm × 15 cm.

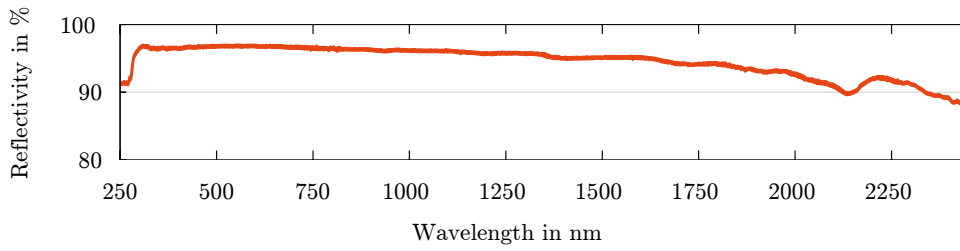


Figure 3.6: Reflectivity in percent of the *Zenith Lite - Diffuse Reflectance Target* over the wavelength in nanometer. Modified by the author; originally obtained from the catalog of the manufacturer *Sphere Optics* [Sph16].

3.4.3 The Screen

With this camera, it is not possible to observe the reflected light spot directly. For this an open CCD (one without optics) that is at least large enough to fit the whole expected PSF is necessary. The cost of a CCD of that size exceeds the allocated funding of this work. So it was decided to project the reflected light spot on a screen and take a photograph from it.

To avoid distortion of the data, the screen has to fulfill several criteria:

- First, the screen has to be highly reflective¹⁰ for light between 300 nm and 550 nm to keep a good signal-to-noise ratio.
- The screen needs to be a diffuse reflector. That means that light entering the screen under any angle has to be reflected in all directions as homogeneously as possible. This reduces the influence of the orientation between screen and principal axis of the mirror facet and between screen and camera to geometric distortion.
- Finally, it is important that the screen is uniform. The position of the light spot on the screen should have no influence on the results obtained by the PTS.

A *Zenith Lite - Diffuse Reflectance Target* from *Sphere Optics* was chosen as screen. It has a high reflectivity in the desired range, as seen in Figure 3.6. The screen has a layer of a polytetrafluoroethylene (PTFE) based polymer [Sph16, p. 2] and is described as a „nearly ideal diffuse Lambertian“ [Sph16, p. 4] reflector.

A Lambertian reflector has the same observed brightness from all directions. An area A with a constant radiance L is called Lambertian if the radiant intensity I depends solely on the cosine of the angle of observation θ ¹¹ [Lambert92]:

$$I(\theta) = LA \cos \theta \quad (3.1)$$

An observer directly in front of the area A has a vanishing angle of observation ($\theta = 0$) and obtains the maximal radiated intensity $I_{\max} = AL$. An observer who observes the reflector under an angle θ recognizes a smaller area $A' = A \cos \theta$. Following the Lambertian law, they get less radiated intensity $I(\theta)$. Since both values A' and $I(\theta)$ depend on the cosine of the same angle, they observe the same radiance

$$L' = \frac{I(\theta)}{A'} = \frac{LA \cos \theta}{A \cos \theta} = L .$$

¹⁰The reflectivity of the screen should be above 95 % [Spangenberg16, p. 38].

¹¹The angle θ is measured between the line of observation and the normal of the area A .

3.4.4 Geometry of Light Sources, Screen and UV-capable Camera

The exact geometry between camera and screen as well as between the various light sources¹² and the screen matters in the PTS. In an ideal setup, the camera is focused to the center of the screen and the principal axis of the objective is normal to both, the screen and the CCD, at all times. Additionally, the light cones from the mirror or from the LEDs would hit the screen orthogonally in its center. This is impossible since the camera would block any light from reaching the screen.

The orientation between objective and CCD is not a variable in this setup¹³, therefore the thoughts, concerning the geometry, focus on the position between camera, as a whole, screen and light sources. For the clearness of wording in this argumentation, it is assumed that the light cones all have circular cross sections; this assumption has no impact on the conclusion or on the actual experiment. Therefore, there are three possibilities to set up the PTS, realistically:

1. The screen is normal to the light cones and a circle is projected on it. Then, the camera takes pictures under an angle, thus distorting all pixels. The resulting image shows an ellipse.
2. The screen is parallel to the objective and the light cones cast ellipses on the screen. The camera takes pictures that represent the screen realistically and therefore show those ellipses.
3. The screen is orthogonal to neither the light cones nor the principal axis. The screen shows ellipses from the light cones and the camera takes distorted images. Therefore, the resulting pictures show ellipses generated through the superposition of both effects.

Each option comes with its own set of issues:

1. The first option presents the structure of the light spot better to the operator of the PTS because it is not distorted on the screen, but this does not translate well into the resulting data. The camera photographs the screen under the angle θ , observing a luminous area A' reduced by $\cos \theta$ with a radiance L' also reduced by $\cos \theta$. This is not an issue because the screen was purposefully chosen to fulfill the Lambertian criterion. Although, it is an issue that parts of the screen are closer to the objective than others¹⁴ and possibly out of focus. This results in blurring and different image scales for different parts of the picture. The blurring is impossible to correct for in software.
2. Option number two is advantageous because the pictures portray the screen naturally; all pixels represent the same area on the screen and the whole picture can be in focus. On the other hand, the picture on the screen is distorted to an ellipse and parts of the screen are closer to the light sources than others are. Since the light sources are considered point-like, their intensity drops with the square of the distance. That means that the closer parts of the screen are brighter lit. This effect is examined in Figure 3.7 and in Section 3.4.4.1.
3. The last option combines the disadvantages of the other two without adding any benefit. It is not considered further.

It was decided to mount the screen parallel to the focal plane of the camera (Option 2) because all problems listed above can be corrected in software except for blurring in parts of the frame. Therefore, when examining the pictures from the PTS, the projection of the (circular) light cone is distorted to an ellipse and some areas of the screen are brighter because they are closer to the respective light source.

¹²The mirror is considered a light source in this context.

¹³It is assumed that the internal geometry of the camera is *perfect* in a sense that it does not add further distortions to the pictures.

¹⁴With a focal length of 2.45 m and a field of view with a diameter of 15 cm, the differences in distances between parts of the screen and the camera can become up to 2.5 % of the focal length for a screen that is tilted not more than 5°.

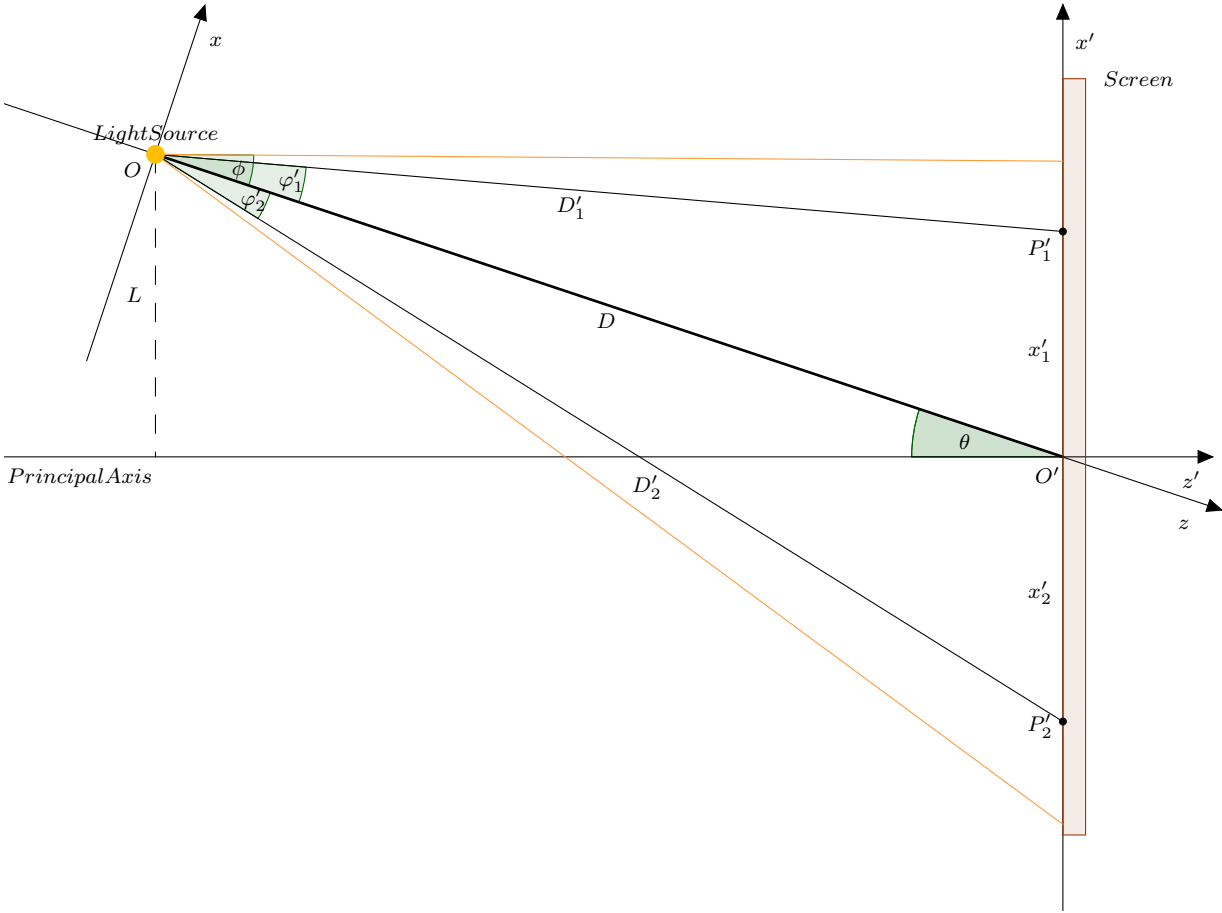


Figure 3.7: Geometry between a point-like light source and a screen that is not set normal to the light cone. The light source is in the origin O of one coordinate system S and the light cone, shown in yellow is symmetrical to the z -axis. The light cone has a half-opening angle of ϕ . The center of the screen is mounted in a distance D to the light source and in the origin O' of another coordinate system S' . The axes y and y' are not shown but are parallel to each other. They are oriented out of the drawing plane. The light source is not directly in front of the center of the screen but instead shifted by a distance L on the side, creating an angle θ between the principal axis of the screen and the direct line between center and light source. Different points P' on the screens surface experience different intensities because of different distances D' to the light source.

3.4.4.1 Issues Due to Uneven Illumination of the Screen

Figure 3.7 shows a sketch of a screen that is illuminated by a light source, with a power P , which is not directly in front of the screen center but a distance L on the side, forming an angle θ . The screen is not evenly illuminated by this light source with an intensity $I = \frac{P}{D^2}$, where D is the distance between the screen center and the light source, because not all points P' have the same distance D' to the light source than the center has. For the points P' that lay on the x - z -plane¹⁵ the distances D' become

$$D' = \sqrt{D^2 - 2Dx' \sin \theta + x'^2}.$$

The y' -axis is not shown in the figure but, since the setup can be considered symmetrical, it can be implemented via the Pythagorean Theorem:

$$D'(x', y') = \sqrt{D^2 - 2Dx' \sin \theta + x'^2 + y'^2} = \sqrt{D^2 - 2Lx' + x'^2 + y'^2} \quad (3.2)$$

¹⁵The x - z -plane is equal to x' - z' -plane.

To normalize for this effect in software, each pixel intensity has to be multiplied by $\frac{D'^2}{D^2}$, according to the pixels x' - and y' -representation on the screen. A pixel with the coordinates (p_x, p_y) on the CCD¹⁶ is represented with the coordinates (x', y') of its geometrical center. Therefore, the conversion factors are $x = a \times (p_x - 511.5)$ and $y = a \times (p_y - 511.5)$, with a as the image scale of the camera¹⁷. Depending on the pixel coordinates, Equation 3.2 becomes

$$D'(p_x, p_y) = \sqrt{D^2 - 2La \times (p_x - 511.5) + a^2 \times ((p_x - 511.5)^2 + (p_y - 511.5)^2)} \quad (3.3)$$

If the distances D and L and the image scale a are measured with uncertainty, the uncertainty of the distance D' becomes

$$\Delta D' = \sqrt{\frac{D^2 \Delta D^2 + a^2 (p_x - 511.5)^2 \Delta L^2 + L^2 (p_x - 511.5)^2 \Delta a^2 + a^2 ((p_x - 511.5)^2 + (p_y - 511.5)^2) \Delta a^2}{D^2 - 2La(p_x - 511.5) + a^2 ((p_x - 511.5)^2 + (p_y - 511.5)^2)}} \quad (3.4)$$

In Table 3.2 are the distances D' calculated for an exemplary setup. For a PTS, in its reference position, it is, indeed, necessary to correct the pictures for different distances to the light source since the maximal difference between actual distance D' and distance to the center D is $D'_{\max} - D = 17.9$ mm. This is larger than the uncertainty of the measurement of the distance D that is commonly $\Delta D \approx 1$ cm. Ignoring this correction factor would lead to an incoming intensity I_{in} that is 1.75 % too low. This is more than the goal of 1 % uncertainty on the reflectivity would tolerate. In the main position, the differences between the maximal and the minimal distance between screen and mirror is $D'_{\max} - D'_{\min} = 1.8$ mm. Since this difference is far smaller than the uncertainty of the distance $\Delta D = 1$ cm, this correction can be ignored for pictures taken in the main position.

The calculated uncertainties of the distance D' are heavily dominated by the measurement uncertainty of the distance D . Table 3.2 shows that the uncertainties diverge only about 1 % in the reference position and less than 0.01 % in the main position. Therefore, it is not necessary to calculate the uncertainties $\Delta D'$ for each measurement for each screen position because the uncertainty ΔD is sufficient.

3.4.4.2 Issues Due to Perspective Distortion of the Structures on the Screen

The distortion of the structures on the screen depends on the angle θ . For a similar *horizontal distance* L between camera and screen, the angle θ becomes much larger in the reference position of the PTS than in its main position¹⁸. Therefore, the distortion may have a more significant influence on the analysis of the reference measurement.

Table 3.2: Examples for the actual distances between light source (or mirror) and position on the screen. The distances are calculated using realistic but exemplary values. The distance D between light source and center of the screen is $D = 2$ m, the distance D between mirror and screen is $D = 32$ m. The distance L between camera and light cone is $L = 40$ cm in both positions and the image scale $a = 0.14786$ mm per pixel. The errors are calculated with an uncertainty of 1 cm on both distances, L and D , and of $0.13 \mu\text{m}$ on the image scale a . The minimal and maximal uncertainties $\Delta D'$ do not necessarily belong to the minimal or maximal distances D' .

D	$D'_{\max}$	$D'_{\min}$	$\Delta D'_{\max}$	$\Delta D'_{\min}$
2 m	2017.9 mm	1986.2 mm	10.1 mm	9.9 mm
32 m	31 999.1 mm	32 001.1 mm	10.0003 mm	9.9997 mm

¹⁶The *E2V CCD47-10* detector of the UV-capable camera has 1024×1024 pixels [Apo].

¹⁷The image scale a of the camera is $a = 0.14786(13)$ mm per pixel (see Section 4.2 on Page 52).

¹⁸The horizontal distance between camera and light source must be large enough that the camera does not cast a shadow on the screen. In the current setup, $L \approx 40$ cm is sufficient. In the reference position, the distance between light source and screen is about 2 m. In the main position, the distance between mirror and screen is 32.14 m. Therefore, the various angles are $\theta_{\text{ref}} = 11.31^\circ$ and $\theta_{\text{main}} = 0.72^\circ$.

For the reference measurement (see Section 3.5) it is important to know which part of the light cone emitted by the light source hits the reflective surface of the mirror. To achieve this, a circular reference region is used that has the same surface area than the (hexagonal) projection of the mirror on the screen.

Assuming no distortion ($\theta = 0^\circ$) this area would be a circle on the screen. Its radius would be calculated via the theorem of intersecting lines and its center would be the center of the light spot. However, in reality this area is the ellipse E that is described as the intersection of the cone C

$$C : x^2 + y^2 = z^2 \tan^2 \phi \quad (3.5)$$

with the plane ϵ

$$\epsilon : x \tan \theta + z = D . \quad (3.6)$$

The cones C represents the light cone from the light source that has an opening angle 2ϕ large enough that the light illuminates the whole mirror but only the mirror in a distance of 32.14 m. The plane ϵ describes the plane of the screen in the coordinates of the system S (see Figure 3.7). The resulting shifted ellipse E is best described as:

$$E : (1 - \tan^2 \theta \tan^2 \phi) \times \left(x + \frac{D \tan \theta \tan^2 \phi}{1 - \tan^2 \theta \tan^2 \phi} \right)^2 + y^2 = D^2 \tan^2 \phi \quad (3.7)$$

This equation is only valid when $\tan^2 \theta \tan^2 \phi \neq 1$. Since the conic section is expected to be an ellipse, one could further require that

$$\tan^2 \theta \tan^2 \phi < 1 . \quad (3.8)$$

For $\tan^2 \theta \tan^2 \phi = 1$ the screen would be parallel to the slope of the cone ($\theta + \phi = 90^\circ$) and the image would show a parabola and for $\tan^2 \theta \tan^2 \phi > 1$ it would be a hyperbola.

From Equation 3.7 one obtains the semi axes of E as

$$a = \frac{D \tan \phi}{\sqrt{1 - \tan^2 \theta \tan^2 \phi}} \quad \text{and} \quad (3.9)$$

$$b = D \tan \theta \quad (3.10)$$

where a is the semi-major axis and b is the semi-minor axis.

This ellipse describes the structure on the screen far better than a circle would. The question occurs if it is necessary to implement the ellipse into the analysis software of the PTS concerning its overall precision.

When the area of a circle $A_{\text{cir}} = \pi r^2$ is used to calculate the incoming intensity I_{in} instead of the area of the ellipse $A_{\text{eli}} = \pi ab$, the possible error in the intensity ΔI_{in} is proportional to the relative error of the area $\frac{\Delta A}{A}$ ¹⁹. The area of an ellipse A_{eli} can be described as a function of its numerical eccentricity ε and its semi-major axis a

$$A_{\text{eli}} = \pi ab = \pi a \sqrt{a^2 - b^2} = \pi a^2 \sqrt{1 - \varepsilon^2} \quad (3.11)$$

with the linear eccentricity $e = \sqrt{a^2 - b^2}$ and the numerical eccentricity $\varepsilon = \frac{e}{a}$. Be the radius r of the circle the same as the semi-major axis a of the ellipse $r = a$ and the relative error becomes:

$$\frac{\Delta A}{A} = \frac{A_{\text{cir}} - A_{\text{eli}}}{A_{\text{cir}}} = \frac{\pi a^2 - \pi a^2 \sqrt{1 - \varepsilon^2}}{\pi a^2} = 1 - \sqrt{1 - \varepsilon^2} \quad (3.12)$$

The numerical eccentricity ε of the ellipse E with a (Equation 3.9) and b (Equation 3.10) is:

$$\varepsilon = \frac{e}{a} = \sqrt{1 - \frac{b^2}{a^2}} = \tan \theta \tan \phi \quad (3.13)$$

¹⁹Assuming a homogeneous intensity distribution in the light cone.

With a circle of a diameter of about 1.26 m in a distance of 32.14 m, the half-opening angle of the cone becomes $\phi \approx \arctan \frac{0.63\text{m}}{32\text{m}} \approx 0.020^\circ$. Together with an angle θ between camera and light source of not more than $\theta = 15^\circ$, the eccentricity of the ellipse E becomes for a typical situation in the reference position $\varepsilon \approx 0.0053$. With this eccentricity, the relative error on the incoming intensity on the mirror I_{int} , due to the wrong approximation of its area on the screen, becomes, according to Equation 3.12, proportional to

$$\frac{\Delta A}{A} = 1.4 \times 10^{-5}.$$

This error is far smaller than the intended precision of the PTS of 1 %. Therefore, it is save to assume that the mirror area, projected on the screen, is circular. Since the perspective distortion on the screen has no significant impact on the reference measurement and since it would have a larger impact in the reference measurement than in the main measurement, it is also save to say that the perspective distortion can be neglected in the whole mirror test.

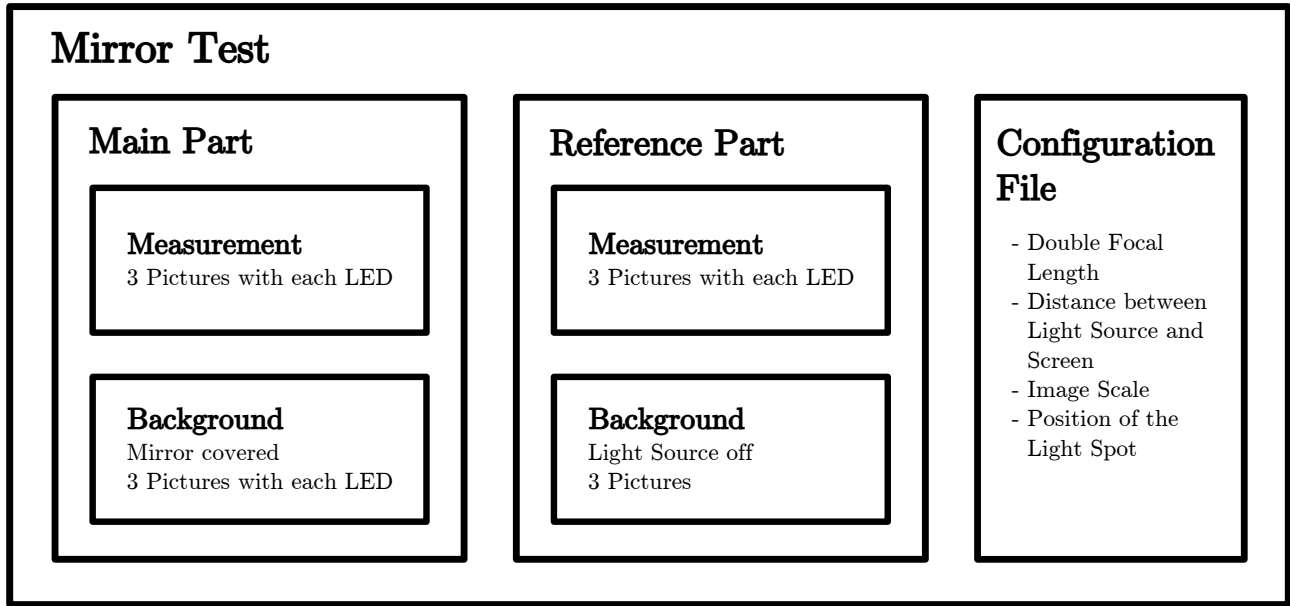


Figure 3.8: Short overview of the mirror testing procedure with the *Projecting Mirror Test Stand A* more in depth overview of the mirror testing procedure and a sketch of the analysis is in Appendix I.

3.5 Mirror Testing Procedure

The (proposed) mirror testing procedure takes about 30 min and an additional 15 min for the analysis. The testing procedure itself is automatized but not autonomous. The BASH script *Mirror Test Stand Assistant* (MIRTESSA) manages the camera and light source, runs the analysis software (ACTAM) and guides the operator during all steps that require human interaction.

As in Figure 3.8 described, each mirror test has two parts: the main and the reference measurement. Every single measurement is also accompanied by a background measurement. The pictures obtained from those measurements should be saved in a directory together with a file that configures the analysis. This configuration file needs the information, which are listed in Table II.2 on Page 80. If the mirror test run is assisted by MIRTESSA, the necessary input for the configuration file is queried by her. The configuration file can be edited manually to correct mistakes or add comments and further information.

Preparation

The camera has to operate at -20°C to minimize thermal noise of the CCD. The camera is cooled permanently because it takes about half an hour to cool it down from room temperature. Furthermore, frequent temperature changes can degrade the vacuum-sealing of the CCD and should therefore be avoided.

At the beginning of the test, the mirror has to be mounted in the cart. The distance between mirror and light source is D_{2f} . The best results are achieved if D_{2f} equals twice the focal length of the mirror facet²⁰. When D_{2f} equals the double of the required focal length of the mirror facet, then the test results are comparable with the optical requirements for this telescope type (see Sections 2.2 and 2.3).

The LEDs are switched on so one can adjust them to illuminate the mirror completely and uniformly. Although the LEDs do not focus at the same point on the mirror (in D_{2f}) they can all be adjusted at once. As yet, the LEDs are adjusted onto the mirror by eye. For the future, it is planned to install photodiodes around the mirror facet to enhance the reproducibility and reliability of the light source

²⁰The double of the focal length of a spherical mirror equals its radius of curvature.

adjustment. The dispersal of the various LEDs and their influence on the final results is investigated in Section 4.5.2.

It has to be verified that the image of the light spot from the mirror is positioned in the center of the camera field of view.

After all components are positioned correctly, MIRTESSA maximizes the signal-to-noise ratio. That means finding an optimal trade-off between sufficient statistics in all pixels that are relevant for the analysis, and avoiding over-exposure of the CCD.

Generally, there are two variables relevant for the signal-to-noise ratio: the exposure time and the LED-current. The current powering the LEDs influences their switch-on characteristic. Since finding an easily usable model for this characteristic is one of the goals of this work, the current is kept constant for all measurements²¹.

The exposure time remains variable. MIRTESSA searches for the longest possible exposure time for each LED. It is important to note that this examination is conducted in the position of the main measurements. This installation guarantees that the most light is concentrated on the fewest pixels. At first a picture with an exposure time of $t_{\text{exp}} = 0.5 \text{ s}$ is taken and a Python script analyses it. If there is any pixel in the picture with the maximal ADC value (65535 or 16383, respectively²²) the file is classified as *overexposed*. Additionally, if there are enough pixels with a sufficiently high²³ ADC value, the file is classified as *overexposed*, too. This second threshold was implemented to avoid working with a exposure time that would overexpose the picture regularly just because the test picture was statistically dim. If the file is not marked as *overexposed*, a new image with additional 0.1 s exposure time is taken. Otherwise, the last exposure time that did not get classified is used for all measurements with this LED, in this mirror test run.

Main Part

In the main measurement part, the pictures of the reflected light spot on the screen are taken. MIRTESSA takes several, usually three pictures of the reflected light spot, for each wavelength. An example for such a picture is shown in Figure 3.9. Additionally, the same number of pictures is taken while the LED is still on, but the mirror is covered²⁴. These pictures are used to subtract the background produced by scattered light from the pictures of the reflected light spot.

After the main measurement but before the PTS is transferred into its reference position, the distance $D_{\text{scale-main}}$ between screen and camera body has to be measured.

An LED has a particular switch-on characteristic. Its intensity is bound to change, as it warms up. To ensure the comparability between these measurements and the measurements in the following reference part, the power output has to be reproducible. Each LED is warmed up prior to the first measurement. The LED forward voltage is read-out, and measurements are taken when the change in forward voltage dropped below a certain threshold²⁵. The forward voltage U_{LED} from this measurement is saved. Before the reference measurement, the LED are warmed up until the same voltage is reached. The correlations between forward voltage, temperature and intensity are examined in Section 4.7.

²¹The currents that are used to power the LEDs are: 25 mA for the 310 nm LED, 300 mA for the 365 nm LED, 130 mA for the 405 nm LED, 200 mA for the 490 nm LED and 200 mA for the 525 nm LED. Those values are chosen because they achieve a good compromise between emitted intensity and thermal power.

²²The dynamic range of the CCD changed from a 16-bit resolution to a 14-bit resolution after repair and maintenance work was conducted.

²³The Python script looks for 50 or more pixels with an ADC value of 60000 or 16000, respectively (about 92 % of the dynamic range) or more.

²⁴There is a radio-controlled shutter installed but it does not feature an interface to the measurement computer and thus has to be operated manually.

²⁵The LED supply-unit can distinguish between a difference in voltage of 1 mV. When the forward voltage of an LED differs just this amount over the duration of the corresponding measurement, the forward voltage is considered to be stable.

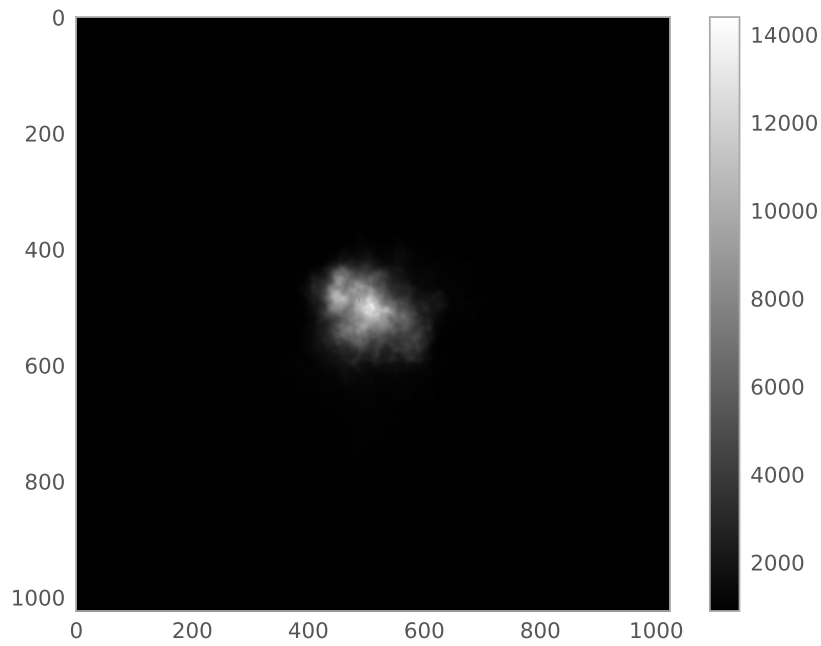


Figure 3.9: Exemplary picture taken from the light spot, reflected by the CTA MST mirror facet prototype KMST4 when illuminated with light with a wavelength of 490nm from the SMB1N-490 LED out of 32.143(66)m distance. The picture was 0.8s exposed. Defect pixels are replaced with the arithmetic mean of neighboring (non-defect) pixel values.

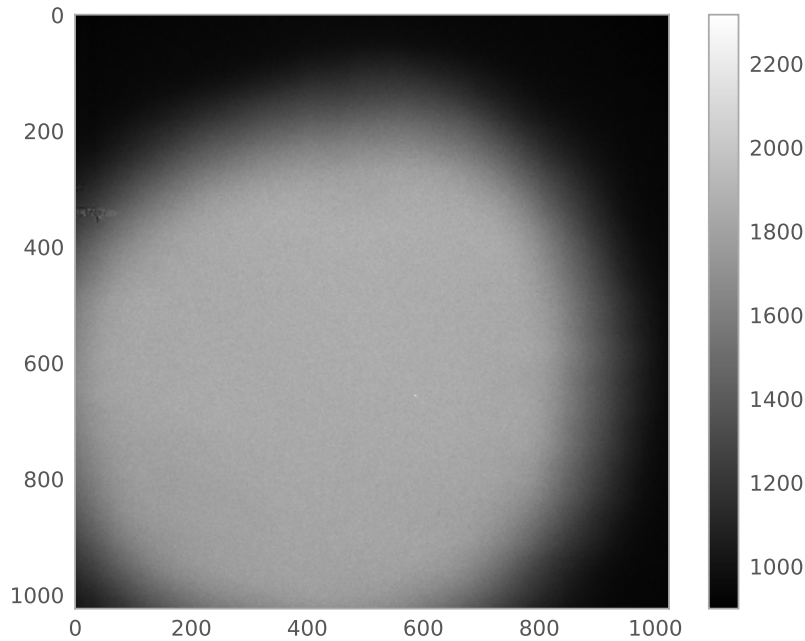


Figure 3.10: Exemplary picture of the reference light spot of the SMB1N-490 LED on the screen in a distance of 1.7889(16) m. This picture is the corresponding reference for the picture in Figure 3.9 and was taken under comparable circumstances.

Reference Part

After the PTS is transferred into its reference position, in which camera and screen are rotated and the LEDs are illuminating the screen directly, the reference measurement can be conducted. The distance D_{ref} between screen and LEDs has to be measured and it is used to calculate the projected area of the mirror on the screen. The distance $D_{\text{scale-ref}}$ between screen and camera body has to be measured to allow to correct the image scale if the telescopic arm of the PTS was not reassembled at the same length.

Each LED is warmed up to reach stability at the same temperature as in the main part before pictures of their light spot are taken ²⁶. An exemplary reference picture is shown in Figure 3.10. Since all reference-background pictures are taken without switched on LEDs, they are interchangeable and it is sufficient to measure them once (for each exposure time).

All the resulting *fits* files (pictures) from one mirror test run have to be saved in a directory together with the configuration file²⁷. The files are saved properly if the current mirror test run is managed by MIRTESSA. In this case, the analysis starts automatically. Otherwise, it needs the directory of the saved pictures as input.

²⁶It can happen that the voltage drop of an LED does not reach stability (with a tolerance of 1 mV at the same voltage as in the main measurement (U_{LED} with an additional tolerance of 3 mV), due to unspecified changes in the environment. In this case, the LED is shut down and allowed to cool. After a minute the LED is switched on again. If the voltage drop does not reach stability now, an error is reported but the mirror test run continues.)

²⁷It is possible that other files interfere with the analysis software if their file names contain strings that are identical to the name-tags from the configuration file.

3.6 Results of a Mirror Test

The software to analyze the gathered data from a mirror test run, conducted with the PTS, is called *Analyze CTA Mirrors* (ACTAM).

The mirror test runs, conducted with the PTS, yield to results that help to decide if the mirrors fulfill the requirements stated in Section 2.2 and Section 2.3 on Page 11 and following.

The mirror facets need to have a directed reflectivity R_{dir} of at least 85 % for light with a wavelength between 300 nm and 550 nm [JAM19, B-TEL-0120 Mirror Reflectivity - V3]. The directed reflectivity R_{dir} is the percentage of the incoming intensity I_{in} that is reflected into a circle with a diameter of 7.5° ²⁸ around the center of mass of the intensity distribution of the reflected light spot.

$$R_{\text{dir}} = \frac{I_{\text{spot}}}{I_{\text{in}}} \geq 85 \%$$

Additionally, the analysis has to estimate the diameter of a circle that contains 80 % of the intensity that is reflected into a circle with a diameter equivalent to an opening angle of 1° ²⁹ [JAM19, CTA-GLOS-202 Theta_80 - V4]. That is the so-called Θ_{80} -width of the PSF. A CTA MST mirror facet is required to have a width of its PSF of $\Theta_{80} = 12 \text{ mm}$ [Garczarczyk18b, p. 8].

Since the PTS does not provide the possibility to measure the intensity over an area with a diameter of 7.5° or even 1° , the directed reflectivity and the Θ_{80} -width are redefined to refer to the reflectivity of a circle with a diameter of $4 \text{ mrad} \approx 0.23^\circ$ around the center of mass of the reflected intensity distribution. In Section 4.3 it is shown that the amount of intensity that is reflected into an area with a diameter of 1° that is not contained in an area with a diameter of 4 mrad is insignificant.

3.6.1 Data Analysis Software for the Projecting Mirror Test Stand

To examine the requested properties, the ACTAM software needs, next to the pictures containing the data itself, further information about the mirror and the mirror test run. Those are listed in Table II.2 on Page 80. In the configuration file that lays, together with the pictures from the mirror test run, in the directory that is indicated as **Base**, are several distances listed that describe the current state of the PTS but also, among others, for with type of telescope the measured mirror is intended for.

The files in **Base** are categorized based on the wavelengths that are stated in the files names. The analysis is then conducted for each wavelength separately.

To calculate the directed reflectivity, two quantities have to be known: All the incoming intensity I_{in} and the intensity I_{spot} , reflected by the mirror into a defined region around the center of the reflected light spot.

Determination of the Incoming Intensity

The incoming intensity I_{in} is the intensity arriving at the mirror. It is not directly measured during the main measurements but it can be derived from the reference measurements. In the reference position of the PTS, the screen is between the light source and the mirror. The according pictures are labeled with the **ref_label** tag in their filename when one of the LEDs is switched on. If all LEDs are off, the pictures get the **refbg_label** tag.

The concept is to measure how much intensity is emitted by the light source in the direction of the mirror.

²⁸An opening angle of 7.5° correlates with the size of the Cherenkov camera in the focal plane of the reflector [Garczarczyk18a, p. 2].

²⁹This circle can lay anywhere in the cFoV. In the context of this thesis, it is implicitly assumed that a circle, with a diameter equivalent to an opening angle of 1° , contains the most reflected intensity when its center lays on the center of mass of the reflected intensity distribution in the focal plane.

- All files with the `ref_label` and with the `refbg_label` are opened and their intensity distributions are added resulting in two arrays of intensities, `ref` and `refbg`, respectively.
- To remove traces from the background illumination from the data, `refbg` gets subtracted from `ref`. After that, `ref` contains only the intensity from the light source ³⁰.
- The intensity arriving at the mirror I_{in} is the same that would pass through the projection of the mirror area on the screen. The distance between light source and screen D_{ref} and the distance between light source and mirror D_{2f} are stored in the configuration file as `ref_dist_in_m` and `F2`. Utilizing the theorem of intersecting lines and assuming that the light source is point-like, the side-to-side diameter of the projected mirror `d_str` is calculated.
- To simplify the analysis, a circle, not a hexagon, is projected on the screen. The area of the projected mirror on the screen is derived from `d_str` and the virtual mirror radius `r_virtual` is the radius of a circle with the same area.
- To calculate I_{in} , which is called `ref_int` in the report file, all ADC values from `ref` are added that lay in a circle with radius `r_virtual` around the center of the projected light spot with the coordinates `center_ref_x` and `center_ref_y` that are provided by the configuration file.

Determination of the Intensity Reflected into a Specific Region

A circle, with the radius of interest ROI, around the center of mass `center` of the intensity distribution of the reflected light spot, contains, according to this definition, an amount of reflected intensity I_{spot} .

- The files tagged with `bg_label` are opened and their intensities are pixel-wise averaged to the array `bg`.
- All other files are opened and averaged to form the `data` array.
- To remove background illumination from the data, `bg` is subtracted from the `data` and the result is an array of intensities, reflected from the mirror.
- The coordinates of `center` result from a function that calculates the center of mass of the intensity in `data`.
- The intensity I_{spot} that is called `spot_int` in the output of ACTAM is the sum of all intensities in `data` that have a distance of ROI or less to `center` ³¹.

Determination of the Total Amount of Reflected Intensity

To determine the total amount of reflected intensity $I_{reflected}$, it would be necessary, strictly speaking, to detect all reflected photons that would hit the Cherenkov camera if the mirror is mounted in a telescope and aligned well [JAM19, CTA-GLOS-204 Mirror Reflectivity - V1]. Since the Cherenkov camera (FlashCam) has a diameter of almost 3 m [Garczarczyk18a, p. 4], that is not feasible.

To get an approximation of the total reflected intensity $I_{reflected}$ of the mirror, the intensity of its reflected light spot is analyzed. In Section 4.3 on Page 55 it is shown that the estimated intensity of the

³⁰The background pictures of the main part are taken with switched on LEDs and a covered mirror. The background pictures of the reference part are taken with switched off LEDs. Therefore, light that is reflected by the walls or inventory of the laboratory is removed from the main measurement (because it is included in the main background) but not from the reference measurements. The influence of this issue is not yet examined.

³¹There is no sub-pixel analysis or interpolation done. Typical values like a radius of interest equivalent to an half-opening angle of 2 mrad, a mirror distance $D_{2f} = 32$ m and the image scale a estimated in Section 4.2.3 yield to a region of interest containing over 500 000 pixels.

reflected light spot I_{spot} does not change significantly for a radius of interest larger than $\text{ROI} = 2 \text{ mrad}$ ³². Therefore, this radius is used to approximate the total reflected intensity of the mirror.

$$I_{\text{reflected}} = I_{\text{spot}}(\text{ROI} = 2 \text{ mrad})$$

Determination of the Width of the Mirror Point Spread Function

To cope with the PTS' technical limitations, the width of the PSF Θ_{80} is defined as the diameter of a circle, around the center of mass of the intensity distribution of the reflected light spot **center**, that contains 80 % of the reflected intensity $I_{\text{reflected}}$.

Θ_{80} is determined by recursively calculating the encircled intensity around **center**. Beginning with a radius **r_psf** that is equal to **ROI** and shorten it one pixel-length after each step. As soon as the encircled intensity drops below 80 % of **spot_int**, the current **r_psf** is recognized as half of Θ_{80} . Mind that the center of gravity is not calculated for each step. Instead, the center of gravity **center** of the original intensity distribution is used as center of the circles for the whole analysis.

³²A radius of interest, that is equivalent to a half opening angle of 2 mrad, is about 6.5 cm on the screen for a usual mirror distance $D_{2f} = 32 \text{ m}$.

3.6.2 Uncertainties of the Analysis

In the previous section, the basic analysis principle is described, without any uncertainties. In reality, mirror test runs, conducted with the PTS, are subject to several uncertainties that can be grouped into four categories:

1. Statistical uncertainties on a measured value, like a distance or the image scale.
2. Systematic uncertainties due to assumptions that may not be valid.
3. Systematic uncertainties due to discrepancies between the ideal and the real PTS.
4. Human errors.

3.6.2.1 Statistical Uncertainties

ACTAM receives uncertainties of its input via the configuration file (see Table II.2 on Page 80).

Uncertainty of the Image Scale a

The image scale a gets corrected with the two distances $D_{\text{scale-main}}$ and $D_{\text{scale-ref}}$ for the main and the reference measurement, utilizing Equation 4.2 and the nominal distances between screen, camera body and objective, D_{scale} and x_0 . The uncertainty of the image scale is corrected with Equation 4.3. Hereinafter it is assumed that the image scale (and its uncertainty) are always properly corrected for the current length of the telescopic arm.

Uncertainty of the Reference Intensity $I_{\text{reference}}$

The uncertainties of the double of the focal length ΔD_{2f} , of the reference distance between light sources and screen ΔD_{ref} and of the image scale Δa are propagated to get the statistical uncertainty of the virtual mirror radius R_{virt} :

$$R_{\text{virt}} = R_{\text{mirror}} \times \frac{D_{\text{ref}}}{D_{2f}} \quad (3.14)$$

$$R_{\text{mirror}} = \sqrt{\frac{\sqrt{3}}{2\pi}} \times S$$

$$\Delta R_{\text{virt}} = \sqrt{\frac{\sqrt{3}}{2\pi}} \times \frac{1}{D_{2f}} \times \sqrt{D_{\text{ref}}^2 \Delta S^2 + S^2 \Delta D_{\text{ref}}^2 + \frac{S^2 D_{\text{ref}}^2}{D_{2f}^2} \Delta D_{2f}^2} \quad (3.15)$$

The flat-to-flat diameter S of a (hexagonal) mirror facet is assumed to be perfect ³³.

The upper and the lower boundary of the reference intensity $I_{\text{reference-max}}$ and $I_{\text{reference-min}}$ result from integrating over the intensity distribution in **ref** with the radii $R_{\text{virt}} + \Delta R_{\text{virt}}$ and $R_{\text{virt}} - \Delta R_{\text{virt}}$ around the center of the reference spot. The expected value of the reference intensity $I_{\text{reference}}$ results from the integration with the radius R_{virt} . The relative uncertainty $\frac{\Delta I_{\text{reference}}}{I_{\text{reference}}}$ of the reference intensity is calculated with the equation

$$\frac{\Delta I}{I} = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \quad (3.16)$$

There is no uncertainty given for the position of the center of the reference spot, which is estimated by eye. In Section 4.5.2, it is shown, that a displacement of the center point of the analysis of up to 50 pixel-diagonals from the real center of the light spot, does not alter the measured reflectivity.

³³If the real flat-to-flat diameter of a mirror facet differs from S to an extent that it would have an influence on this measurement than this reason would suffice to dismiss the mirror facet. Admittedly, this would not be realized by testing the mirror facet with the PTS.

Uncertainty of the Reflected Intensity I_{spot}

The uncertainties of the double of the focal length ΔD_{2f} and of the image scale Δa are propagated to get the statistical uncertainty of the radius of interest ΔR_{roi} :

$$R_{\text{roi}} = \frac{1000 D_{2f} \tan(\rho_{\text{roi}})}{a} \quad (3.17)$$

$$\Delta R_{\text{roi}} = \sqrt{\left(\frac{1000 \tan(\rho_{\text{roi}})}{a} \times \Delta D_{2f}\right)^2 + \left(\frac{1000 D_{2f} \tan(\rho_{\text{roi}})}{a^2} \times \Delta a\right)^2} \quad (3.18)$$

With R_{roi} the radius of interest in pixel-lengths and ρ_{roi} the radius of interest in radians. Since the radius of interest is set to be $\rho_{\text{roi}} = 2 \text{ mrad}$, there is no uncertainty.

The upper and lower boundary of the reflected intensity $I_{\text{spot-max}}$ and $I_{\text{spot-min}}$ result from the integration over the intensity distribution **data** around its center of mass **center** with the radii $R_{\text{roi}} + \Delta R_{\text{roi}}$ and $R_{\text{roi}} - \Delta R_{\text{roi}}$, respectively. The expected value of the reflected intensity I_{spot} results from the integration with the radius R_{roi} and the relative uncertainty $\frac{I_{\Delta\text{spot}}}{I_{\text{spot}}}$ of the reflected uncertainty is determined with Equation 3.16.

Neither the center of mass **center** nor the intensity distribution are considered to have uncertainties. Since the intensity distribution is heavily dominated by one peak (see Figure 3.9) an uncertainty in the exact position of said peak has a diminishing effect on the estimated intensity. The intensity distribution in **data** is the average of at least three frames. The standard deviation of this averaging is not considered further since it would inflate the processing time of the analysis by about a factor of ten and the change in results is, for now, statistically insignificant.

Uncertainty of the Reflectivity R

The reflectivity R is the ratio of reflected intensity I_{spot} and incoming intensity I_{in} . Since the PTS cannot measure the incoming intensity I_{in} onto a mirror facet, the reference intensity $I_{\text{reference}}$ is used instead. The lower boundary of the reflectivity R_{min} is the ratio between $I_{\text{spot-min}}$ and $I_{\text{reference-max}}$ whereas the upper boundary of the reflectivity R_{max} is the ratio between $I_{\text{spot-max}}$ and $I_{\text{reference-min}}$. Furthermore, the expected value of the reflectivity is the ratio between the two expected values of the intensities. The relative uncertainty of the reflectivity $\frac{\Delta R}{R}$ is the geometric sum of the relative uncertainties of the intensities and the absolute uncertainty ΔR is therefore:

$$\Delta R = R \times \sqrt{\left(\frac{\Delta I_{\text{spot}}}{I_{\text{spot}}}\right)^2 + \left(\frac{\Delta I_{\text{reflectivity}}}{I_{\text{reflectivity}}}\right)^2} \quad (3.19)$$

With R being the expected value of the reflectivity in this case. Several independent mirror tests show that the relative uncertainty of the reflected intensity $\frac{\Delta I_{\text{spot}}}{I_{\text{spot}}} \approx 1.8 \times 10^{-4}$ is usually considerably smaller than the relative uncertainty of the reference intensity $\frac{\Delta I_{\text{reflectivity}}}{I_{\text{reflectivity}}} \approx 1.5 \times 10^{-2}$.

Uncertainty of the Θ_{80} -Width of the PSF

The Θ_{80} -width of the PSF is the diameter of a circle around the center of mass **center** of the intensity distribution **data** that contains 80 % of the intensity that is reflected into a circle with a diameter of 1° around the same point. The PTS cannot measure intensities over an angle of 1° in a distance D_{2f} from the mirror. Therefore, only the diameter of a region, containing 80 % of the intensity, reflected into a circle with a diameter of 4 rad (R_{spot}) is measured (see Section 4.3).

The uncertainty of the Θ_{80} -width of the PSF depends on the uncertainties of the radius of interest R_{roi} , of the image scale a and of the double of the focal length D_{2f} . The upper boundary $\Theta_{80\text{max}}$ of the Θ_{80} -width of the PSF is calculated by comparing the lower boundary of the intensity, deposited

in a circle with a radius r^{34} with 80 % of the upper boundary of the reflected intensity $I_{\text{spot-max}}$. $\Theta_{80_{\text{max}}} = 2 \times r$ for the largest r for which the (lower boundary of) the encircled intensity is still smaller than 80 % of the (upper boundary of) the reflected intensity. The lower boundary $\Theta_{80_{\text{min}}}$ of the Θ_{80} -width of the PSF is twice the largest radius r for which the upper boundary of the encircled intensity is smaller than 80 % of the lower boundary of the reflected intensity $I_{\text{spot-min}}$. During the transformation of the boundaries of the Θ_{80} -width from the unit of pixel-lengths into millimeters the uncertainty of the image scale a is taken into account. Although the uncertainty Δa is not propagated throughout the transformation to avoid statistical uncertainties on the boundaries of the Θ_{80} -width, but rather the boundaries of the image scale $a + \Delta a$ and $a - \Delta a$ are used.

3.6.2.2 Systematic Uncertainties due to Invalid Assumptions

There are assumptions made for the analysis. If they are not, or not completely, met, the results from the analysis may differ from the real properties.

The Internal Structure of the Camera is Perfect

It is assumed that the camera and its optics have no impact on the pictures taken with it. If this is not the case, it can have an unestimable effect on the results from the analysis. Flat-fielding, as described in Section 4.1, is a method to tackle misrepresentations of the data that are due to a difference in sensitivity between CCD pixels. This can also correct features created by the optics to a limited amount but issues like shadowing, lens distortions or damages to the CCD casing would still be a major issue.

A Maximal Radius of Interest of 2 mrad Suffices

For technical reasons, it is not possible to measure the deposited intensity of an area that is much larger than a circle with a radius corresponding to $\rho_{\text{roi}} = 2 \text{ mrad}$. In Section 4.3 it is shown in Figure 4.9 that most intensity, reflected by a typical mirror under test, is reflected into an area with a radius of less than 1 mrad. Still, there is reflected intensity in the area with a radius of 2 mrad that are not in the area with a radius of 1 mrad. It is plausible that there is further intensity reflected by the mirror into an area $0.5^\circ \approx 8.7 \text{ mrad}$ or $3.75^\circ \approx 65 \text{ mrad}$ away from its principal axis.

In this scenario, the reflectivity of a mirror, measured with the PTS, is systematically lower than its actual reflectivity (when measured strictly according to the CTA requirements) (see Section 5.2 on Page 76). The Θ_{80} -width of its PSF is smaller, too, since it is compared to a lower amount of reflected intensity.

Without information about the typical angular dependency of the reflectivity of the mirror facets over at least 4° , it is impossible to correctly estimate the influence of this systematic uncertainty. In Section 3.6.4 and Appendix III a possibility to estimate the surface reflectivity of a mirror is introduced. The difference between reflectivity measured with the PTS and surface reflectivity is the upper boundary for systematic uncertainty of the PTS.

The Reference Intensity $I_{\text{reference}}$ is a Proper Substitute for the Incoming Intensity I_{in}

For technical reasons, it is not possible to measure the intensity I_{in} reaching the mirror surface directly. Therefore a reference intensity $I_{\text{reference}}$ is measured and used to calculate the reflectivity R . During the reference measurement, a screen is positioned in front of the light sources and pictures are taken of the light spots on the screen. During the analysis, the deposited intensity in an area that is equivalent to the area of the projection of the mirror surface on the screen is used as reference. There are three assumptions necessary to justify this practice, which are explained in the following paragraphs.

³⁴The function that calculates the deposited intensity of a circle is only working with a resolution of whole pixel-lengths. To get boundaries of this analysis, the intensity is estimated again, with a radius that is 1 pixel-lengths larger or smaller.

The Intensity of the Light Source is the Same

It is assumed that the light source has the same power output P and therefore the same intensity I during the main and the reference measurement. This is not necessarily the case since the power output and the optical efficiency of an LED are temperature depended. Section 4.7 examines the temperature (and time) dependencies of the LED intensity and introduces a correction factor V due to the inability to reproduce the intensity exactly. Equation 4.9 shows the addition of this factor to the equation of the uncertainty of the reflectivity, Equation 3.19.

Light is Emitted from the a Point-Like Source

If the light used in the reference measurement does not follow the propagation characteristic of a point-like light source (see Equation 4.7), then the amount of intensity inside the geometric projection of the mirror on the screen would not be the same as the intensity actually reaching the mirror. In Section 4.6 it is show that the LEDs used in the PTS are sufficiently point-like.

The Intensity in the Cut-Out is Equal to the one Reaching the Mirror

It is assumed that the intensity density in the cut-out section of **ref** is the same as the intensity density of the portion of the light cone that actually hits the mirror. This assumption can be justified if either is true: The intensity distribution of the light sources is isotropic in an angular range large enough to contain the mirror and the reference region. Or it can be ascertained that the reference area is geometrically identical to the projection of the mirror.

The isotropy of the light source or rather the homogeneity of the intensity distribution in the reference light spot is analyzed in Section 4.5.1. Without further investigations (see Section 5.1.3.1), the light source cannot be considered homogeneous enough to ignore the actual positioning between mirror positioning and reference region.

The quality of the match between the reference region and the projection of the mirror is difficult to quantify. Both, the adjustments of the light cones onto the mirror and the assessment of the center of the light spot are done by eye. In theory, the center of the light cone aligns with the center of the mirror but the deviation between theory and reality is unknown.

3.6.2.3 Uncertainties due to a Non-Ideal Setup

There are further sources for uncertainties and errors that are related to the individual PTS of the IAAT.

The construction of the PTS does not comply fully with the 2f-setup described in Section 3.4.4. Its deviations are examined in Section 4.4 and have been found to be insignificant.

The background reduction of the intensity distributions of the main and the reference measurement are not completely comparable. During the main background measurement, the light source is still on and the mirror is covered but during the reference background measurement, the light source is switched off. Therefore, the intensity scattered from walls and inventory is still present in the main background pictures and can be removed from the main (data) pictures but not from the reference pictures. If this has a significant influence on the quality of the data remains to be examined.

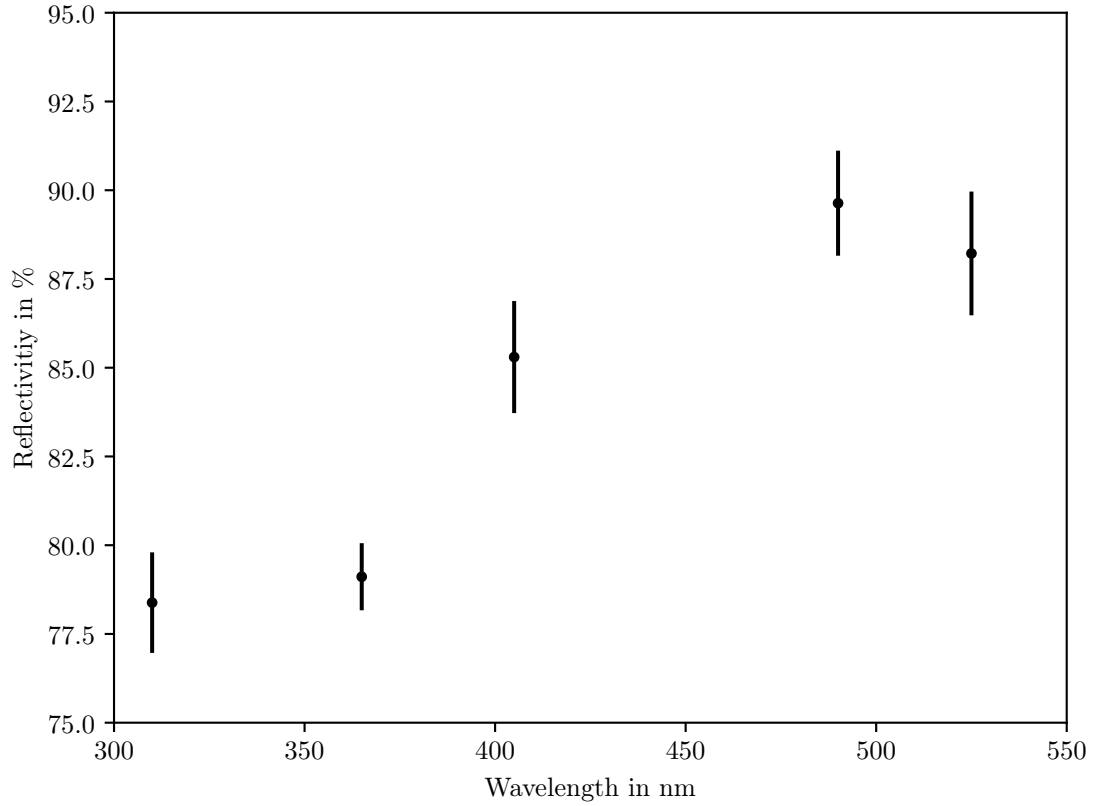


Figure 3.11: Reflectivity R of a CTA MST mirror facet over wavelength. Analysis was conducted with respect to a possible change in image scale a and light source intensity I . The (un)isotropy of the light source is not yet corrected for.

3.6.3 Results of a Mirror Test Run

The mirror $KMST_4$ is a CTA MST mirror facet prototype. It was developed by the *Institute of Nuclear Physics* (INF) at the *Polish Academy of Science*. Its design is similar to the one, described in Section 2.4.

The results from a mirror test run with this mirror are presented in Figure 3.11 and Figure 3.12. The results from this mirror test run are corrected for the systematic that is based on the angle between the axes light source to screen and camera to screen (see Section 3.4.4). The results show measurement uncertainty based on the uncertainties in measuring the various distances (D_{2f} , D_{ref} , $D_{scale-main}$, $D_{scale-ref}$ and D_L) (see Table II.2 on Page 80) and the image scale a (see Section 4.2) of the camera as well as the uncertainty in reproducing the light source intensity of the main measurement for the reference measurement ΔV (see Section 4.7).

According to this mirror test run, the mirror $KMST_4$ is not suitable to be used in a CTA MST.

It is required that every reflective surface of a CTA MST reflects at least 85 % of the incoming intensity with wavelengths between 300 nm and 550 nm [JAM19, B-TEL-0120 Mirror Reflectivity - V3]. In Figure 3.11 it is clearly visible that the reflectivity measured at 310 nm is $R_{KMST_4}(310 \text{ nm}) = 78.38(142) \%$ and at 365 nm it is $R_{KMST_4}(365 \text{ nm}) = 78.11(94) \%$. This is too low.

It is further required that the Θ_{80} -width of the PSF of a mirror facet does not exceed 12 mm, measured in twice the focal length of the mirror facet [Garczarczyk18b, p. 8]. In Figure 3.12, the

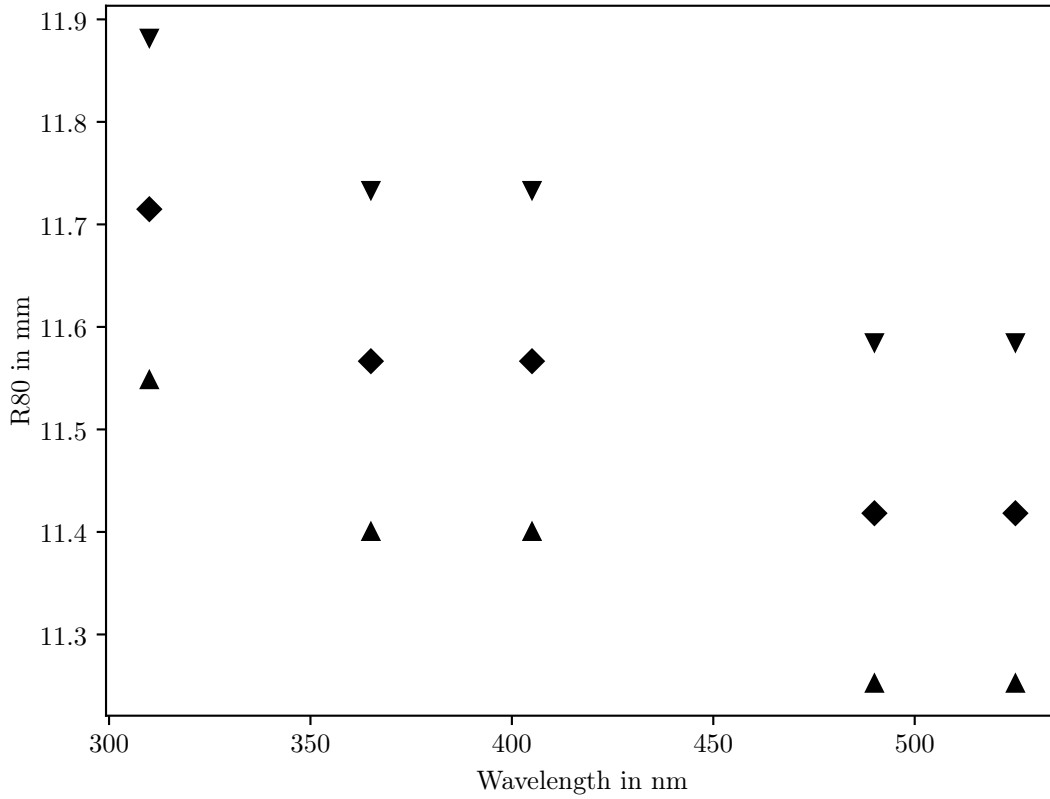


Figure 3.12: Width of the PSF of a CTA MST mirror facet over the wavelength. R80 is half of the Θ -width. The diamonds indicate the expected values and the triangles the boundaries. Analysis was conducted with respect to a possible change in image scale a and light source intensity I . The (un)isotropy of the light source is not yet corrected for.

width of the PSF is given as the R80-width that is the radius of a circle that contains 80% of the intensity reflected into a region with a radius of about 64 mm, roughly equivalent to 2 mrad. In this work it is assumed that this are valid parameters to estimate the width of the PSF of a mirror and that $\Theta_{80} = 2 \times R_{80}$ (see Section 4.3). Therefore, KMST4 does not fulfill the requirement on the PSF for any wavelength.

It is no reason for concern that this particular mirror fails the requirements stated from the CTA Project Office, since it is an early prototype and the institutes developing mirror facets made several improvements and changes in design in the meantime.

3.6.4 Comparison between Measurements

To set the results from the PTS into perspective and to test their general plausibility, they are compared with results from the RTS and from a commercially available reflectometer. A detailed description of the reflectometer is in Appendix III.

Both mirror testing stands measure the directed reflectivity R_{dir} . This is the ratio between the intensity I_{spot} reflected into a certain region of interest and a reference intensity $I_{\text{reference}}$ as a stand-in for the intensity reaching the mirror I_{in} . The reflectometer measures the surface reflectivity R_{surface} . Although, the reflectometer itself has a field of view, it does not compare the intensity reflected by the

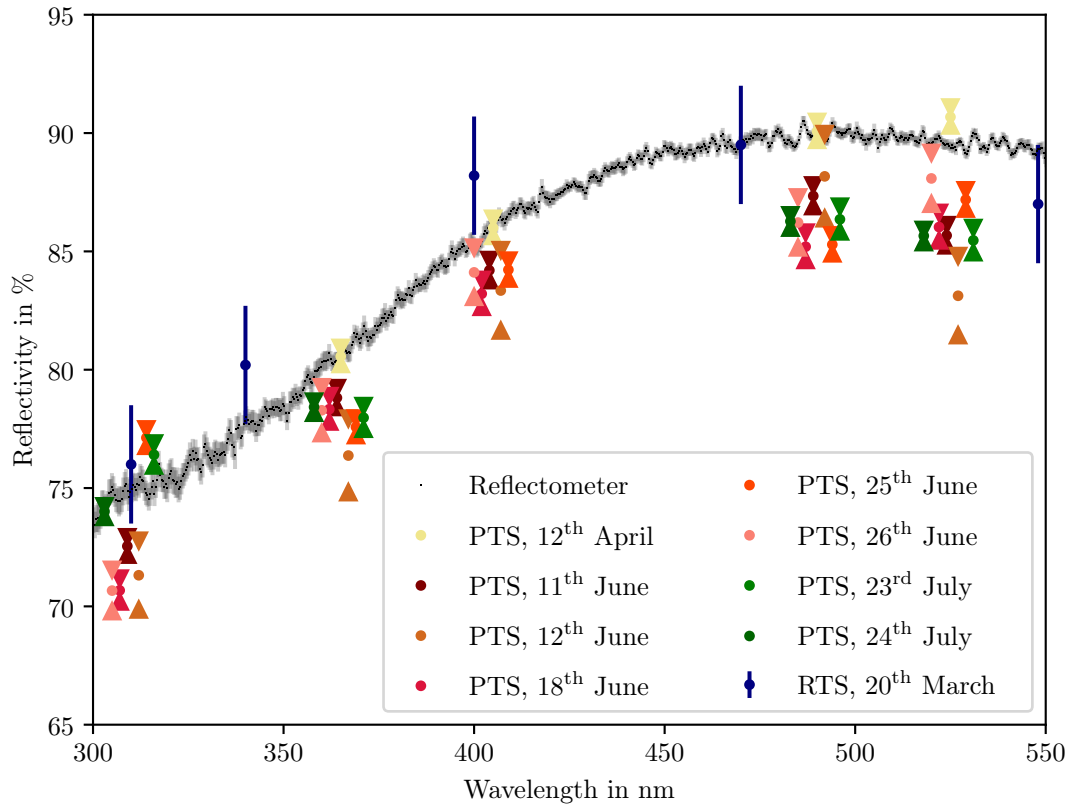


Figure 3.13: Reflectivity over wavelength. Comparison between multiple mirror test runs with both, the PTS and the RTS, and a measurement with the hand-held reflectometer (see Appendix III) of the same CTA MST mirror (KMST4). The uncertainties of these data are not yet corrected for a possible change in light source intensity between the main and the reference measurements because no voltage drop is reported. The various measurements of the PTS are slightly shifted on the wavelength axis to enhance readability.

mirror into its field of view with the intensity of its light source but it compares the reflected intensity of the mirror in test with the reflected intensity of a reference mirror with a known reflectivity (see Paragraph 4 in Section III.3). Assuming the reflectivities of the mirror in test and of the reference mirror have comparable radial dependences, this method is capable of estimating the total reflectivity of the mirror. Therefore, the surface (or total) reflectivity R_{surface} is always larger than the directed reflectivity R_{spot} (as long as the spot does not cover the whole field of view of the mirror).

In Figure 3.13, the results of several mirror test runs are compared with the surface reflectivity estimated with the hand-held reflectometer.

To begin with, it is apparent that the results of the reflectometer do not comply with the results of the RTS. As of now, the results of the RTS are not as reliable as the results of the reflectometer and several of the issues of the RTS are not understood, yet.

The results of the PTS are, with exceptions, below the results of the reflectometer. The spread of the results of the various PTS mirror test runs is about as large as the uncertainty of the individual results. The results of the measurements, conducted with the 310 nm LED, comply less with each other and with the results from the reflectometer. This LED has a lower intensity than the other ones (see Table 3.1). This leads to a lower signal-to-noise ratio and it is assumed that this is responsible for the worse results. A more powerful ultra-violet LED is already prepared to be used in the PTS.

Additionally, there are several possible sources of uncertainty when operating the PTS that are not discussed in this work. In Section 5.1 are recommendations for further studies on the PTS.

Chapter 4

Investigations Concerning the Mirror Test Stand

In this chapter describes several investigations that were conducted to improve the overall performance of the *Projecting Test Stand* (PTS). At the beginning, investigations concerning the CCD camera are described and subsequently the setup of the PTS and the light sources are examined.

4.1 CCD Camera Flat-Fielding

Flat-fielding is a procedure to improve the quality of digital images. Generally, there are three effects that reduce the quality of a digital image: First, the read-out electronics may have a *bias* that occurs in every pixel, in every image taken, independent of the exposure time or illumination. Second, thermal effects produce a so-called *dark-current* at the CCD even when it is not exposed [Mackay86, p. 267]. Given a constant CCD temperature, the dark-current is proportional to the exposure time. Finally, there can be variations in sensitivity between different CCD pixels [Mackay86, p. 276]. Since they register the same signal differently, the impact position of a photon on the CCD would matter. This has to be corrected, before the data of the whole CCD becomes comparable. Furthermore, even distortions of the camera optics can get corrected by flat-fielding.

Principally, it would be equally important to correct the electronic bias and the dark-current of the CCD camera. During a mirror test run, background pictures are taken, with the same bias and dark-current. When the background is subtracted from the data, the bias and dark-current correction happens implicitly. The flat-field correction has a proportional influence on the data and therefore it is not corrected by subtracting the background.

The idea behind flat-fielding is that any structure that is visible on a photograph of a completely uniform scene has to come from the camera itself [Mackay86, p. 276].

Although the electronic bias and the dark-current are not an issue for the data taken in the measurement, they are present in the pictures, used for flat-fielding (so-called *flats*). To avoid overcompensating them, they have to be removed from the flats.

Correction of Electronic Bias

To calculate the electronic bias of the readout electronic, the CCD camera has to be operated in a special mode. The shutter of the camera does not open and the exposure time is set to 0 s. Therefore, no pixels get illuminated and they do not have the time to collect charge from thermal processes either. The average pixel content is used as electronic bias.

In Figure 4.1 one of the zero-exposure pictures, used for the determination of the bias, is shown. The whole frame seems to be homogeneous and the individual pixel values spread around their average

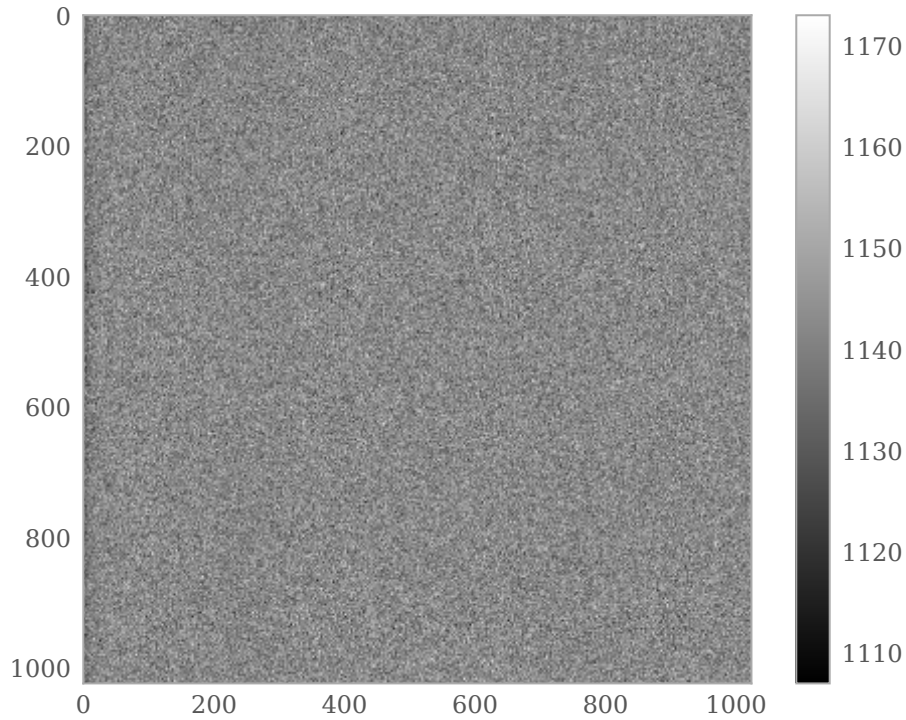


Figure 4.1: Zero-exposure picture of the *Alta U47-UV* camera. Shutter remained closed, exposure time was set to 0 s.

of 1143 ADC-counts, according to the Poisson distribution, with a width of almost 34 ADC-counts.

Correction of Dark-Current

The dark-current is examined from so-called *dark-frames* like the one in Figure 4.2. The camera has to be set to expose but keeps the shutter closed. Dark-frames contain the already discussed electronic bias, the thermal dark-current and random hits by cosmic ray particles. To examine the dark-current these operations have to be conducted:

- Subtract the electronic bias from every pixel in the dark-frame.
- Identify and remove all pixels from the data that were hit by a cosmic ray particle.
- Average the accumulated charge of every remaining pixel.
- Divide the average charge through the exposure time of the dark-frame to get the dark-current.

It is possible to remove the pixels that were hit by cosmic ray particles from the data because the accumulated charge varies greatly¹. It is assumed that the variations in the accumulated charge between the pixels, not hit by cosmic ray particles, follow a symmetric Poisson distribution (see Figure 4.3). To get the threshold that separates the pixels that solely accumulated charge through thermal processes,

¹The pixels hit by cosmic ray particles can have an order of magnitude more accumulated charge than the majority of the pixels.

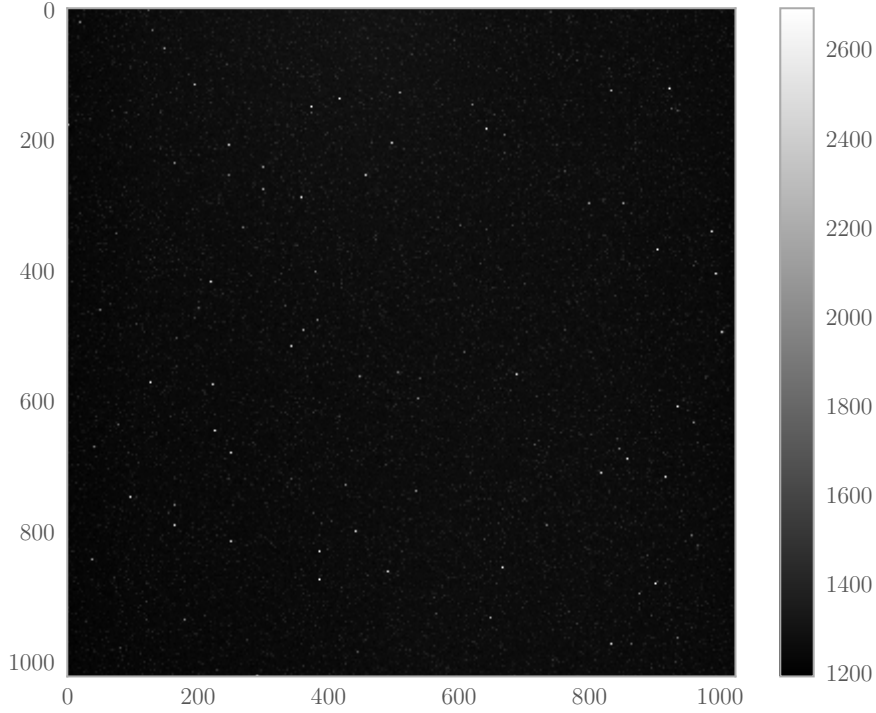


Figure 4.2: Dark-frame of the *Alta U47-UV* camera. Shutter remained closed, exposure time was set to 100 s. This picture is already corrected for electronic bias and cosmic ray particle hits.

from the ones that got hit by cosmic ray particles, the difference between median and minimum of the distribution was added to the median.

For the dark-frame, shown in Figure 4.2, the threshold is roughly 200 ADC-counts. With this method, about 0.03 % of all pixels get flagged as hit by cosmic ray particles and removed from the data.

In Figure 4.2, the pixels, hit by cosmic ray particles, are replaced with the double of the threshold. This is to make them visible, while still remaining resolution for the structure beneath them.

Creating a Flat-Fielding Correction Matrix

Flats are usually taken from a homogeneous surface that is illuminated uniformly or from a cloudless sky during twilight [Mackay86, p. 277].

Multiple n flats F_n are taken with exposure times $t_{\text{exp } n}$ and corrected for their electronic bias Q_{bias} and dark-current J_{dark} .

$$C_n = \begin{pmatrix} f_{n0,0} & \cdots & f_{n0,1023} \\ \vdots & \ddots & \vdots \\ f_{n1023,0} & \cdots & f_{n1023,1023} \end{pmatrix}^2 - Q_{\text{bias}} \times \mathbb{1} - J_{\text{dark}} t_{\text{exp } n} \times \mathbb{1}$$

The results C_n are added together (to C) to further reduce statistical effects. To generate the correction matrix M , each entry of the corrected flats stack C is inverted and divided through the mean $\langle \frac{1}{C} \rangle$

²It is implied that a CCD with 1024×1024 pixels is in use.

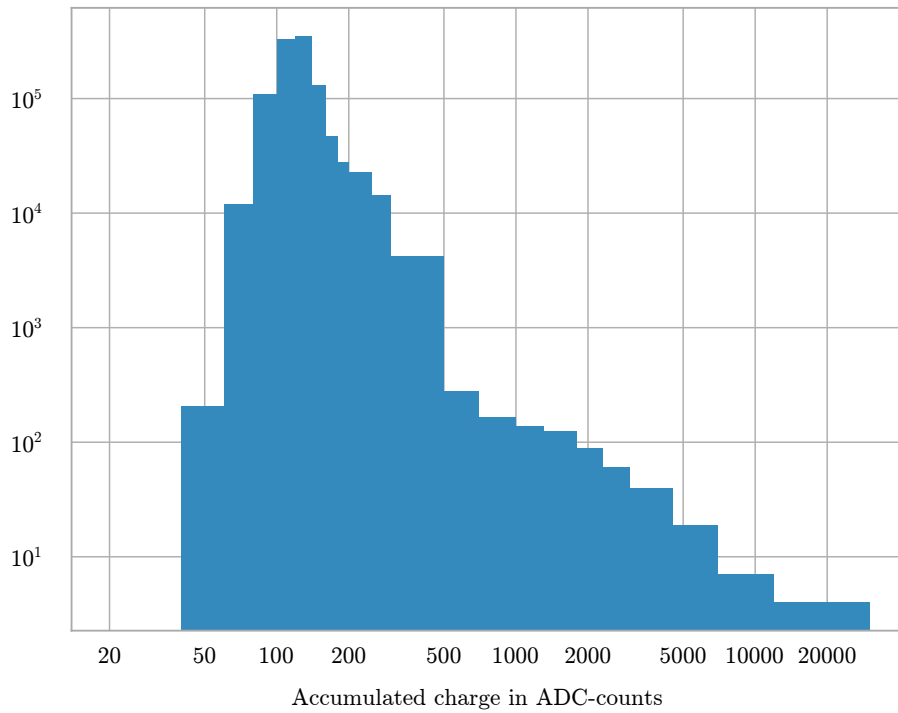


Figure 4.3: Histogram of the accumulated charge in the dark-frame, which is shown in Figure 4.2. The entries of this histogram are already corrected for the electronic bias but not for the pixels that were hit by cosmic ray particles.

of the whole inverted stack ³.

$$M = \begin{pmatrix} \frac{1}{c_{0,0}} < \frac{1}{C} >^{-1} & \dots & \frac{1}{c_{0,1023}} < \frac{1}{C} >^{-1} \\ \vdots & \ddots & \vdots \\ \frac{1}{c_{1023,0}} < \frac{1}{C} >^{-1} & \dots & \frac{1}{c_{1023,1023}} < \frac{1}{C} >^{-1} \end{pmatrix}$$

The average value of the resulting correction matrix M is $\langle M \rangle = 1$.

Such a correction matrix M is shown in Figure 4.4. Pictures of both, the reflected light spot and the projection of the mirror area, are comparable in size with the yellow area. As long as the field of view of the camera is perfectly aligned with the light spots, a correction would not be necessary.

The mirror focuses the reflected light at the same position on the screen for all measurements. But the LED light spots in the reference measurements are not at the exact same position for each LED since they are not aligned perfectly. Additionally, the reference spot is not necessarily at the same position as the reflected light spot. Figure 4.4 shows that the sensitivity of the CCD differs about 5 % between the ocker area in the center and the red area at the edges.

If for example, the reflected light spot of a mirror test run hits the central area of the frame with its highest intensity and only one quarter of the region used to evaluate the reference intensity is positioned in the red area, the reference intensity would be estimated about 1 % smaller.

This effect should be corrected for. Due to technical shortcomings of the available camera CCD, it was not possible to create any meaningful correction matrix for the pictures utilized in this work.

³Due to internal approximations of the software used to evaluate the correction matrix (numpy), the inversions and the averaging do not commute.

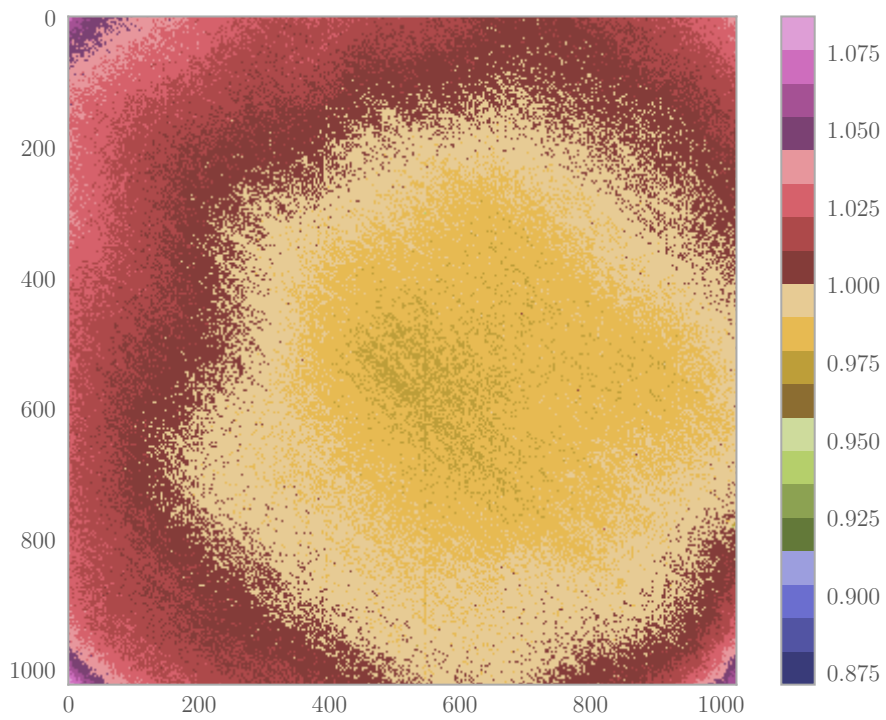


Figure 4.4: Exemplary flat-fielding correction matrix. The color-scheme was chosen to highlight the boundary between the region where the camera is more sensitive (beige) and less sensitive (red). The diagonally striped pattern is possibly due to small variations in pixel size [Mackay86, p. 277]. The zero-exposure, dark, and flat frames used to create this correction matrix, were taken with the *Alta U47-UV* camera in Spring 2018.

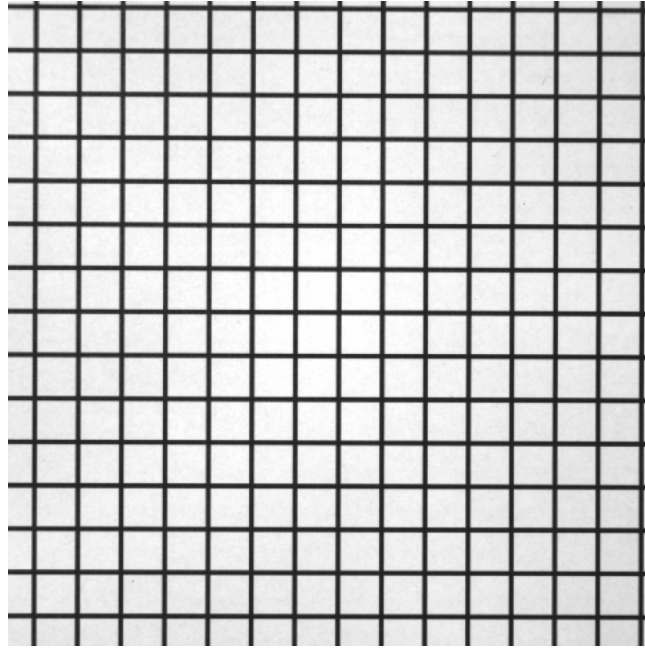


Figure 4.5: Grid-pattern of white squares with an edge length of 9 mm separated by black lines with a thickness of 1 mm. This pattern was designed by Jürgen Dick. Patterns like this are used in the MTS to examine how many pixel-widths (on the photograph) are corresponding to 1 cm on the screen.

4.2 Image Scale

The camera takes pictures of the screen. Each CCD pixel contains a particular intensity and represents a specific area on the screen. It is important to know which area on the screen is represented by one pixel to calculate the local intensity and to project the shape of the mirror on the screen in the reference measurement. This *image scale* can be calculated in two different ways:

4.2.1 Determination via Testgrid

To determine the image scale of the setup, a picture with precisely known features is photographed. An example would be the grid in Figure 4.5. This pattern was printed on a sheet of paper and taped onto the screen. After the precise scale is examined, the pattern can be removed and a mirror analysis can be conducted with this scale.

A script was written to analyze such pictures. To get the distance on the screen, which is represented by the width of a CCD pixel, the script has to do the following for each line of pixels:

- Dismiss it if it has too many dark pixels because this line would be from one of the horizontal lines from the grid.
- Find all peaks in intensity over the number of pixels in a line (see also Figure 4.6).
- Lay a Gaussian fit at the peaks and calculate their extremum to get a representation of the borders between the squares.
- Calculate the distances between neighboring peaks (in pixel-widths) to get the equivalent of 1 cm on the screen.
- Take the average of the results for all peak-pairs in all lines to get the mean image scale and use the standard deviation as uncertainty.

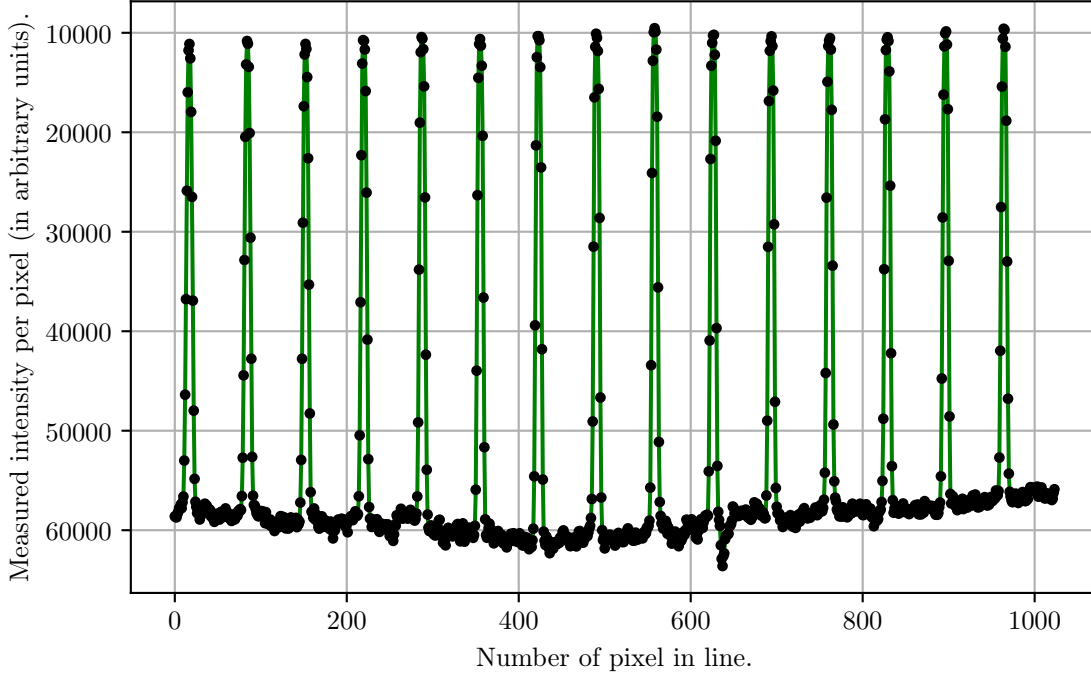


Figure 4.6: Horizontal cut through a grid like in Figure 4.5, avoiding an all-black line. The black points are the ADC-counts representing the intensity deposited in the corresponding pixels. The green line is the connection between the data-points and it is there to make the order of the points better visible.

Then the script transposes the grid and repeat the analysis to get the equivalent of 1 cm (vertically) in pixel-heights.

4.2.2 Determination via Distanzmeter

If the image scale a' is known for a given distance D' between camera-body and screen it is possible to calculate it for other distances D , too. The new image scale a can be calculated via the intercept theorem:

$$a = a' \times \frac{d}{d'} \quad (4.1)$$

The lengths d and d' correspond to the distance between the screen and the intersection point. The intersection point is the central plane of the objective. Since the distances, D and D' , are measured from the body of the camera they need to be corrected by the distance x_0 between camera-body and objective:

$$a = a' \times \frac{D - x_0}{D' - x_0} \quad (4.2)$$

The distance between camera body and central plane of the objective x_0 was obtained by taking pictures of the grid at various distances. In Figure 4.7 are the results of those measurements displayed over the distance D . A straight line was fitted on this data. The value of the abscissa at the point of interception with the fit is the distance between camera body and objective $x_0 = 173.3489(49)$ mm.

The uncertainties for the new image scale a depend on the known properties and on the uncertainty of the distanzmeter used to measure D .

$$\Delta a = \sqrt{\left(\frac{D - x_0}{D' - x_0} \Delta a'\right)^2 + \left(\frac{a'}{D' - x_0} \Delta D\right)^2 + \left(a' \frac{D - x_0}{(D' - x_0)^2} \Delta D'\right)^2 + \left(a' \frac{D - D'}{(D' - x_0)^2} \Delta x_0\right)^2} \quad (4.3)$$

It is necessary to reevaluate the image scale if the arm, carrying the camera and screen, was shortened and reassembled. Equation 4.2 shows that the image scale a is not any different from a' if the distanzmeter measures the same length D as the previous distance D' (with a constant x_0). Although, the uncertainty of the image scale Δa becomes larger since the distanzmeter itself has an uncertainty and cannot guarantee that the distanced D' and D are equal. With the assumptions of $a = a'$ and $D = D'$ the equation for the uncertainty of the image scale Δa simplifies to:

$$\Delta a = \sqrt{\Delta a'^2 + \left(\frac{a'}{D - x_0}\right)^2 \times (\Delta D + \Delta D')^2} \quad (4.4)$$

4.2.3 Comparison between the two Methods

It is necessary to determine the image scale with the test-grid sometimes, since the second method needs a reference scale. Especially after the zoom-factor of the CCD-camera is changed, a reevaluation of the image scale becomes inevitable. To ensure the reliability of the image scale calculated from the distance between camera and screen, both methods were used on the same data.

In Figure 4.7, eleven red points are visible. Those points are the result of the analysis using a picture of the grid at the corresponding distance. The red rectangles are the uncertainties of the points. The uncertainty on the image scale is taken from the analysis and the uncertainty on the distance is the standard derivation of the distribution from five measurements with the distanzmeter. The point marked with a darker tone of red was measured with a more extensive sample size to serve as a more reliable reference for the second method of analysis.

The black line is the theoretical development of the image scale over the distance using Equation 4.2 with the data from the reference point: $D' = 2.4474(20)$ m and $a' = 0.14786(13)$ mm.

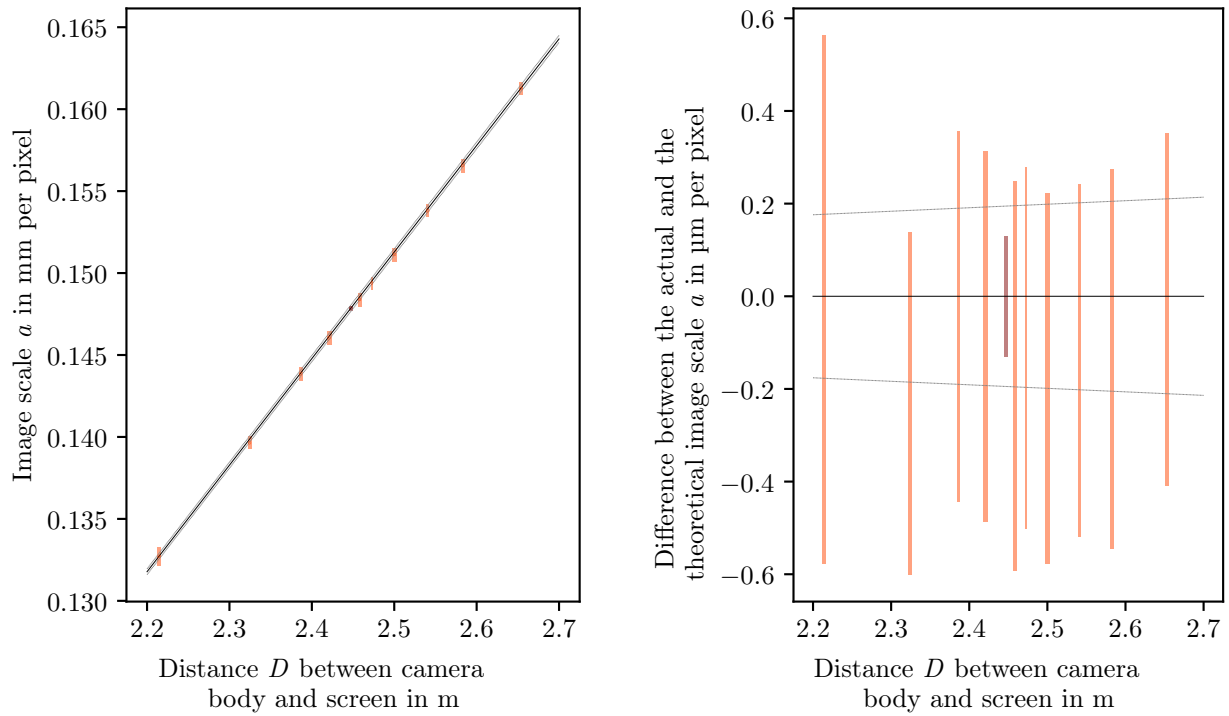
4.2.4 Relevance of the Scale

What is the impact of the uncertainty of the image scale on the final result? Is it beneficial to analyze it so carefully? Figure 4.8 shows the dependency between the uncertainty of the image scale and the uncertainty of the final result from the analysis, the reflectivity.

In Figure 4.8 are results from several different analysis-runs. All of them were obtained using the same software and the same data of the mirror KMST4 from the 24th July, 2018. The image scale a of this run was $a = 0.14775(43)$ mm and it resulted in a reflectivity (averaged over all wavelengths) of $R_{\text{spot-2mrad}} = 79.7\%$. The analysis also uses two other distances, the reference distance (between the LEDs and the screen when the mirror testing stand is in its reference position) $D_{\text{ref}} = 1.793(1)$ m and the double focal length $D_{2f} = 32.271(10)$ m.

The runs that are represented by green marks were done using only the shown uncertainties of the image scale. On the other hand, the runs represented by the red marks had also uncertainties in their reference distance and their double focal length. Those uncertainties were guessed based on the quality of the distanzmeter and the distance as $\Delta D_{\text{ref}} = 1$ mm and $\Delta D_{2f} = 1$ cm.

So the data shows a direct, linear relationship between the uncertainties of the image scale and the reflectivity. Since the PTS should operate with an uncertainty of $\Delta R = 1\%$ it is important to measure the image scale carefully. Image scales with uncertainties in the area of $\Delta a \approx 0.25 \mu\text{m}$ were already achieved.



(a) Image scale a over distance D between camera body and screen.

(b) Uncertainties Δa of the image scale, estimated with the distanzmeter over distance D between camera body and screen.

Figure 4.7: Relation between the image scale a and the distance between camera and screen D . The red points are the results from the direct method to determine the image scale. The point in darker red was measured with multiple pictures to serve as a reference value. The red rectangles are the uncertainties of the values. The black line is the theoretical development based on Equation 4.2. The broken lines are created from the most extreme values out of the uncertainty intervals from the reference point. The graphic on the right shows the same data but the theoretical values (black line) are subtracted from all data points to emphasize the uncertainties.

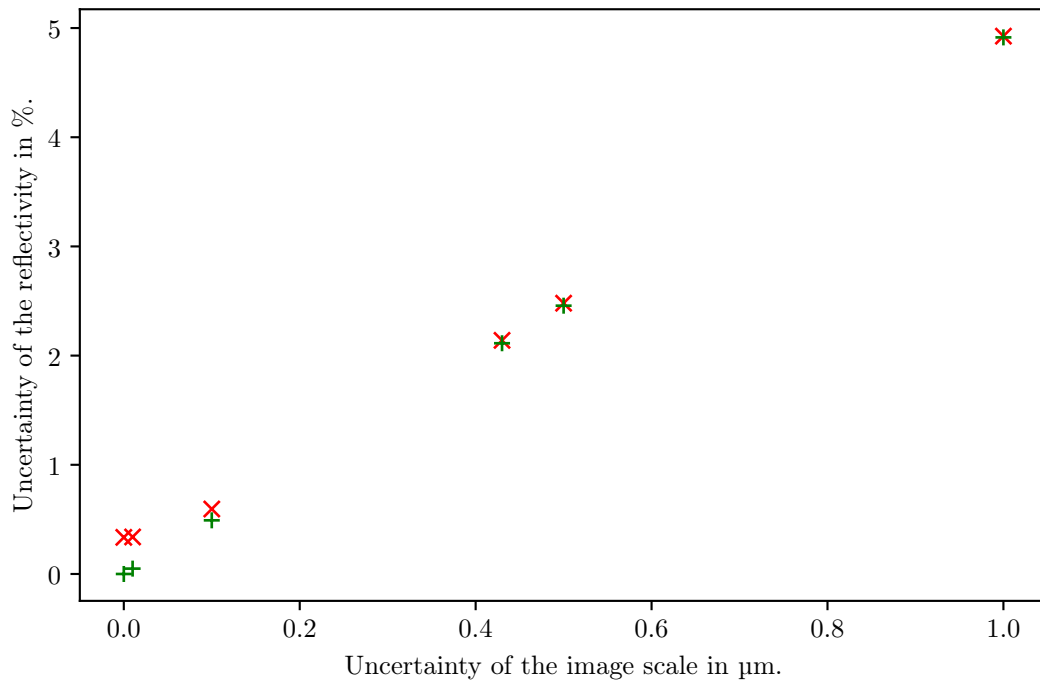


Figure 4.8: Relation between the uncertainty of the image scale and the uncertainty of the reflectivity of a mirror. The uncertainty of the reflectivity is given in *percent points* not as a percentage. The red 'x' marks show the results from the analysis under normal conditions with only the uncertainty of the image scale altered. The green '+' marks show the results if the uncertainties of the reference distance D_{ref} and the double focal length D_{2f} are set to zero ($\Delta D_{\text{ref}} = 0$ and $\Delta D_{2f} = 0$).

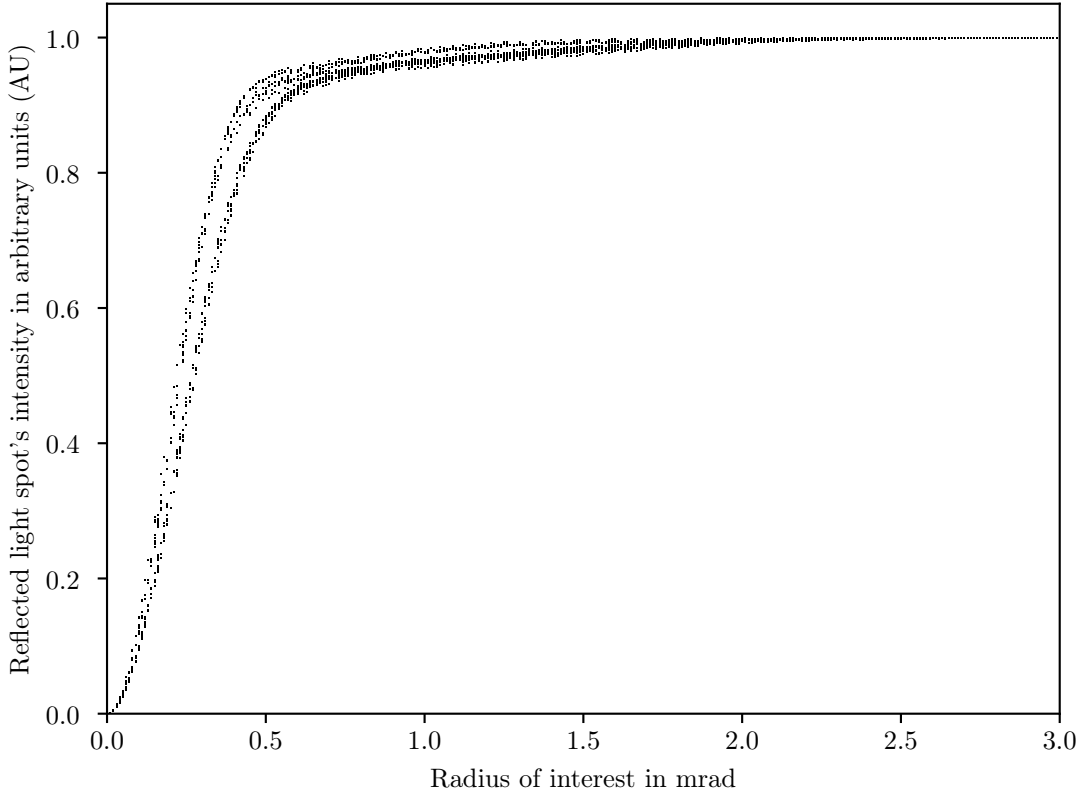


Figure 4.9: Integrated intensity reflected into a circle around the center of mass of the intensity distribution over the its radius $I_{\text{spot}}(R_{\text{roi}})$. The radius of interest is given as an angle that is equivalent to the radius, observed from the mirror. The development of the intensity is normalized to the value at a radius of $R_{\text{roi}} = 3 \text{ mrad}$ so different measurements can be plotted together. The measurements are taken from different mirror test runs and different wavelengths. The uncertainties of the individual measurements are smaller than the spread of all of the measurements. The uncertainties of the individual measurements are therefore not shown in this figure because they convey no additional information and only reduce the readability further.

4.3 Region of Interest around the Center of the Reflected Light Spot

The radius of the region of interest R_{roi} is used to draw a circle around the center of mass of the intensity distribution of the reflected light spot, in ACTAM. To get the reflected intensity of a spot $I_{\text{spot}}(R_{\text{roi}})$, all ADC values inside this circle get added up. On the official website of the CTA Observatory, jama.cta-observatory.org, it is stated that all the intensity reflected into a region with an opening angle of 3.75° has to be considered. This is too broad to be used with the PTS.

Since the mirrors in test produce a reasonable narrow light spot where most intensity is reflected into (see Figure 4.9), it is possible to approximate the total reflected intensity of the mirror $I_{\text{reflected}}$ with the intensity of a reflected light spot $I_{\text{spot}}(R_{\text{roi}})$, even with a radius of interest $R_{\text{roi}} < 0.5^\circ$.

Figure 4.9 shows the intensities of a multitude of reflected light spots (from different mirror test runs and observed in different wavelengths), analyzed with various radii of interest up to $R_{\text{roi}} = 3 \text{ mrad}$.

It is clearly visible that the vast majority of the intensity is reflected into an area with a radius of about 0.5 mrad . If the analysis already encircles the majority of the reflected light spot, the intensity shows a linear relationship with the radius of interest. As a hypothesis, the linear relationship can be

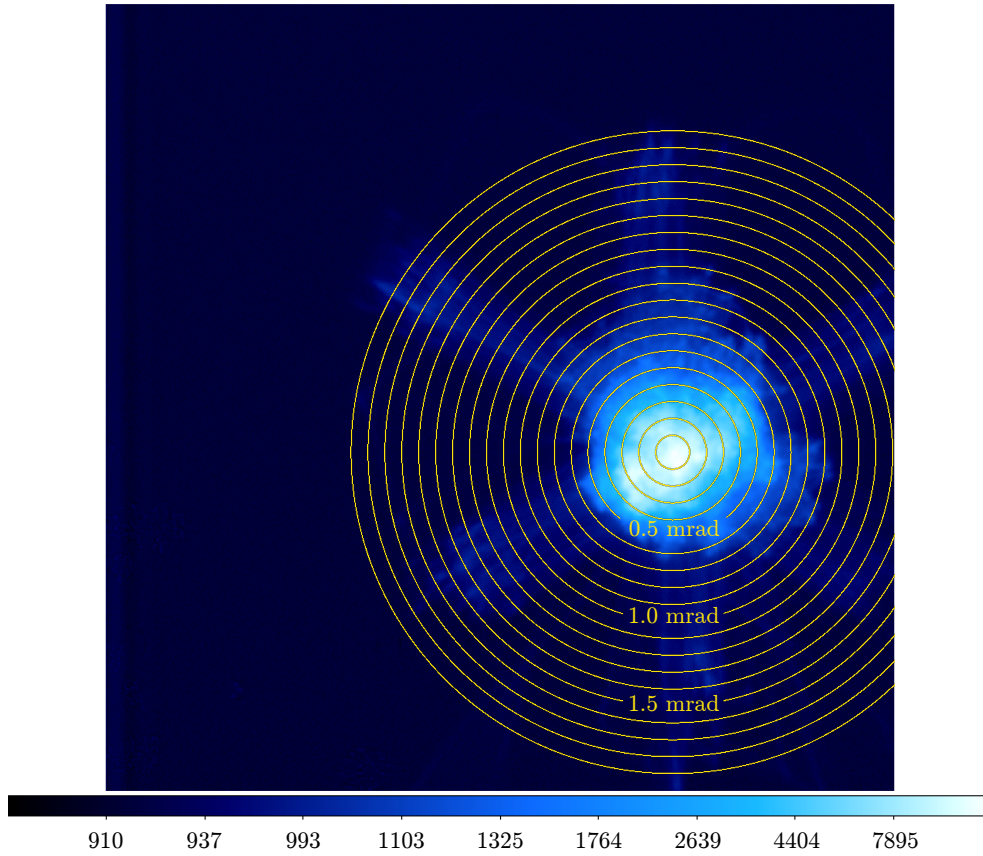


Figure 4.10: Exemplary picture of a reflected light spot of the mirror facet KMST4. The integration circles are shown for various radii of interest, given as angles observed from the mirror. The unit of the nonlinear scale are ADC counts.

explained with the six rays emitting from the center that this specific mirror under test shows. Since these rays have an approximately even width, they add a constant amount of intensity per radius to the total intensity.

In Figure 4.9 is a second break of slope at a radius of interest between $R_{\text{roi}} = 1.5 \text{ mrad}$ and $R_{\text{roi}} = 2 \text{ mrad}$ visible. This also hints at the relevance of the rays to about 5% of the total reflected intensity, since those rays typically exceed up until an angular distance of about 2 mrad from the center of mass (see Figure 4.10). Additionally, it has to be stated that integration circles start to hit the edge of the frame for radii between $R_{\text{roi}} = 1.5 \text{ mrad}$ and $R_{\text{roi}} = 2 \text{ mrad}$.

Figure 4.10 shows that the PTS has the possibility to analyze the light spot with a radius of interest of $R_{\text{roi}} = 2 \text{ mrad}$ if the reflected light spot is centered well into the CCD camera field of view. Since Figure 4.9 shows that there is no significant intensity added to the final result for radii of interest larger than $R_{\text{roi}} = 2 \text{ mrad}$, the total reflected intensity of the mirror can be approximated with this spot intensity

$$I_{\text{spot}}(R_{\text{roi}} = 2 \text{ mrad}) \lesssim I_{\text{reflected}} .$$

4.4 Orientation between Camera, Screen and Light Source

The Ideal Setup Geometry

The ideal geometric setup of the PTS is described in Section 3.4.4. Ideally, the centers of the camera, screen, light source and mirror are on the same height above the ground.

In the main position, the light source and the screen are both twice the focal length of the mirror D_{2f} away from it and as close to the principal axis of the mirror as possible without having the camera blocking the light to or from the mirror. In the reference position, the screen is positioned between the light source and the mirror. It is at a distance D_{ref} as far away from the light source as possible whilst keeping the entire light spot from the light source in the field of view of the camera. At all times, the camera is positioned on the principal axis of the mirror and it is oriented normally to the screen. The angle θ between the incoming *light rays* from the light source and the normal of the screen kept as small as possible by maximizing the distance D_{ref} and by minimizing the divergence between light source and principal axis of the mirror.

When the PTS is set up ideally, all effects from the geometry between camera, screen, light source and mirror can be corrected for. These effects are, on the one hand, the perspective distortion of the structure of the light spot on the screen and, on the other hand, the inequalities in distance between the surface of the screen and the light source.

On Page 29 it is shown that the influence of the perspective distortion on the measured reflectivity is negligible. On Page 27, it is described that the inequalities in distance D' to the light source for different points P' on the screens surface can be corrected by multiplying each pixels collected intensity with the ratio $\frac{D'^2}{D_{\text{ref}}^2}$. Figure 4.11 highlights the impact of this correction. Several independent mirror test runs were analyzed with and without correcting their reference pictures with this scheme. The differences in the results are of the order of 0.3 %. This cannot be neglected when aiming at a total measurement uncertainty of the reflectivity of 1 %.

The Actual Setup Geometry

The PTS of the IAAT is not exactly set up following the ideal geometry. The centers of the camera, screen, light sources and mirror are not on the same height but the centers of the camera and the light sources are 106 cm above the ground, the center of the screen is 118 cm above the ground and the center of the mirror is only 100 cm above the ground.

The positioning of the screen is uncritical since the screen is large enough so it can be used normally in the plane of camera and light sources.

The influence of the positioning of the mirror center depends on the angle that the screen and light sources have to the principal axis. On Page 27 it is explained that the angle to the optical axis of the screen that results from a horizontal displacement of the screen from the principal axis of 30 cm to 40 cm does not affect the analysis significantly. Therefore, a vertical displacement of less than 10 cm cannot have an effect either.

4.5 Alignment of Light Source and Mirror

The alignment between light source and mirror is important. In the case that the light cone from the light source does not hit the complete mirror, two things happen. First, the un-illuminated part of the mirror does not get analyzed during a mirror test run and second, the intensity reaching the mirror is over estimated.

For the time being, the alignment of the light sources onto the mirror is done *by hand*. It is just verified that the mirror is completely inside the distinctly visible light cone.

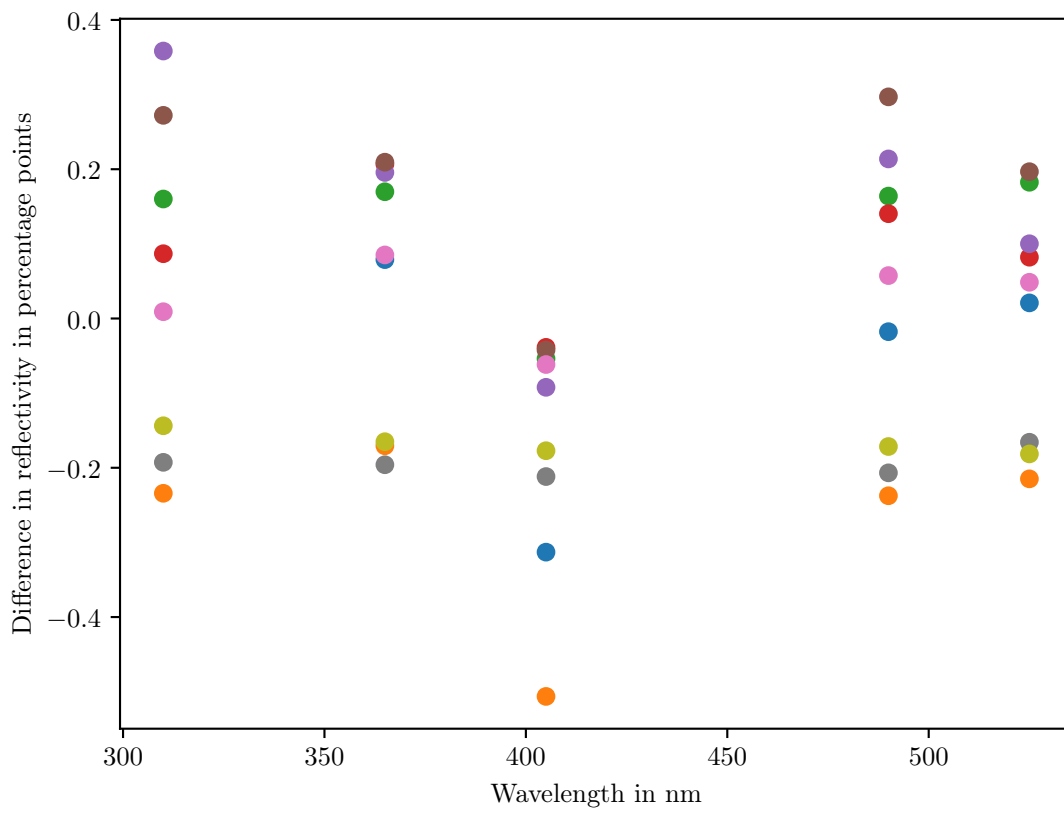


Figure 4.11: Difference between the results of the reflectivity analysis with and without correcting for the inequalities in distance (and therefor local intensity) between the screen surface and the light source. Different colors indicate independent mirror test runs with the mirror facet KMST4.

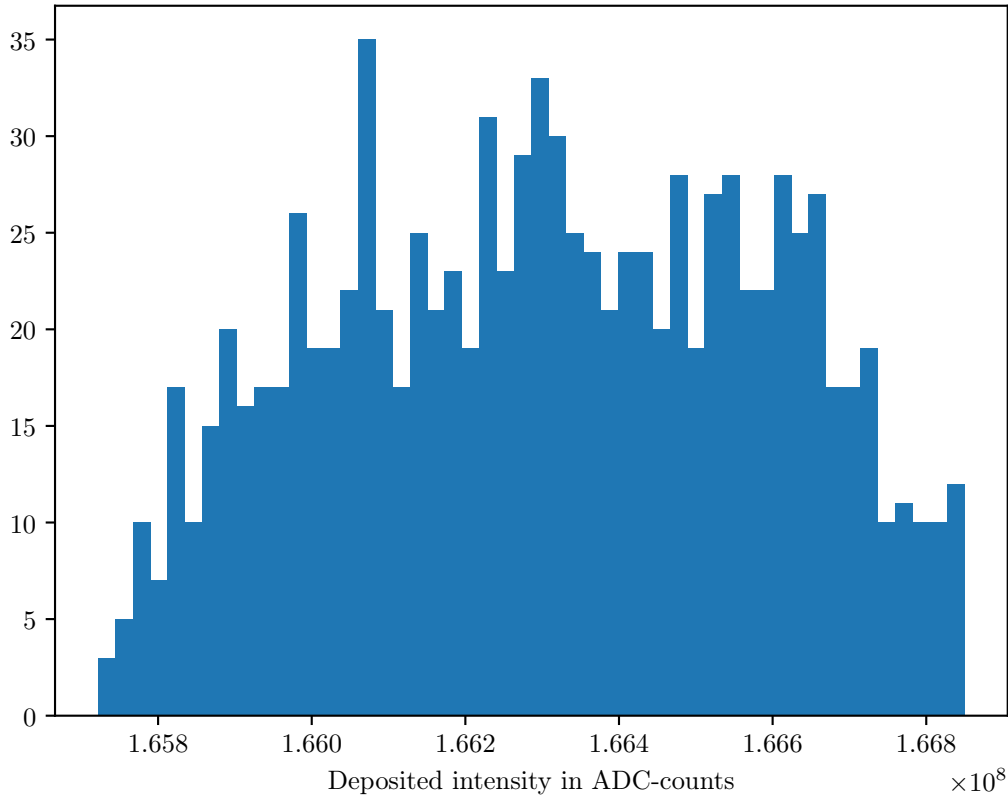


Figure 4.12: Exemplary histogram of the deposited intensities in an area equivalent to the area of the projection of the mirror on the screen. This data results from the averaging of eighteen measurements with the 525 nm LED.

4.5.1 Isotropy of the Light Source

If the light sources are emitting intensity isotropically (inside the light cones defined by the collimators), it is not relevant where, in the light cone, the mirror is positioned and further which part of the reference pictures is used to estimate the reference intensity.

To estimate the degree of anisotropy of the light sources, the homogeneity of the light spots on the reference pictures were analyzed. An exemplary reference picture can be seen in Figure 3.10 on Page 33. Pictures like this were used to estimate the center point and the radius of the projected light spot by hand. Only the homogeneity of the intensity distribution inside this circle is examined.

The homogeneity of any distribution is connected to an observation scale. In this experiment, the relevant scale is the size of the mirror. Therefore, the size of the projected mirror is calculated with the distances between light sources, screen and mirror, corresponding to the reference pictures. Finally the total deposited intensity is calculated for 1000 mirror-projection sized circles, which are randomly distributed inside the light spot. These intensities are then plotted as histograms like the example in Figure 4.12.

The histogram in Figure 4.12 shows that the reference intensities $I_{\text{reference}}$ differ about 1 % from each other ⁴.

⁴For this individual measurement with the 525 nm LED. Other measurements, sometimes with the same LED, differ from this result for reasons that are not yet understood.

Therefore, the estimated reference intensity $I_{\text{ref-est}}$ can differ from the actual reference intensity $I_{\text{ref-act}}$ depending on the positioning of the mirror projection and the reference region inside the perceived to be homogeneous light spot. To account for this, the homogeneity H is introduced. $H = 1$ when there is no trend in the dependency between deposited intensities and locations of the analyzed regions. $\Delta H = 0$ if the deposited intensities, in all examined areas in the light spot, are identical; equating to a single peak in the histogram. Otherwise, ΔH is the width of the distribution in the histogram divided through the average value.

The reflectivity of a mirror is the ratio between the reflected intensity I_{spot} and the actual reference intensity $I_{\text{ref-act}}$

$$R = \frac{I_{\text{spot}}}{I_{\text{ref-act}}}.$$

Since the actual reference intensity is not measured, this equation is modified to be used in accordance with the estimated reference intensity:

$$R = \frac{I_{\text{spot}}}{H \times I_{\text{ref-est}}} \quad (4.5)$$

The uncertainty of the reflectivity that is presented as Equation 3.19 becomes:

$$\Delta R = R \times \sqrt{\left(\frac{I_{\text{spot}}}{\Delta I_{\text{spot}}}\right)^2 + \left(\frac{I_{\text{ref-est}}}{\Delta I_{\text{ref-est}}}\right)^2 + \left(\frac{H}{\Delta H}\right)^2} \quad (4.6)$$

Assuming, all variations in the estimated reference intensity are statistical, the homogeneity of the examined region is $H = 1$ with variations ΔH equal to the relative width of the distribution. The distribution, shown in Figure 4.12 has a relative width of about 0.66 %. The influence of the inhomogeneities on the resulting uncertainty of the reflectivity is not negligible when compared with common relative measurement uncertainties of intensities like $\frac{I_{\text{spot}}}{\Delta I_{\text{spot}}} \approx 0.02 \%$ and $\frac{I_{\text{reference}}}{\Delta I_{\text{reference}}} \approx 1.45 \%$ (see Section 3.6.2 on Page 38).

The relative width of the distribution of the estimated reference intensities exceeds 1 % in a not negligible share of the examined reference pictures. Their histograms do not support the assumption that the spread is purely statistical. In the case that the examined region contains the border of the light spot, the estimated reference intensity becomes dependent on its location. Since the radius and the center of the examined region is estimated by a human with no other criterion than their perception of homogeneity, the reproducibility and reliability can be challenged.

It is not acceptable for the PTS to have a measurement uncertainty of several percent solely due to inhomogeneities in the reference light spot. Until an analysis scheme is developed that can reliably identify the homogeneous region in the reference light spot, it is not suitable to place the mirror and the reference region anywhere in the light spot and correct for their possible deviation in intensity.

It is necessary to align the reference region with the projection of the mirror as closely as possible. Therefore, the center of the light cone of the light sources is aligned with the mirror center and the reference region is positioned around the center of the light spot in the reference pictures.

4.5.2 Center of the Reference Circle

To calculate the reflectivity R of a mirror with Equation 2.1 on Page 9 it is necessary to know the incoming intensity I_{in} and the reflected intensity I_{spot} . The reflected intensity I_{spot} is measured in the main part of the mirror test run. The incoming intensity I_{in} cannot be measured directly with this setup. Therefore, the reference measurement has to be conducted. In Section 4.7, it is shown that the power output of the LEDs is reproducible. To get the same amount of energy in both measurements, only the intensity deposited in a particular circle on the screen is used for the analysis. This circle should have the same area than the projection of the mirror on the screen. Ideally, this circle and the

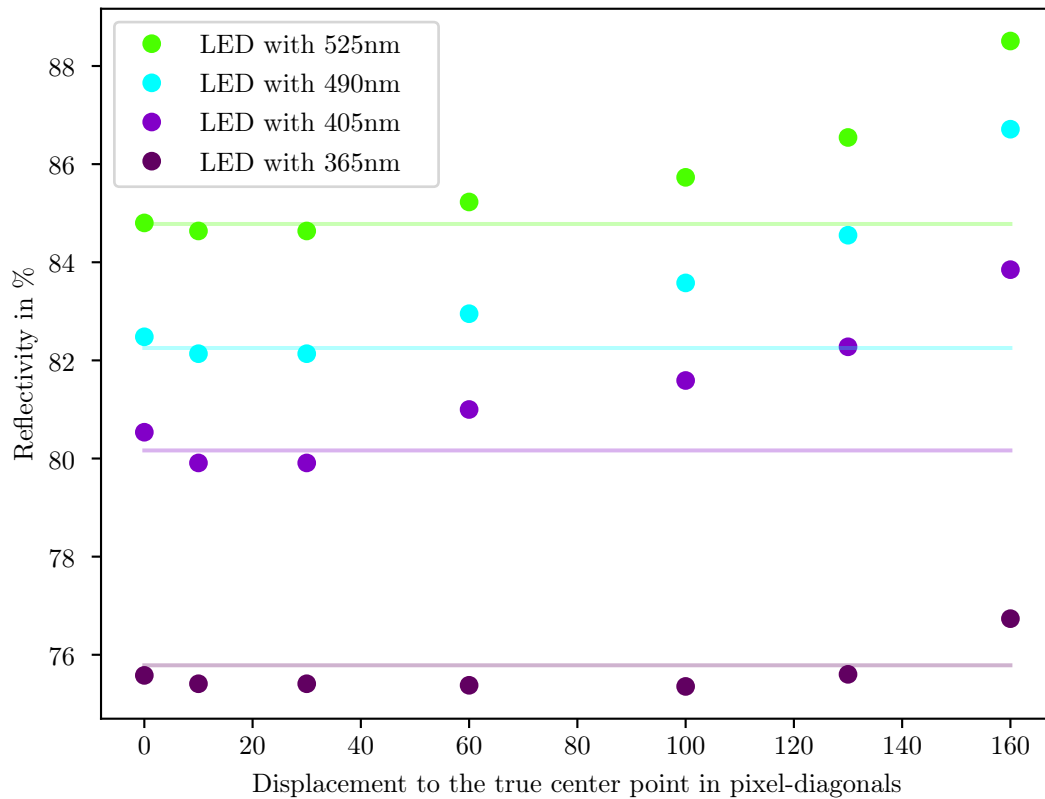


Figure 4.13: Comparison between the results of several analysis runs of a test with the mirror KMST4. The difference of the various runs was that the ACTAM got different values for the center of the reference position as input. The horizontal lines are the results for when the ACTAM uses the same position (500|500) for all wavelengths. This was common practice before the start of this work. The data-points at a displacement of 0 pixel-diagonals are the result of an analysis run with spot-centers that were obtained manually using a draw-circles tool.

light cones from the LEDs are concentric and it is assumed that the LEDs are aiming on the mirror center.

The LED-collimator array does not aim all LEDs equally well on the mirror. Therefore, their reference-circles may have different centers. The analysis program *ACTAM* uses reference-circle centers that have to be identified manually. Since the light cone, reaching the mirror, is not necessarily concentric to the light cone, emitted by the light source, and since the center of this light cone is estimated by hand, there is a possible gap between the true center of the reference-circle and the used one.

The impact of this discrepancy has to be analyzed. All reference pictures from one mirror test were inspected closely and the center point of their light spot was examined with the circular tool from ds9. The center points from pictures belonging to the same measurement were averaged and those results are the best available guesses for the real center points. In the scope of this analysis those best guesses are used as the true center points. Figure 4.13 shows the results from the analysis using the true center points on the one hand compared with several wrong ones. The reflectivity of the analysis with the true center points has to be compared with the reflectivities from other analysis runs. The other points in this diagram are generated by adding the stated value to both the x- and y-coordinate of the

center point. Those analysis runs are all performed with an individual center point for each wavelength. Uncertainties are not shown in this figure to maintain readability. Since the uncertainty of all of the shown data points is approximately equal (about 2 %), it is not interesting for this evaluation. The horizontal lines are the result from an analysis with the same center point (500|500) for all wavelengths.

This center point was commonly⁵ used because it was convenient. The data in Figure 4.13 shows that this practice is reasonable because the difference between the results, gained this way, is not significant compared with the uncertainty of reflectivity $\Delta R \approx 2\%$ gained from the analysis itself. Using an universal center point is far less effort for the person carrying the analysis out.

Because the data is flat (with respect to the 2 % uncertainty) up to a displacement of circa 50 pixel-diagonals, the reflectivity would not change if there were slender mistakes done while determining the central point of the reference pictures. The co-developer of an earlier version of ACTAM did a similar study with multiple mirrors and stated that the results are comparable between them. The data she presented in [Göggelmann17, p. 41] is similar to the data in Figure 4.13. This data seconds her conclusion that it is accurate enough to examine the center point position manually.

4.6 Point-Like Characteristic of the Light Source

The intensity I of a point-like light source depends on their power output P and on the inverse of the squared distance D between light source and detector:

$$I(D) = \frac{P}{4\pi D^2} \quad (4.7)$$

This equation is also used in ACTAM to calculate the incoming intensity I_{in} on the mirror based on the intensity measured in the reference measurement. For extended light sources with an area $A = \pi R^2$, the intensity drops less rapidly with increasing distance D .

$$I(D) = \frac{P}{4\pi R^2} \times \ln \left(1 + \frac{R^2}{D^2} \right) \quad (4.8)$$

The LEDs have square chips with an area A of 1 mm^2 . To examine if the approximation of the LEDs as point-like light sources is valid, their intensity I was measured with a photodiode in several distances D . The data was analyzed twice, once with the model of a point-like light source and once with the model of a round light source with their actual area of $A = 1 \text{ mm}^2$. This comparison is shown in Figure 4.14. The absolute values for the intensity I are not stated since conversion rate between incoming intensity and the generated current on the photodiode is unknown. It is sufficient to state the intensity in arbitrary units, because only the change in intensity I with distance D is interesting, in this measurement. The data seems to follow the inverse quadratic law and the fitted function using this law describes the data well. In fact, the function, fitted using Equation 4.8, that describes spread-out light sources is visually, indistinguishable from the other one. Additionally, both fits result in a comparable value for the total power of the LED with $790.24 \text{ m}^2 \text{ AU}^7$ and $790.27 \text{ m}^2 \text{ AU}$, respectively. This corresponds to a relative error in the intensity reaching the mirror of 3.8×10^{-5} . Therefore, the approximation of the LEDs as point-like light sources is valid.

Figure 4.15 shows the development of the intensities of four LED with increasing distance. The fitting routine does not take into account the uncertainties of the individual measurements because they are small in relation to the distance between the measurements. The quality of the fitted function

⁵It was used in the analysis show in Figure 4.8 among others.

⁶Approximation only valid in a vacuum.

⁷The arbitrary power unit is an arbitrary intensity unit (AU) multiplied by an area of 1 m^2 .

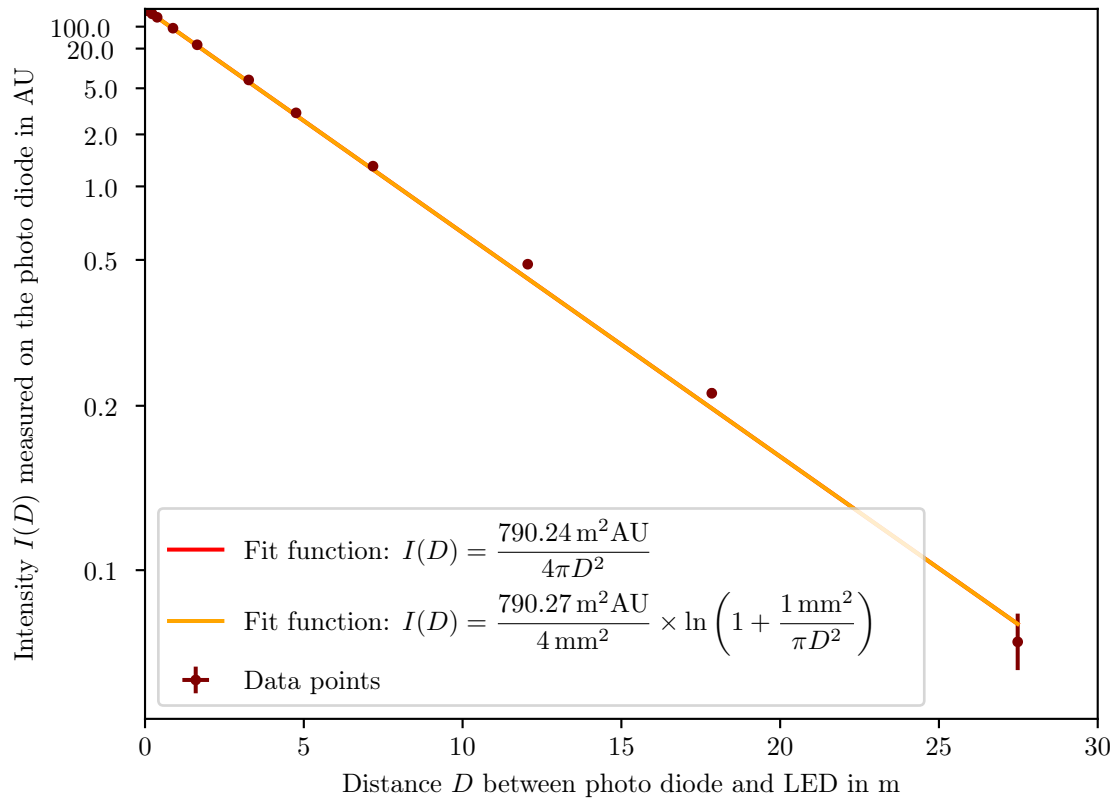


Figure 4.14: Intensity I measured at a photodiode in arbitrary units (AU) over the distance D in meter between the LED and the photodiode. This measurement was conducted with the 365 nm LED with a current of 300 mA, but the measurements of the other LEDs show the same tendency. There are two fits on this data: One assumes a point-like light source and uses Equation 4.7 and the other assumes a circular light source with an area $A = 1 \text{ mm}^2$ and uses Equation 4.8. Both fits describe this data equally well since their graphs are identical.

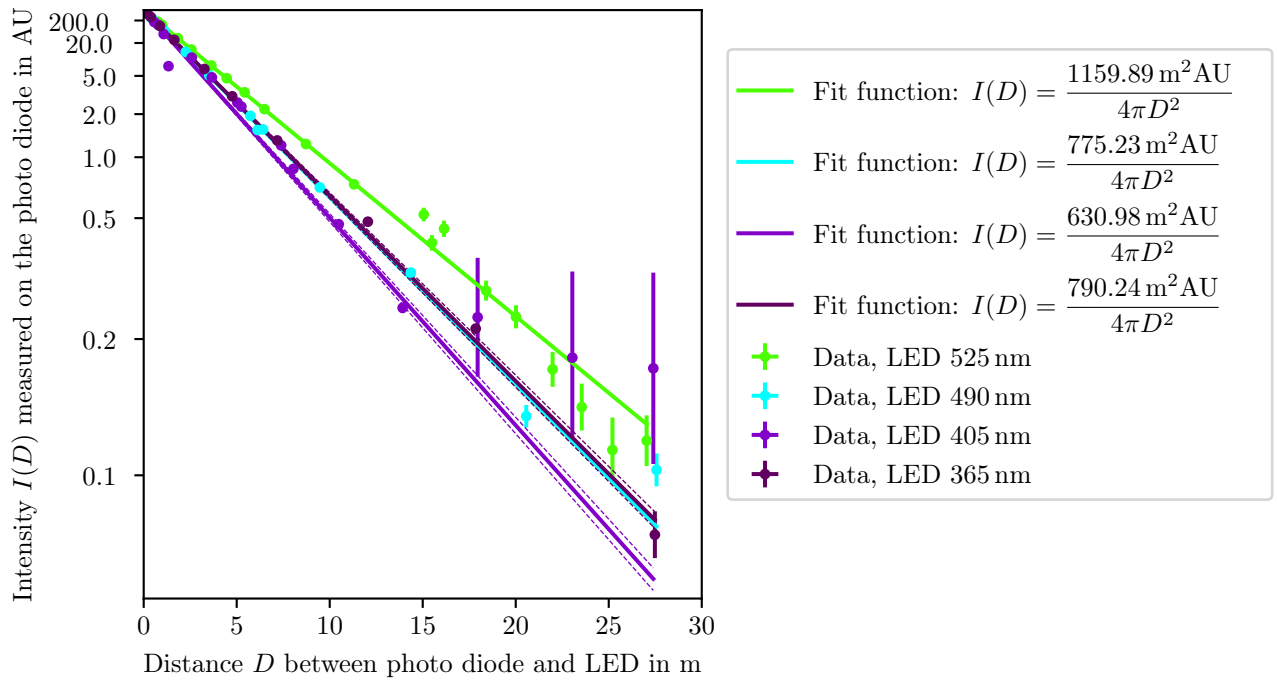


Figure 4.15: Intensity I measured at a photodiode in arbitrary units (AU) over the distance between photodiode and LED. Data of four LEDs interpreted as point-like light sources with the fit function from Equation 4.7. In the data of the measurements with both, the 405 nm LED and the 490 nm LED was one point each that is removed from the measurement because they were measured incorrectly.

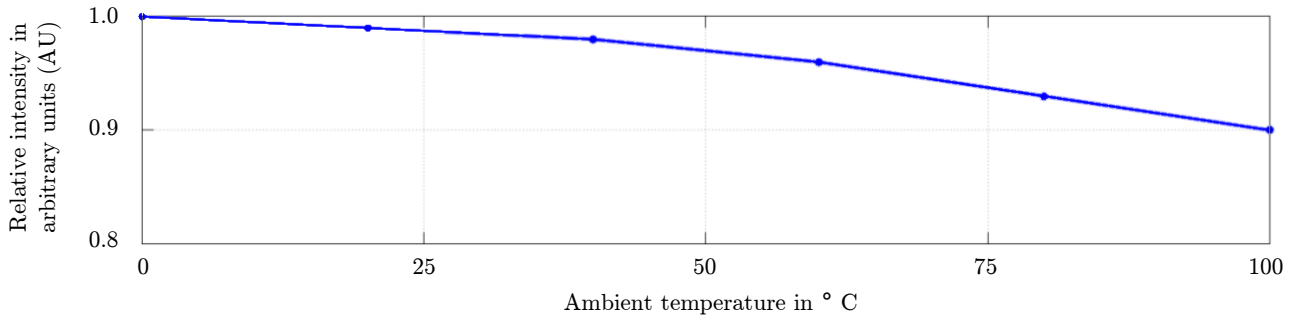


Figure 4.16: Relative intensity over ambient temperature for an LED supported with 250 mA by a constant current source. Exemplary data for the SMB1N-490 490 nm LED [Roi14c]. Originally obtained from [Dick], modified by the author.

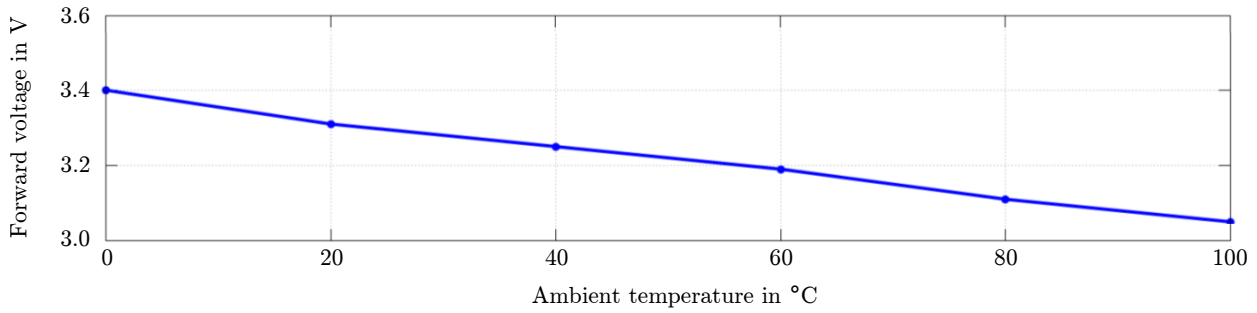


Figure 4.17: Forward voltage over ambient temperature for an LED supported with 250 mA by a constant current source. Exemplary data for the SMB1N-490 490 nm LED [Roi14c]. Originally obtained from [Dick], modified by the author.

and the precision of the distanzmeter⁸ were considered and the resulting uncertainties are displayed as broken lines in the plot. They are not always visible because they are very small for the measurements with the 525 nm LED and the 490 nm LED. The data confirms that the development of the LEDs intensities are described well with Equation 4.7.

4.7 Stability of the Light Source Intensity

To calculate the reflectivity of the mirrors two values need to be known, the intensity I_{spot} , reflected from, and the intensity I_{in} , reaching onto the mirror. The reflected intensity is examined in the main part of a mirror test run. For technical reasons, the incoming intensity cannot be measured directly at the PTS. To examine a reference intensity $I_{\text{reference}}$, as a stand-in for the incoming intensity I_{in} , a reference measurement is conducted at a later time. To draw conclusions about the reflectivity it is assumed that the intensity of the light source is the same during both measurements.

Even if powered with a highly constant current, the emitted intensity of an LED is not constant, as seen in Figure 4.16, but it depends on its temperature. Immediately after the LED was switched on, it still has ambient temperature. Since the density of charge-carriers is lower in a colder semiconductor, it has less conductivity. More voltage drops at the cold LED (see Figure 4.17). The power output depends on the current and the voltage; therefore, it is also larger on a colder LED. As the LED warms

⁸The precision of the *Leica Disto X310* distanzmeter is 1 mm [Lei13]. Although, it is assumed that the precision of any given measurement, of a distance D , with this distanzmeter, does not exceed $D \times 10^{-3} + 1$ mm due to human uncertainty.

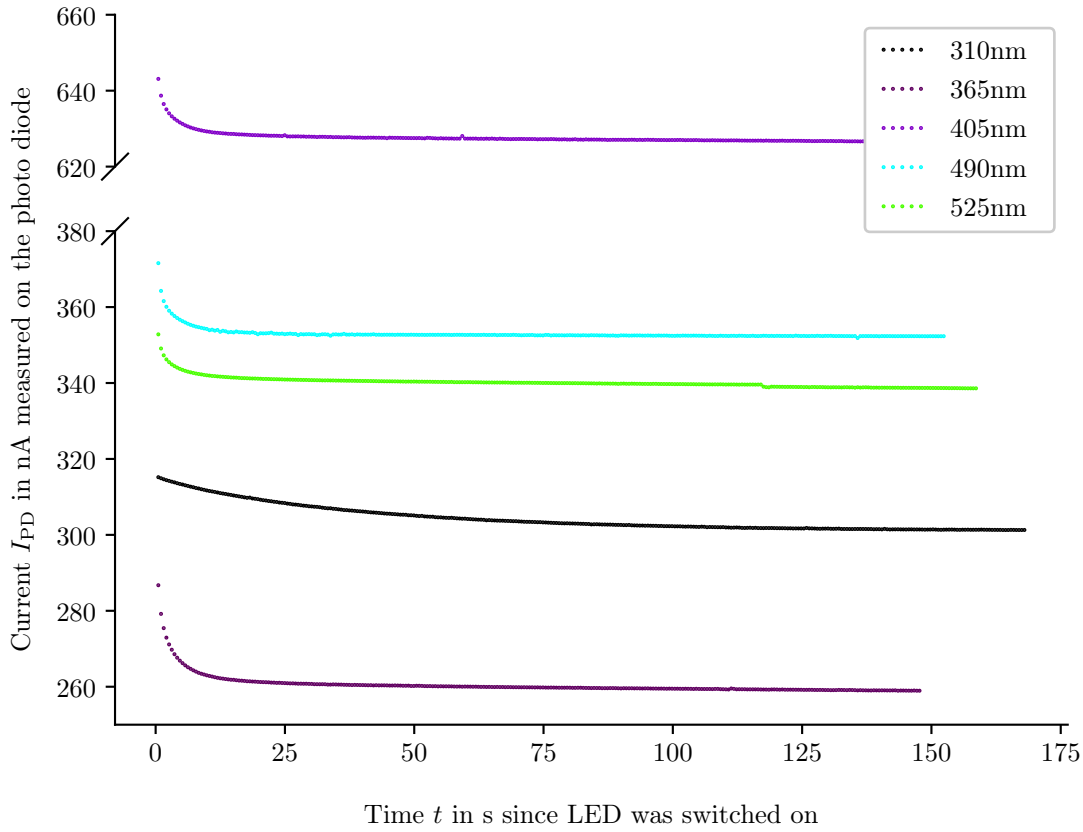


Figure 4.18: Development of the intensity with time for the five LEDs. All of those LEDs besides the UV-LED (with 310nm) were powered by a 350mA constant current source. The UV-LED got 50mA. The distance between the photodiode and the LEDs was not the same for each measurement. All five measurements were synchronized to the moment of switch-on.

up its conductivity rises and its power drops. When electric heating and cooling over the surroundings reach a state of equilibrium, the intensity becomes constant.

4.7.1 Simultaneous Measurements of Voltage Drop and Intensity

The absolute temperature of the LED is unknown but the LED supply-unit can report the forward voltage, dropped at the LED. This forward voltage is proportional to the temperature of the semiconductor (see Figure 4.17).

The change of intensity over time was measured with a photodiode⁹ for the five LEDs and the results are shown in Figure 4.18. It is clearly visible that the ultra-violet LED (310nm) does not reach a state of equilibrium in the first three minutes after the LED was switched on. For the other LEDs, the situation is not clear. It is not reasonable to wait as long as three minutes prior to each measurement since this would add up to at least half an hour and this is the planned time frame for the whole mirror test. The LEDs get enough time to reach a state where their change in temperature during one measurement is reasonably small¹⁰. The forward voltage, equivalent to this temperature, is saved and

⁹It is assumed that the current J_{PD} , generated by the photodiode, is proportional to the intensity that illuminates it.

¹⁰In this context *reasonably small* means that over the duration of one measurement (5s up to 90s for three pictures taken at each step) the change in temperature is equivalent to a 1 mV change in the forward voltage, or less.

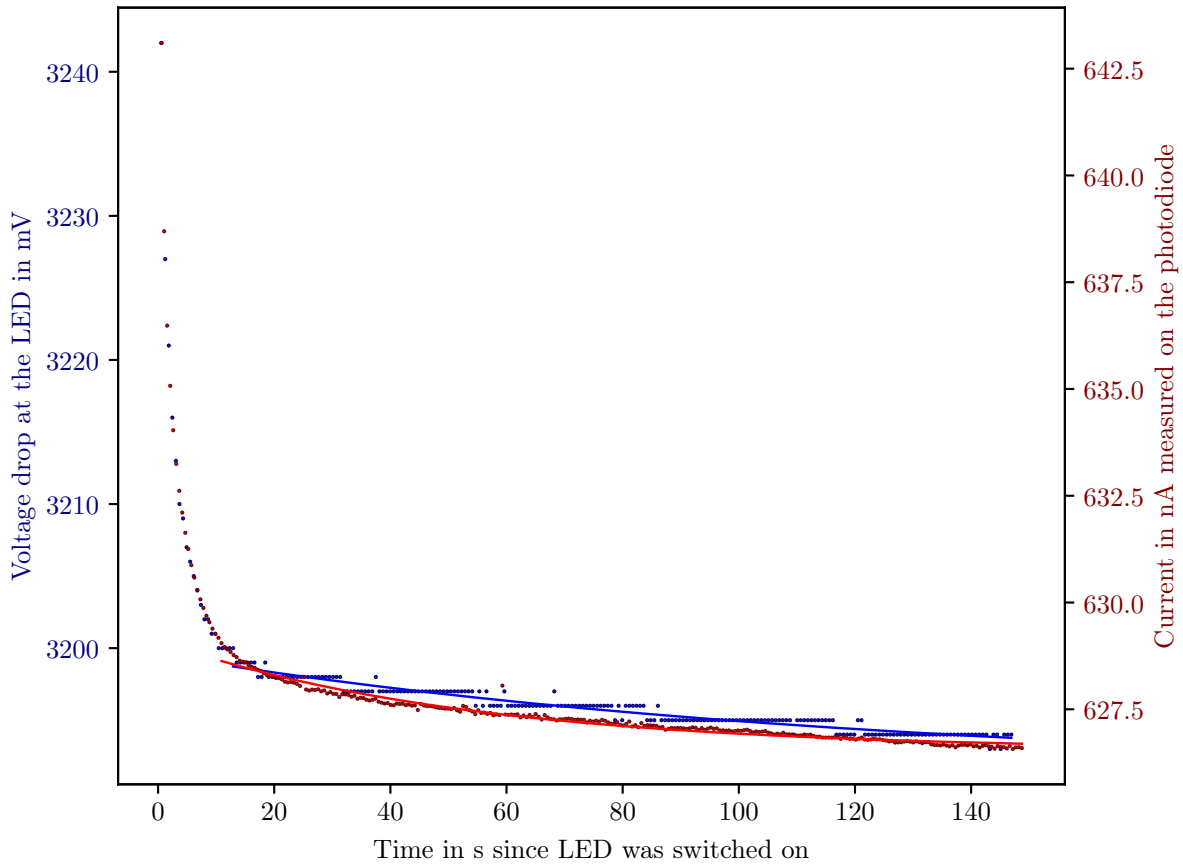


Figure 4.19: Visualization of the development of the forward voltage and the current in the photodiode with time. The forward voltage is reported by the LED supply-unit. The dark points are the measured values, synchronized to the moment of switch-on. The lines are exponential functions that are fitted to the data but not to the first 20 points. This data was taken from the blue 405 nm-LED powered with a current of 350 mA.

the reference measurement will be conducted after the LED reached the same forward voltage ¹¹.

Figure 4.19 shows both, the forward voltage U_{LED} and the current J_{PD} , measured simultaneously. This measurement was conducted at room temperature in a light-tight container with the blue LED of 405 nm, supported with a constant current of 350 mA. Both, the LED supply-unit and the photodiode, were taking data before the LED was switched on. This *dark values* were not subtracted from the data because they were several orders of magnitude smaller than the signal. The two data sets were synchronized to the moment the LED was switched on and the axes were scaled to optimally resolute the range between initial peak and convergence value. It is visible that both data sets follow the same trend and that measuring the forward voltage is a way to estimate the LED intensity.

4.7.2 Theoretical Impact of a Difference in the Intensity

In order to deduce the impact of an intensity change of the light source between spot and reference measurement, $I_{reference}$ stands for the estimated incident intensity from the reference measurement and I_{in} is the true incident intensity.

¹¹Due to unreproducible changes of the ambient temperature of the LEDs, it is possible that the temperature of an LED does not stabilize itself at exactly the same value during the reference measurement than during the main measurement. In those cases, a difference of 3 mV has to be accepted.

To calculate the directed reflectivity R_{dir} , using not the unknown incoming intensity I_{in} but the measurable reference intensity $I_{\text{reference}}$, a ratio V is implemented.

$$R_{\text{dir}} = \frac{I_{\text{spot}}}{I_{\text{in}}} = \frac{I_{\text{spot}}}{V \times I_{\text{reference}}} \quad (4.9)$$

Therefore, it is possible to use the reference intensity $I_{\text{reference}}$ instead of the real incoming intensity I_{in} if their ratio $V = 1$.

$$V = \frac{I_{\text{in}}}{I_{\text{reference}}} \quad (4.10)$$

It is not possible to measure the intensities of the LEDs directly at the mirror testing facility. Only the forward voltage U_{LED} can be read out on site. Therefore, it is important to find a relation between the voltage and the power.

To find this relation the development of the forward voltage of an LED and their power output are measured simultaneously. This is shown in Figure 4.19. The development of the current J_{PD} , which is measured with a photodiode, is proportional to the intensity of a light source for a constant distance between photodiode and light source.

The temporal development of the intensity and the forward voltage of an LED follow an exponential decay. Therefore exponential functions of the form

$$f(t) = A \times \exp^{-Bt} + C \quad (4.11)$$

are fitted on data¹² comparable to Figure 4.19. A relation between the forward voltage U_{LED} and the current J_{PD} can be formulated using the fit parameters A_{U} , A_{I} , B_{U} , B_{I} , C_{U} and C_{I} :

$$J_{\text{PD}}(U_{\text{LED}}) = \frac{A_{\text{I}}}{A_{\text{U}}} \exp^{\frac{B_{\text{I}}}{B_{\text{U}}}} \times (U_{\text{LED}} - C_{\text{U}}) + C_{\text{I}} =: c \times (U_{\text{LED}} - C_{\text{U}}) + C_{\text{I}} \quad (4.12)$$

Because of the proportionality of the power output P and the current J_{PD} , the ratio V from Equation 4.10 becomes:

$$V = \frac{J_{\text{PD}}(U_{\text{main}})}{J_{\text{PD}}(U_{\text{reference}})} = \frac{c \times (U_{\text{main}} - C_{\text{U}}) + C_{\text{I}}}{c \times (U_{\text{reference}} - C_{\text{U}}) + C_{\text{I}}} =: \frac{U_{\text{main}} - K}{U_{\text{reference}} - K} \quad (4.13)$$

where the fit parameters are combined into a constant

$$K = C_{\text{U}} + \frac{C_{\text{I}}}{c} = C_{\text{U}} + C_{\text{I}} \times \frac{A_{\text{U}}}{A_{\text{I}}} \times \exp^{-\frac{B_{\text{I}}}{B_{\text{U}}}}.$$

Now, it is possible to estimate the influence of an uncertainty in the reproducibility of the LED forward voltage between main part and reference part of the mirror test run. The relative uncertainty of the reflectivity (Equation 4.9)

$$\frac{\Delta R_{\text{dir}}}{R_{\text{dir}}} = \sqrt{\left(\frac{\Delta I_{\text{spot}}}{I_{\text{spot}}}\right)^2 + \left(\frac{\Delta I_{\text{reference}}}{I_{\text{reference}}}\right)^2 + \Delta V^2} \quad (4.14)$$

depends on the relative uncertainty of the intensity of the reflected light spot I_{spot} , the relative uncertainty of the reference intensity I_{ref} and on the uncertainty of the ratio V .

$$\Delta V = \sqrt{\left(\frac{\Delta U_{\text{main}}}{U_{\text{ref}} - K}\right)^2 + \left(\frac{U_{\text{main}} - K}{(U_{\text{ref}} - K)^2} \times \Delta U_{\text{ref}}\right)^2 + \left(\frac{U_{\text{main}} - U_{\text{ref}}}{(U_{\text{ref}} - K)^2} \times \Delta K\right)^2} \quad (4.15)$$

If both forward voltages U_{main} and $U_{\text{reference}}$ are the same (U_{LED}), the uncertainty of V becomes:

$$\Delta V = \frac{\sqrt{2} \Delta U_{\text{LED}}}{U_{\text{LED}} - K} \quad (4.16)$$

¹²The first 20 data-points since the moment of switch-on were not used for the fit. This led to better fitting functions. The measurements are not taken in the realm of quick heating.

4.7.3 Measured Impact of a Difference in the Light Source Intensity

To examine the impact of a (possible) deviation between the LED voltage drop during the main measurements and during the reference measurements, the voltage drop U_{LED} has to be measured during the whole mirror test run. In Figure 4.19 it is apparent that the resolution of the voltmeter of the LED supply-unit is 1 mV. In the first warm-up period of a mirror test run, MIRTESSA is programmed to let the LEDs warm up until their voltage U_{main} is stable within 1 mV over the duration of the measurement. For the reference measurement, the LEDs warm up until they are stable (with fluctuations of 1 mV over the duration of the measurement) around a voltage $U_{\text{reference}}$ that can differ up to 3 mV from U_{main} . The resulting uncertainty of the reproducibility of the voltage is therefore $\Delta U_{\text{LED}} = 5 \text{ mV}$. The difference between U_{main} and $U_{\text{reference}}$ has to be accepted since unspecified changes in the environment of the PTS light source lead to a different temperature when the thermal equilibrium is reached.

The constants K that are presented in Table 4.1 originate from experiments like in Section 4.7.1, where the development of the voltage drop U_{LED} and of the current J_{PD} are simultaneously recorded (see Figure 4.19). For this experiment, the LEDs are supplied with the same constant current as in a mirror test run.

The uncertainty of the factor V has the same influence on the relative uncertainty of the reflectivity of a mirror as the (relative) uncertainty of the reflected intensity I_{spot} and the (relative) uncertainty of the reference intensity $I_{\text{reference}}$ (see Equation 4.14). Utilizing the data, which are presented in Table 4.1, the uncertainty ΔV of the factor V can be estimated with Equation 4.16. Adducing the statistically insignificant data of four independent mirror test runs, a trend arises:

- The relative uncertainty of the reflected intensity is $\frac{\Delta I_{\text{spot}}}{I_{\text{spot}}} = 5.2(27) \times 10^{-5}$.
- The relative uncertainty of the reference intensity is $\frac{\Delta I_{\text{reference}}}{I_{\text{reference}}} = 6.056(86) \times 10^{-3}$.
- The uncertainty of the factor V is $\Delta V = 1.63(32) \times 10^{-2}$.

Therefore, the uncertainty of the reflectivity measurement of a mirror test run is dominated by the reproducibility of the intensity of the light sources. In Figure 4.20 the results of a single mirror test run are shown. The influence of the uncertainty of the reproducibility of the LED intensity, is apparent.

Table 4.1: Table of the supplied operating current J_{LED} , the voltage drop U_{LED} at which stability is achieved, and the fit constant K .

Note: The listed voltage drop is the average value at which MIRTESSA reports the voltage drop to be stable in a regular mirror test run. This is not identical to the voltage drop, to which the voltage, in the experiments that were used to evaluate the fit constant K , convergences.

Note: This data is not supported by enough repetitions (about eight each) to be statistically sound and is rather to be interpreted as orientation values.

LED	Operating current J_{LED}	Voltage drop U_{LED}	Fit constant K
DUV310-SD256	25 mA	4793(2) mV	5104(14) mV
SMB1N-365V	300 mA	3340(10) mV	3600(1700) mV
SMB1N-405V-02	130 mA	3045(2) mV	2460(590) mV
SMB1N-490	200 mA	2840(10) mV	3700(1000) mV
SMB1N-525V	200 mA	2710(2) mV	3180(190) mV

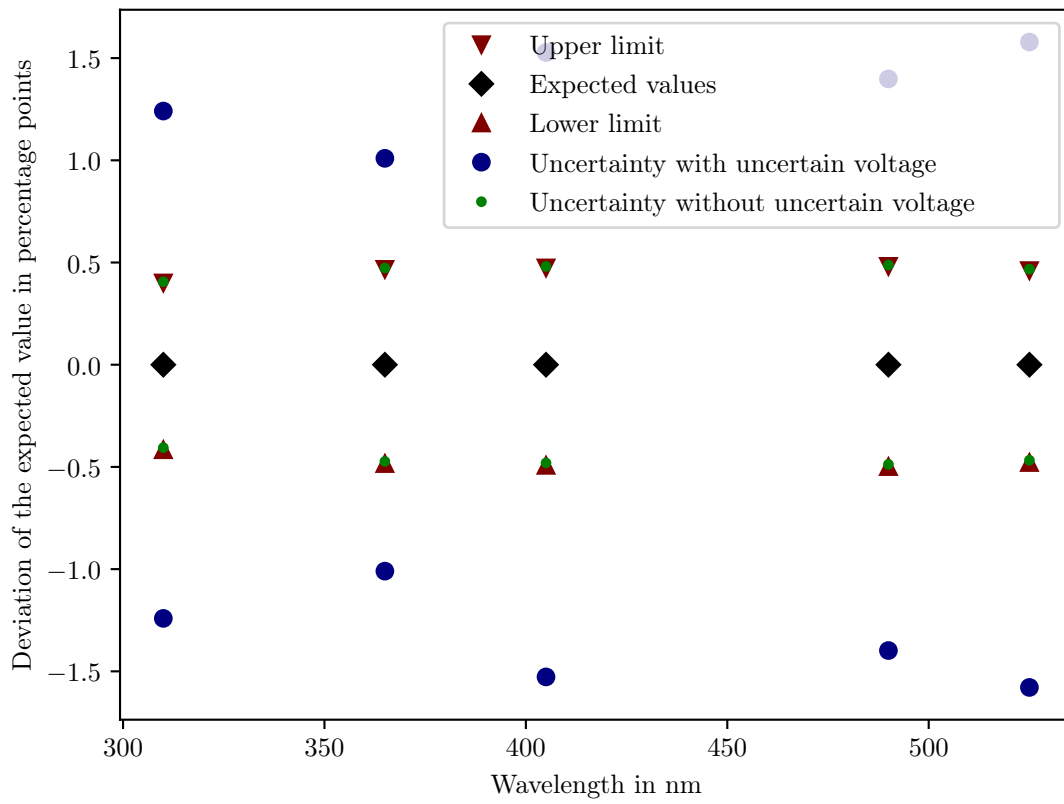


Figure 4.20: Uncertainties in the reflectivity of a mirror test run. Visualized as the deviation from the expected value in percentage points. The upper and lower limit of the reflectivity are calculated with no regard to an uncertainty in the voltage drop U_{LED} .

Chapter 5

Outlook and Conclusions

5.1 Outlook on Possible Improvements of the PTS

During the course of this work, a multitude of further improvements of the PTS, were discussed. They may regard the structure of the PTS, the quality of already conducted experiments or new analysis procedures. As of now, their significance or even their viability are not determined but they are outlined in the following sections:

5.1.1 Suggested Changes to the Setup of the PTS

5.1.1.1 Suggested Improvements to the Structure of the PTS

Mechanical Rigidity of the PTS

The mechanical setup of the PTS has several issues that curb the quality of an optical experiment.

The most heavyweight issue is the telescopic arm, on which the CCD camera and screen are mounted. This arm was originally implemented to enable the experimentalist to change the PTS from its main position into its reference position without changing the relative orientation between camera and screen by rotating the arm. Since the arm has to be collapsed and extracted in order to rotate in the limited space the laboratory provides, it does not comply with its original goal. Furthermore, the arm is not stiff enough to support the weight of the screen without a deformation of several centimeter. For now, the screen is supported by an extra tripod in both measurement positions.

A possible improvement would be to install fixed mounting positions at which the camera and the screen can be mounted for the main and the reference measurement. The telescopic arm could be completely removed. This setup would be more convenient for the experimentalist since it takes comparatively long to change the PTS with the telescope arm from one position into the other. On the other hand side, the telescopic arm adds security: Neither the camera nor the screen are at any time not firmly mounted to the structure of the PTS and the risk of dropping one or the other is greatly reduced.

Although, suggesting to keep the arm but to install the fixed support structures seems to be redundant, it is assumed that the quality of the reference pictures can be improved this way. Since changes in the position of the mirror are negligible with regards to its absolute distance, the alignment of the light sources require only minimal readjustments between mirror test runs. Having a clearly defined position for the screen in the reference position of the PTS, removes the necessity to find the optimal positioning between screen and light sources from the mirror testing routine.

Another minor source of possible uncertainty is the mounting of the CCD camera, itself. It is mounted in a way to be continuously variable in its angular orientation towards the screen. It would be an improvement to design a camera mounting that keeps the camera orthogonally oriented to the

surface of the screen. Since this is already assumed to be true (see Section 3.4.4) there is no need for variability at this point.

Mounting all Elements on the Same Level

In Section 4.4 it is explained what parts of the PTS geometry differ from its idealized geometry in Section 3.4.4. Although the influence of the varying level of the optical elements of the PTS is minimal on the final results, there is no reason not to align the centers of the camera, screen, light sources and mirror, should the PTS ever be redesigned.

Mirror Facet Mounting in the PTS

A CTA MST mirror facet is mounted in the cart of the IAAT mirror testing facility with its corners pointing directly up and down. Compared with the mounting in an MST, the facet is rotated by 90° . This is due to spacial limitations in the laboratory. Further work, addressing this issue, is already in planning to finally qualify the PTS to measure CTA MST mirrors according to [Chadwick18].

5.1.1.2 Replacement of the CCD Camera

The specific camera, of the *Alta U47-UV* series, utilized at the IAAT PTS has several minor deficiencies. First, there are about 700 pixels in one row of the CCD that do not report any signal at all. This is not an issue since their impact is negligible in comparison with the over one million working pixels of the CCD.

Secondly, the dynamic range of the camera plummeted from a 16-bit resolution to a 14-bit resolution after maintenance and refurbishment work was conducted with this camera. There is no definitive explanation for this, yet. This is also not an issue because the signal-to-noise ratio of the PTS is usually large enough to be displayed well with only 14-bits of dynamic resolution.

Finally, there are features, visible on some of the pictures, taken with this camera (see Appendix IV). Those features are usually the same or at least quite similar for any time between a few hours up until several days but can also change in appearance for no particular reason. If the CCD is warmed up from its operational temperature (-20°C) and cooled down again, the features changed definitely. There can be pictures taken, completely void of these kinds of features, too. It is hypothesized, due to the putative relationship between those features and the CCD temperature, that there is a liquid inside the CCD housing of the camera.

It is not possible to correct for these features with flat-fielding since the CCD has to be warmed up to room temperature before the camera can be removed from the PTS to take flat-frames. Up until this phenomenon is understood, no meaningful flat field correction can be conducted with this camera at all. In Section 4.1 it is examined that flat-fielding can alter the reflectivity measured with the PTS significantly.

Therefore, it is suggested to replace or repair this camera.

5.1.1.3 Replacement of the Light Sources

The intensity of the DUV310-SD356 310 nm LED is considerably lower than that of the other LEDs (see Table 3.1). This yields to a worse signal-to-noise ratio of measurements taken in the ultra-violet. To address this issue a more powerful ultra-violet LED was bought and installed in a new light source. Therefore, future measurements with the PTS can be conducted with a better signal-to-noise ratio in the ultra-violet regime.

Additionally, this new light source has a laser attached to it. As soon as the proper alignment of laser and LED light cones is verified, the laser can be used for an easier alignment of the LEDs on the mirror and to mark a point in the reference pictures.

5.1.2 Necessary Repetitions of Already Conducted Experiments

5.1.2.1 Examination of the Properties of the Current Light Sources

Even by continuous operation of the PTS with the old light source, experiments have to be repeated:

So is the comparison between the relative size of uncertainties of the measurements of the reflected intensity, of the reference intensity and of the factor V just based on four independent measurements (see Section 4.7.3). The dominance of the uncertainty of the factor V over the uncertainties of the two intensities is a major motivation in further studies regarding the development of the light source intensity with time (see Section 5.1.3.2). Incidentally, no dedicated experiments are necessary to examine the relationship between those three uncertainties, because they are monitored during normal mirror testing.

Also in Section 4.7.3, the fit constant K that is necessary to calculate the uncertainty of the ratio V (see Equation 4.16) was examined. Table 4.1 makes it apparent that the constant K has to be examined with a higher accuracy, especially for the LEDs SMB1N-365V and SMB1N-490. As a reminder, the constant K describes the relation between the developments of the forward voltage of an LED and its intensity with time. Both of those quantities are temperature dependent. The forward voltage or the intensity of an LED may converge to a different value depending on the ambient temperature when the LED was switched on. The fit constants K presented in Table 4.1 were examined based on data collected in an environment between 5 °C and 10 °C warmer than the environment of the PTS. It is therefore recommended to either investigate the dependency of K with the ambient temperature or to use data, collected in a more suitable environment, to reexamine K .

Furthermore, it is advised to conduct examinations regarding the isotropy and the point-like characteristic of the DUV310-SD256 310 nm LED, too. In the Sections 4.5 and 4.6, these properties were examined for the four other LEDs and since their results were similar, it was assumed that this would be the case for the 310 nm LED, too. This assumption should be verified.

5.1.2.2 Possible Examination of the Properties of a New CCD Camera

In case that the PTS' current model of CCD camera gets replaced or repaired, it becomes necessary to repeat camera specific investigations.

According to a flat-field correction (see Figure 4.4) that was conducted before the camera CCD begun showing unspecified structures (see Section 5.1.1.2), flat-fielding has the potential to improve the quality of the results from the PTS, further. Therefore, flat-fielding should be conducted according to the description in Section 4.1, as soon as a suitable CCD is available.

The image scale a (and the distance x_0 between camera body and objective) has to be reevaluated, every time the objective was removed from the camera body or the focal length of the objective was changed. In these cases, it is necessary to determine those values using a testgrid (see Section 4.2.1).

5.1.2.3 Possible Examination of the Properties of the New Light Sources

Before starting to utilize the new light source for the PTS, all light source specific investigations have to be repeated. Those are the developments of the LED intensities with time (see Section 4.7) and the influence of the anisotropies of the LEDs on the final results (see Section 4.5.1). The investigation concerning if the light sources can be considered point-like, are dominated by the setup of the PTS (see Section 4.6) and since LEDs of the new and the old light source have the same size, it is not necessary to repeat these investigations ¹.

¹The ultra-violet LED used in the new light source is of a different design than the one in the old light source. Even if the measurements from Section 5.1.2.1 were conducted, since the time of this writing, it remains important to test all characteristics of the new ultra-violet LED.

5.1.3 Suggested Means to Further Improve the Quality of the PTS

5.1.3.1 Isotropy of Light Sources

It is suggested to find a reliable and ideally quick solution for finding the center point and radius of the light spot in the reference pictures (see Figure 3.10 on Page 33). This would remove the human interaction further from the analysis of mirror test runs. Currently the center of the reference spot is still estimated by eye.

In the case that all reference pictures are suitable for this kind of analysis and that the light sources happen to be isotropic in a sufficiently large area, the alignment of light sources and mirror becomes less relevant. The correction for inhomogeneities in the reference light spot that is described in Section 4.5.1 can be implemented into the ACTAM software. This would increase the estimated uncertainties of the reflectivity of the PTS but it would make its results more reliable.

If it is possible to find a model for the dependency between estimated reference intensity and the position of the examined region on the reference picture, then Equation 4.5 can be used without the assumption of $H = 1$. This is a promising opportunity to reduce the measurement uncertainty of the PTS since an assumption can be removed.

5.1.3.2 Temporal Stability of the Light Sources

In Section 4.7 it is shown, that the uncertainty ΔU_{LED} in the reproducibility of the light source intensity has major influence on the uncertainty of the reflectivity determined by the PTS. Therefore, the pursuit of improving the PTS measurement uncertainty is futile without improving the reproducibility of the light source intensity. To curb its influence on the final uncertainty of the reflectivity measurement until it is roughly on the same level than the relative uncertainty of the reference intensity, the voltage drop of the main measurement U_{main} has to be reproducible in the reference measurement with a precision between $\Delta U = 4 \text{ mV}$ and $\Delta U = 1.5 \text{ mV}$ (see data on Page 69 and Equation 4.16).

Accept Different Voltages in Main and Reference Measurement

The PTS cannot achieve this goal with the current hardware. The resolution of the voltmeter of the LED supply-unit is 1 mV . Therefore it can not measure the difference between two voltages with a higher precision than 2 mV . Furthermore, each LED does not converge to the same exact voltage drop U_{LED} every time it is switched on but U_{LED} lays in an interval with a width of 5 mV up to 20 mV^2 .

The impact of the temporal stability of the light source intensity on the resulting reflectivity could be reduced, when both forward voltages, the LED stabilizes itself at, before the main measurement U_{main} and before the reference measurement U_{ref} are saved individually (with an uncertainty of $\Delta U = 1 \text{ mV}$ each). In that case, the ratio V would not be assumed to be $V = 1$ but would be estimated with Equation 4.13. Neither could the ratio V be assumed to be always $V = 1$ instead of its exact form from Equation 4.13; nor could its uncertainty ΔV from Equation 4.15 be simplified to Equation 4.16.

The exact benefits of this new approach have to be investigated further. On the one hand it is clearly beneficial to remove assumptions from an equation and replace them with measurable quantities, on the other hand the fit constant K gains a higher influence. Equation 4.13 shows that the constant K gets influence, not only on the uncertainty of the measured reflectivity, but also on the reflectivity itself. In Equation 4.15 even the uncertainty ΔK of K becomes relevance.

The problem is that for now the fit constant K is not reliably known. In Table 4.1 on Page 69 it is shown that the uncertainty ΔK is in the same order of magnitude than K itself. In Section 5.1.2.1, it was already discussed that it would be beneficial to get better measurements of the fit constant K .

²To achieve a reproducibility of $U_{\text{LED}} = 5 \text{ mV}$, a tolerance of 3 mV at which the voltage drop stabilizes itself is accepted. The reached value does not always lay in this range. Currently this range is used because experience has proven that it provides a good compromise between success rate and quality of the resulting data.

Find a Better Fitting Function for the Warm-Up Characteristic

It is entirely possible that, even with a much larger sample size, the uncertainty of the fit constant K does not improve significantly for all wavelengths. Since an exponential function (like in Equation 4.11) is most likely not the best fitting function for all of the LEDs warm-up characteristics.

Figure V.1 on Page 91 shows a comparison between the development of the forward voltage U_{LED} of an LED and the current generated by a photodiode J_{PD} with time. At least development of the current J_{PD} does not follow a simple exponential decay. It is strongly advised to examine the reason for this behavior and to find a better fitting function.

It is expected that the uncertainty ΔK of the fit constant K (or of an comparable constant resulting from the new fit functions) can be improved this way.

Monitor Voltage Drop of LEDs Continuously

It is already possible to monitor the forward voltage of the LEDs during the entire mirror test run. A coherent relation between forward voltage of an LED U_{LED} and the current generated at a photodiode J_{PD} , which is supposed to be proportional to the intensity of the LED I_{LED} would be interesting. As soon as it is found, it will be possible to correct each individual picture taken for the current intensity of the LED.

5.2 Conclusions

The primary goal of this work was to show that it is possible to accurately examine if the reflectivity and the width of the PSF of a mirror facet fulfill the requirements for being used in a CTA MST, in less than an hour. In the requirements, it is explicitly stated that a mirror test must be conducted in the ultra-violet regime, too, to qualify the mirror for usage in CTA [JAM19, B-TEL-0120 Mirror Reflectivity - V3]. The IAAT operates a mirror testing facility, including the PTS, the *Projecting Mirror Test Stand*. Since in previous works on the PTS the estimation of measurement uncertainties was not systematically investigated, it became a secondary goal of this thesis to examine the uncertainty of the PTS. To conduct reliable tests, an uncertainty of 1 % was set as an optimal goal for measuring the reflectivity of a mirror facet.

During the course of this work, the MIRTESSA software was developed from scratch to automate mirror test runs with the PTS. Not only is the amount of training, a new experimentalist needs, reduced but also the reproducibility and reliability of the results is enhanced.

The software for the analysis of measurements with the PTS (ACTAM) got a general overhaul. It was adjusted to the latest requirements and the code was streamlined. Furthermore, ACTAM is now capable of propagating the uncertainty of input values and estimating the uncertainty of its analysis scheme (see Section 3.6.2).

Multiple possible sources of uncertainty were examined and their influence on the final results was estimated (see Chapter 4). Thereby, various properties of the CCD camera (e.g. its image scale etc.) and the light sources (e.g. their switch-on characteristic etc.) were examined. The setup geometry between CCD camera, screen, light sources and mirror in test was discussed in detail and, where necessary, was corrected for in ACTAM (see Sections 3.4.4 and 4.4).

Finally, a commercially available hand-held reflectometer was examined to get reference values to compare the results of the PTS with that were obtained by completely independent means (see Appendix III).

For the time being, the PTS measures the reflectivity of a mirror facet with a relative uncertainty of up to 2 % and the width of its PSF with a relative uncertainty of up to 1.5 %. The uncertainties are still slightly higher than what was set as optimal at the beginning, but there are several approaches

discussed in Section 5.1 that promise further improvement. The results of multiple mirror test runs with the same mirror facet show that the results of the PTS are generally reproducible (see Figure 3.13 on Page 43) but there are still outliers that can be due to human error, or due to features of the PTS that are not examined, yet. A reliable examination of measurement uncertainties is by all means preferable to uncertainties that could only be guessed, previously.

To decide if the PTS is qualified to conduct mirror tests for CTA MST, its parameters have to be compared with the official requirements from [Chadwick18], stated in Section 3.2: The pixelated detector (screen and CCD camera) cover about 130 % of the required angular pixel size of the Cherenkov camera of 0.18° [Garczarczyk18a, p. 3]. The detector has about 675 pixels per axis over the length of $\frac{2}{3}$ of these pixels. A mirror test run contains means to measure the reference intensity and to reduce the background of the data but the mirror facets are not mounted exactly the same way in the PTS as they would be in a CTA MST, due to technical limitations.

There is work in progress to find a solution regarding the last point. Furthermore, the PTS is, strictly speaking, only able to approve mirror facets to be used in a CTA MST, not to dismiss them. Due to the fact that the maximal radius of interest for the reflected intensity is $\rho_{\text{roi}} = 2 \text{ mrad} \approx 0.11^\circ$ and not the $\rho_{\text{roi}} = 3.75^\circ$ required for reflectivity measurements [JAM19, CTA-GLOS-204 Mirror Reflectivity - V1 and CTA-GLOS-195 Camera Field of View (cFoV) - V2] and [Garczarczyk18a, p. 3] or even the $\rho_{\text{roi}} = 0.5^\circ$ required for PSF measurements [JAM19, CTA-GLOS-202 Theta_80 - V4]. This yields to systematically too low reflectivities and systematically too narrow Θ_{80} -widths of mirrors. Therefore, if a mirror facet that was measured with the PTS does not comply with CTA's optical requirements, it is not yet to be dismissed but has to undergo further tests at other mirror testing facilities.

Further improvements of the PTS that exceed the scope of this work are in development. Some of the most advanced ideas are already outlined in Section 5.1. In the long run, a reliable, fast and predominantly automated Mirror Testing Facility is crucial to efficiently support CTA with affordable high quality mirror facets.

Appendix I

Outline of the Mirror Testing Procedure

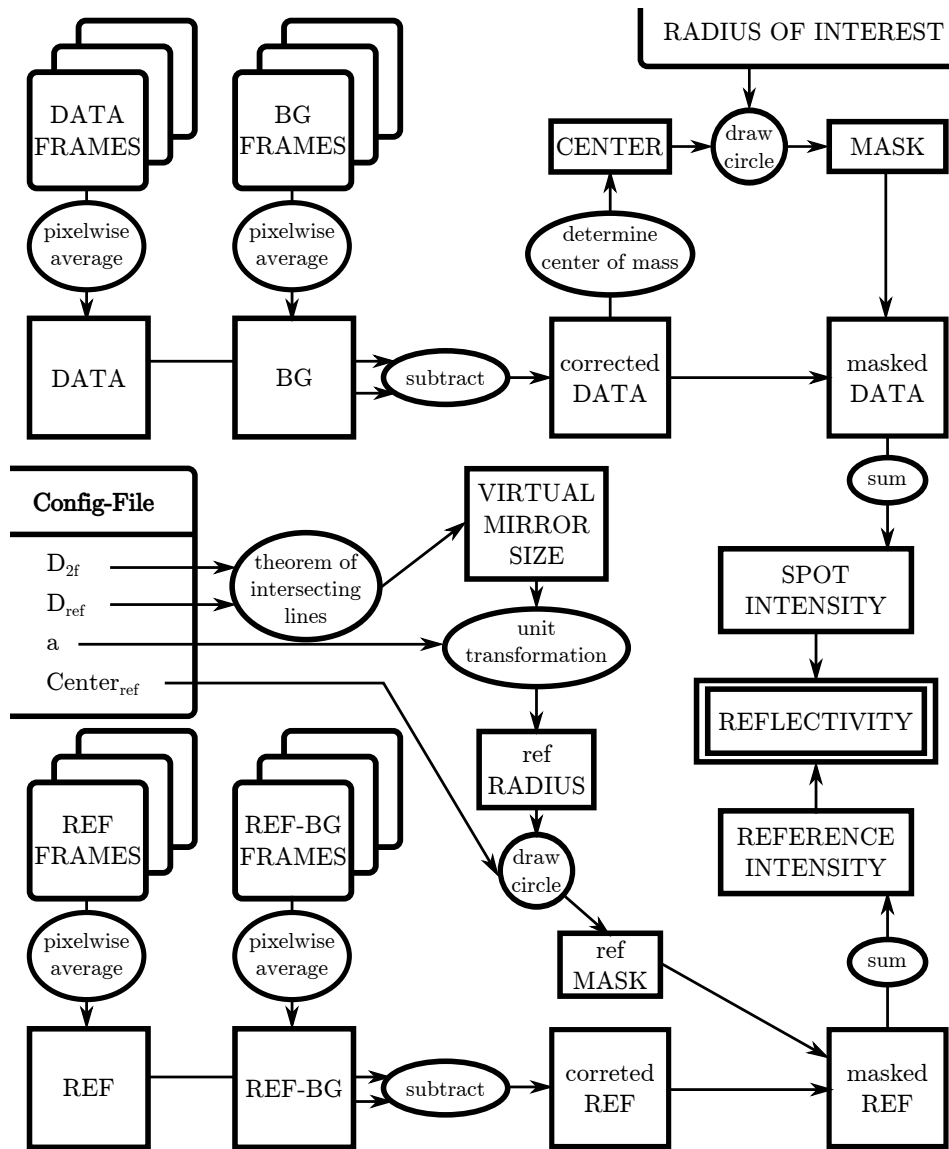


Figure I.1: Simplified sketch of the mirror testing procedure (and analysis of the reflectivity). The various steps are described in the Sections 3.5 and 3.6.1. This sketch does not show the analysis of the width of the PSF or any estimations of measurement uncertainties.

Appendix II

Input for and Output from ACTAM

Table II.1: Output, provided by the data analysis software ACTAM for the *Projecting Test Stand* (PTS), for each wavelength separately.

Description	Name
Center of mass of the intensity distribution in the whole data (in pixels)	center
Upper and lower boundary of the intensity reflected into a circle with the radius R0I around center (in ADC-counts)	spot_int
Relative uncertainty of the reflected intensity	spot_int_rel_err
Upper and lower boundary of the reference intensity $I_{\text{reference}}$ (in ADC-counts)	ref_int
Relative uncertainty of the reference intensity	Ref_Int_Rel_Err
Relative error of the approximation of the incoming intensity I_{in} with the reference intensity $I_{\text{reference}}$	dV
Reflectivity R (in %)	R_tot
Uncertainty ΔR of the reflectivity (in %)	Reflectivity_Error

Table II.2: Necessary input for the data analysis software (ACTAM) for the *Projecting Test Stand* (PTS). The first line is the only input that has to be provided when starting the analysis software. All other information are either in the configuration file (second section of the table) or implied in the analysis software (last section of the table). As an example, the inputs for the analysis of a mirror test run from 24th July, 2019 are given.

Description	Variable name	Example
Directory with the configuration file and data	Base	'20190724'
Mirror name or number	mirror_name	'KMST4'
The telescope the mirror is intended for	mirror_type	'MST'
Double of the focal length D_{2f} (in m)	F2	32.103
Uncertainty ΔD_{2f} of this length (in m)	error_F	0.01
Reference distance D_{ref} between LEDs and screen (in m)	ref_dist_in_m	1.77
Uncertainty ΔD_{ref} of this distance (in m)	error_ref_dist	0.002
Image scale a of the camera (in mm per pixel)	pixel_size_in_mm	0.14786
Uncertainty Δa of the image scale (in mm per pixel)	error_pixel_size	0.00013
Distance $D_{\text{scale-main}}$ between screen and camera body with the PTS in main position (in m)	arm_length_main_in_m	2.452
Uncertainty $\Delta D_{\text{scale-main}}$ of this distance (in m)	error_arm_length_main	0.002
Distance $D_{\text{scale-ref}}$ between screen and camera body with the PTS in reference position (in m)	arm_length_ref_in_m	2.452
Uncertainty $\Delta D_{\text{scale-ref}}$ of this distance (in m)	error_arm_length_ref	0.002
Distance L between light source and principal axis of the mirror (in cm)	deviation_LED_axis_in_cm	28
Uncertainty ΔL of this distance (in cm)	error_deviation	1
List of the wavelengths of the LEDs used in this mirror test run	wavelengths	[310,365,405,490,525]
List of the x-coordinate of center the reference spot (in pixels)	center_ref_x	[550,430,390,475,500]
List of the y-coordinate of center the reference spot (in pixels)	center_ref_y	[660,500,700,670,660]
List of the voltage drops U_{LED} at which the measurements were conducted (in mV)	voltages	[4795,3376,3049,2839,2711]
String used to identify reference measurements	ref_label	'ref'
String used to identify background measurements	bg_label	'bg'
String used to identify the reference background	refbg_label	'rb'
Nominal distance D_{scale} between screen and camera body (in m)	Arm_Length_Nominal	2.4474
Uncertainty ΔD_{scale} of this distance (in m)	Arm_Length_Error	0.0022
Nominal distance x_0 between camera body and objective (in m)	X_0	0.1733489
Uncertainty Δx_0 of this distance (in m)	X_0_Error	0.0000049
Flat-to-flat diameter of the mirror (in m)	mirror_size	1.2
Radius of interest (in radians)	ROI	0.002
Range of x-coordinates of defective pixels of the CCD	b_x	(302, 1023)
y-coordinate of defective pixels of the CCD	b_y	546
Change ΔU of the voltage drop U_{LED} between the two measurements (in mV)	error_U	5

Appendix III

The Hand-Held Reflectometer

III.1 Description of the Reflectometer

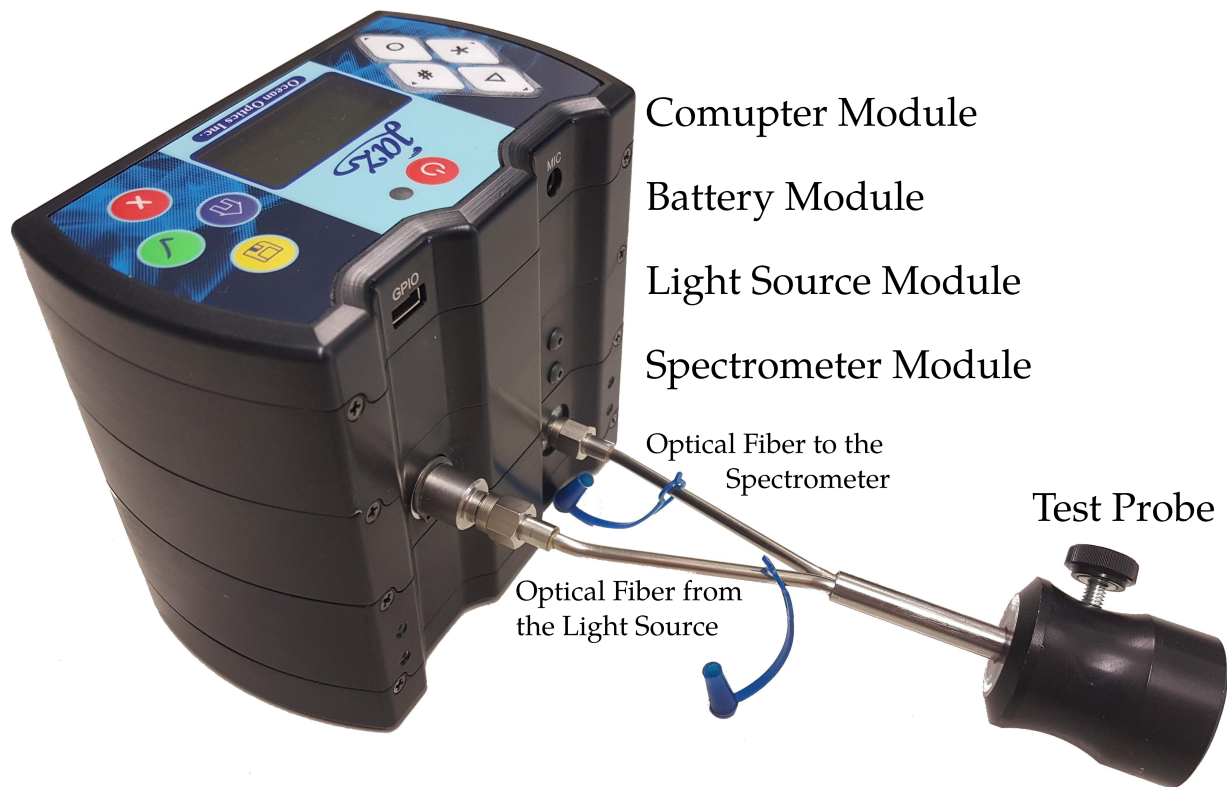


Figure III.1: The hand-held reflectometer *Jaz* by Ocean Optics. It consists of four modules: the actual spectrometer (module JAZ-S), a light source (module JAZ-PX), a battery (module JAZ-B) and a computer (JAZ-DPU). It is equipped with a test probe and the short set of optical fibers.

The reflectometer used for measurements of the surface-reflectivity is the hand-held model *Jaz* from Ocean Optics. The reflectometer consists of the *Display and Processing Unit* (JAZ-DPU) and several modules as shown in Figure III.1. When the reflectometer is not powered directly over the 5 V input it can run on the *Battery & Memory Module* (JAZ-B) up to two hours in this configuration [Oce, p. 4]. It is equipped with the *Pulsed Xenon Module* (JAZ-PX) as light source. This is a „pulsed, short arc xenon lamp for UV-Vis applications“ with a pulse frequency of 200 Hz emitting light between 190 nm

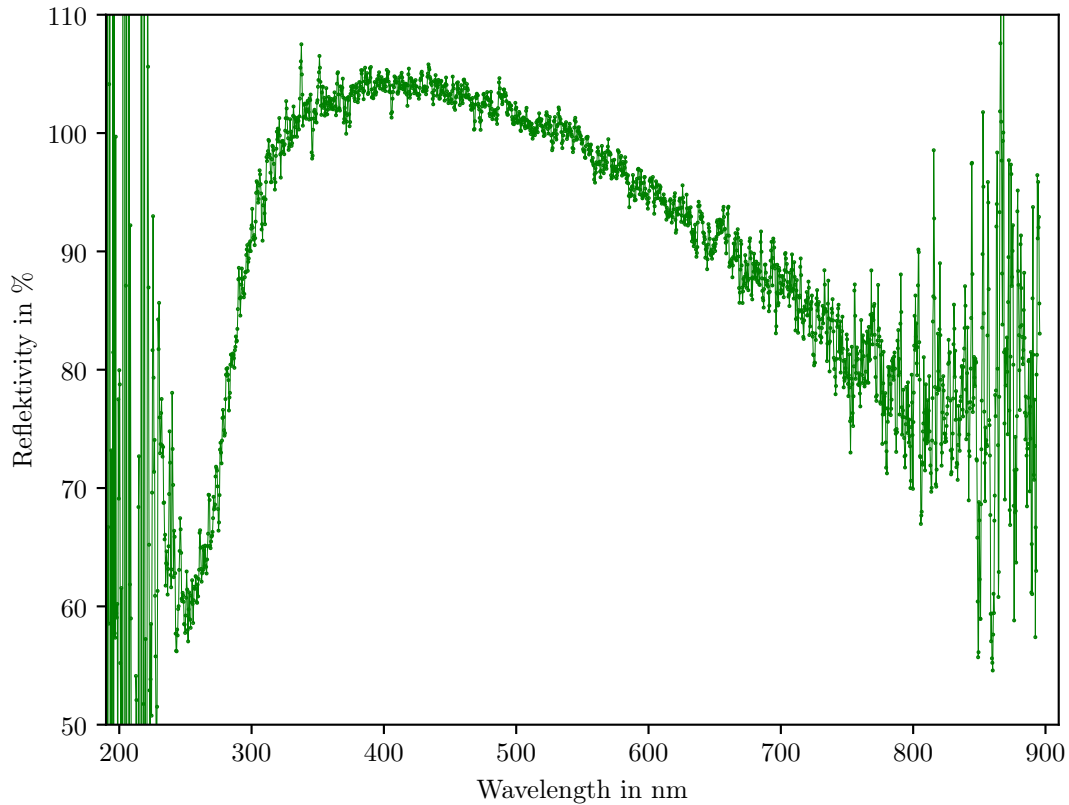


Figure III.2: This shows the whole range of data, collected by the reflectometer. It is clearly visible that the data loses significance with a wavelength below 250 nm or above 750 nm. The measurement was taken with the mirror *MST MLT 124* on the 31st January, 2019. This data is not yet processed by the algorithm described in section III.2.

and 1100 nm [Oce, p. 24]. The light emitted by the lamp is conducted through an optical fiber to the test probe. Reflected by a surface, it is conducted through another optical fiber into the *Spectrometer Module* (JAZ-S).

The detector of the reflectometer is a linear silicon CCD [Oce, p. 3] and is unreliable below 250 nm or above 750 nm because it measures too much noise in this regime (see Figure III.2). In the manual it is stated that the detector has a wavelength range of up to 650 nm located anywhere between 200 nm and 1100 nm [Oce, p. 3]. The resolution is, depending on the wavelength, between 0.3 nm and 10 nm FWHM [Oce, p. 3]. Both, the wavelength range and the spectral resolution of JAZ-S suffice the requirements¹ from the Mirror Qualification Test Plan [Chadwick18, p. 12].

III.2 Surface Reflectivity Measurement Procedure

When measuring a mirror with the Jaz hand-held reflectometer, the following procedure was established:

After powering up the reflectometer, the red button has to be pushed to go to the main menu. *PX-lamp* has to be selected with the green button. The #-button has to be pushed twice to go through *setup* and *timing control*. *free running* has to be confirmed with the green button. Pushing the red

¹Minimal wavelength range: 300 nm to 550 nm, preferred wavelength range: 200 nm to 700 nm, spectral resolution: < 10 nm [Chadwick18, p. 12].

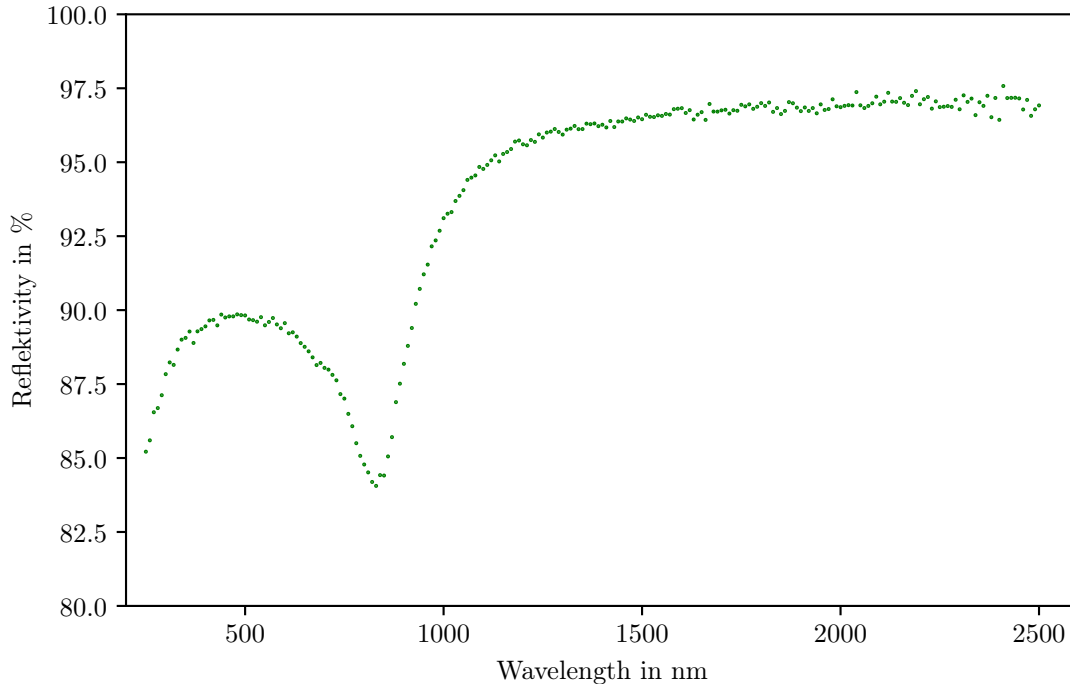


Figure III.3: The nominal spectrum of the Ocean Optics *STAN-SSH-Reflectance-Standard*, obtained by their website oceanoptics.com/product/stan-series-reflectance-standards/.

button twice yields to returning to *px-lamp*. The light source can now be turned on or off with the arrow keys on the reflectometer. A running lamp produces a noticeable sound.

It is important to complete the setup of the reflectometer before connecting it with a computer via cable. It is necessary to connect the reflectometer and the computer before running the analysis software OceanView² because it would create a simulation, otherwise.

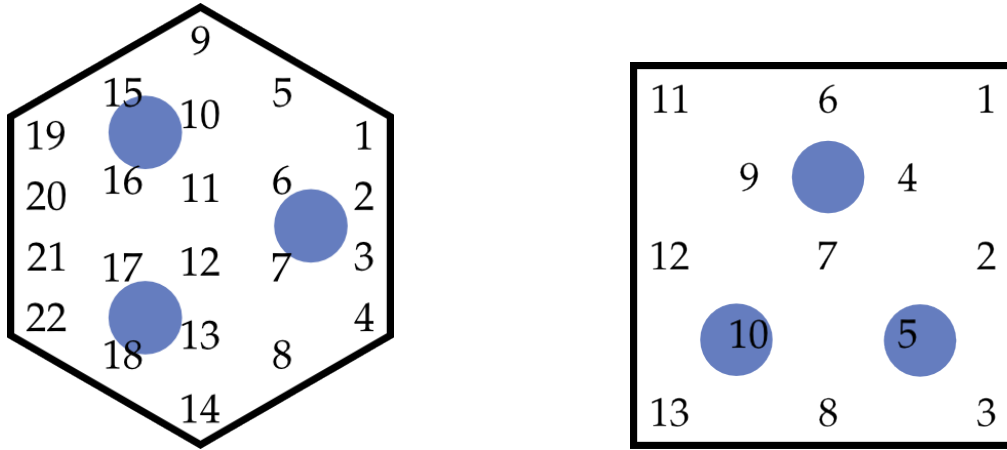
The software has a template for reflectance measurements in its *Create new spectroscopy application*-menu. These instructions of the program should be followed until it asks for the integration time of the measurement. It is necessary to set it to a multiple of 5 ms to avoid interference with the pulse frequency of the lamp of 200 Hz [Oce, p. 24]. The best integration time is as long as possible without overexposing the detector of the reflectometer. This can be controlled, by setting the boxcar width to $w_{\text{box}} = 1$ and comparing the maximum of the intensity with the threshold of about 56 000 ADC counts.

The program asks to take a reference and a dark spectrum. Therefore, the test probe has to be pressed on the reference mirror. In the context of this work, all surface reflectivity measurements were taken using a *STAN-SSH-Reflectance-Standard*. Its nominal spectrum is shown in Figure III.3.

It is required to take at least 10 surface reflectance measurements distributed over the surface of the mirror. At least one measurement must be taken in the center of the mirror and one at the point most distant from the center [Chadwick18, p. 12]. It is advised to take reference measurements frequently between different surface measurements of the mirror so instabilities from the light source or spectrometer can be corrected for, if necessary.

All surface reflectivity measurements of CTA MST mirrors, in the context of this work, were conducted with the test pattern from Figure III.4a. Figure III.4b shows the test pattern for MAGIC mirrors. Reference measurements with the STAN-SSH reference mirror are taken before and after each measurement run and after the eighth, the 14th and the 18th measurement for an MST mirror. Each

²OceanView 1.6.3 by Ocean Optics



(a) The pattern for MST surface reflectivity measurements. The orientation of a mirror is defined by the two sides with name-tags being the vertical sides and the single mounting point being on the right (as observed from in front of the mirror).

(b) The pattern for MAGIC surface reflectivity measurements. The orientation of a mirror is defined by the single mounting point being on top.

Figure III.4: The layout of the test patterns of surface reflectivity measurements, conducted in the context of this work. The three blue circles represent the mounting points on the backside of the mirrors. The numbers indicate the position of the measurement points and the order of measurements. The mirrors are represented facing their reflective surface.

measurement consists of five pictures taken without removing the test probe from the reflective surface to deal with statistical variations from the detector.

III.3 Analysis and Results

The data of the surface reflectivity measurement runs is analyzed by a Python script following this routine:

1. Take the mean and standard deviation of five measurements that were taken at the same spot. If the measurement procedure is known, label the reference measurements as such.
2. Separate the reference measurements from the rest.
3. Assign each measurement to the reference measurement that was taken in the closest temporal proximity.
4. For each measurement, multiply the intensity of every given wavelength with the intensity in the nominal spectrum (Figure III.3) at that wavelength and divide the result with the intensity of the closest reference measurement at that wavelength³. Use Gaussian propagation of errors.
5. For a spectrum of the whole mirror, over all wavelengths, take the average over all measurement points.

³The reflectometer assumes that the internal reference measurement was taken with a reference mirror with exactly 100% reflectivity over all wavelengths. This is not the case, so it is necessary to normalize the data with the nominal spectrum of the reference mirror (see Figure III.3). All reference measurements taken with the same mirror should also show exactly 100% over all wavelengths. This is not the case, too. The detector or the light source are not completely stable. The measurements need to be corrected for this degrading with their respective reference measurement (see Section III.4.2).

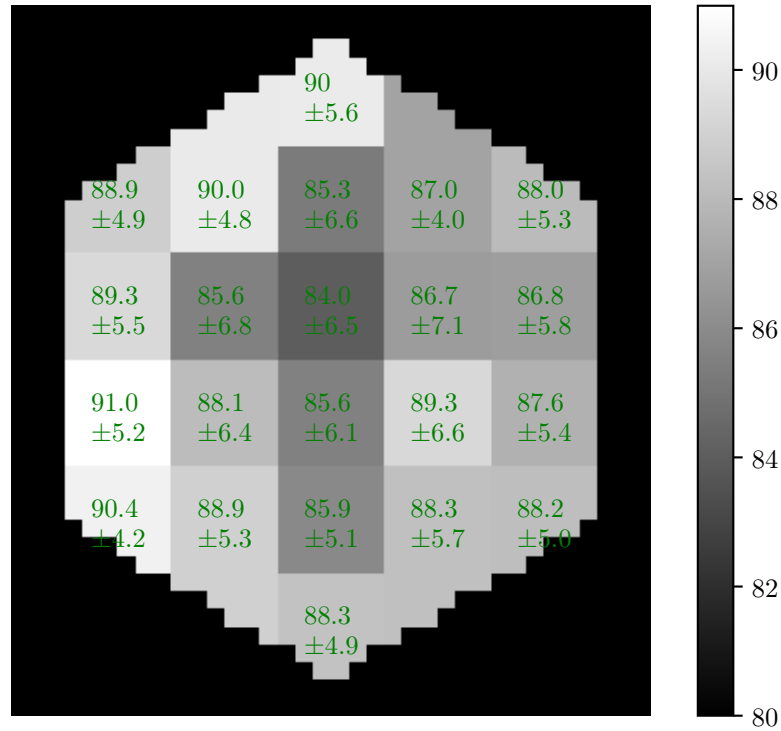


Figure III.5: Surface reflectivity measurement of the mirror *MST MLT 124*, taken on the 8th January, 2019. All reflectivities are averaged over all wavelengths. Reflectivities are given in percent.

6. For a comparison between different areas of the mirrors surface, like in Figure III.5, take the average over all wavelengths for each measurement point.

III.4 Reliability of the Reflectometer

III.4.1 Uncertainty Caused by the Optical Fibers

There are two sets of optical fibers. The short (and stiff) set is shown in Figure III.1. The other set has optical fibers embedded in a flexible tube with a length of about 1.2m. It is more convenient to work with the long set of fibers because the reflectometer can sit on a table. On the other hand side, moving an optical fiber has influence on the signals flowing through it. A comparison between the results of surface reflectivity measurements with both sets of fibers is shown in Figure III.6. The difference between two measurements with different optical fibers is about 1 % with a spread of about 1 %. Therefore, measurements taken with any set of optical fibers are comparable.

III.4.2 Drift of the Lamp or Detector

To set up the hand-held reflectometer for surface reflectivity tests, it has to take a reference measurement that is saved internally. When there is another measurement taken, later but with the same mirror, the reflectometer does not measure the same reflectivity. There are two hypothesis to explain this under

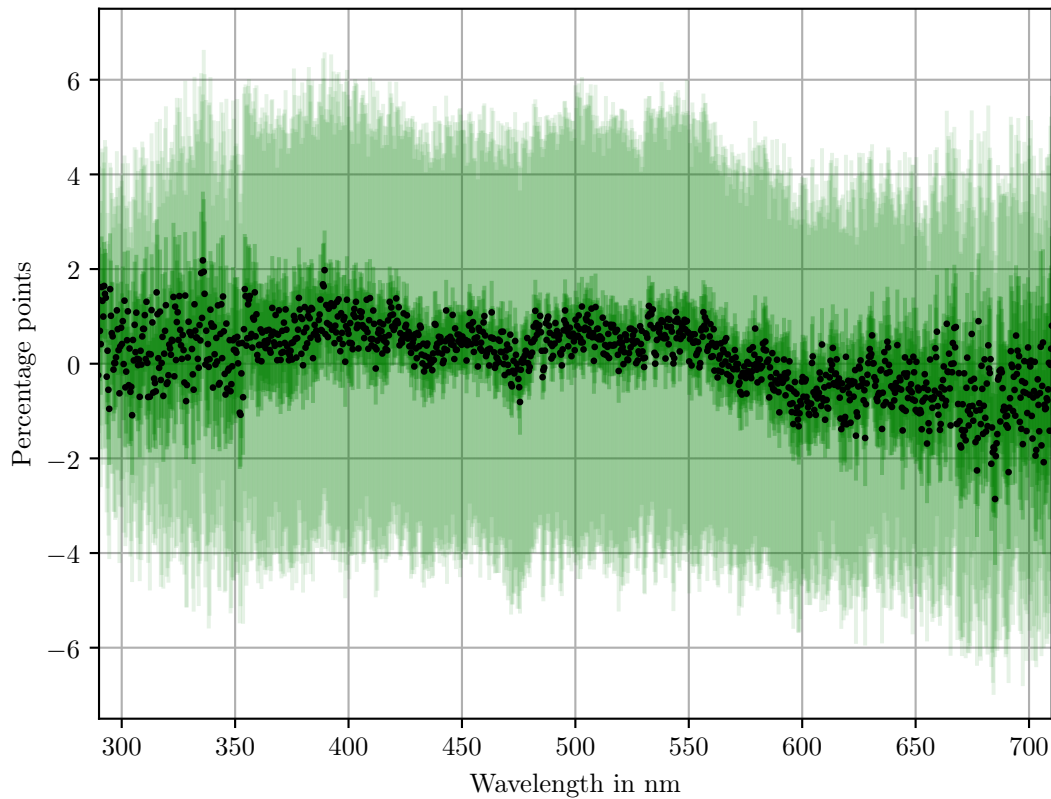


Figure III.6: Difference between the spectra of surface reflectivity measurements with different sets of optical fibers. The measurement, taken with the short set of optical fibers is subtracted from the measurement with the long set. The dark green error bars are the sum of the uncertainties of the mean of the two measurements. The light green error bars are the sum of the standard deviations of the two measurements. The mirror in test is MAGIC 225.

the assumption that the analysis-software of the reflectometer handles all data equally: Either the sensitivity of the detector or the intensity of the light source changes.

To observe the change of the measurements over time, a mirror was firmly connected to the test probe. The reflectometer was not in use for several days and it is reasonable to assume that the light source was at room temperature. Immediately after powering up the reflectometer, one measurement was taken and saved as internal reference. Then, the reflectometer was set up to measure the surface reflectivity of this mirror once every 15 s over the course of an hour. The data in Figure III.7 shows that the measured reflectivity drops to about 93 % of the value stored as the internal reference.

Since the light source of the reflectometer was used to probe its detector and vice versa, it is not possible to draw a conclusion about the quality of either. It is necessary to decouple those two units. Figure III.8 shows the data from such an experiment. The reflectometer was prepared similarly, thus the mirror was still attached to the test probe and a new internal reference spectrum was taken. The reflectometer was also set to save one measurement every 15 s for an hour but its light source was not turned on. The data has a bias from about -0.2% but no significant trend. One possible explanation is that the baseline of the readout electronic of the CCD shifted somehow over its lifetime. Nevertheless, this effect does not explain the degrading, visible in Figure III.7.

The experiment in Figure III.9, in contrast, was designed to isolate the effects of the light source

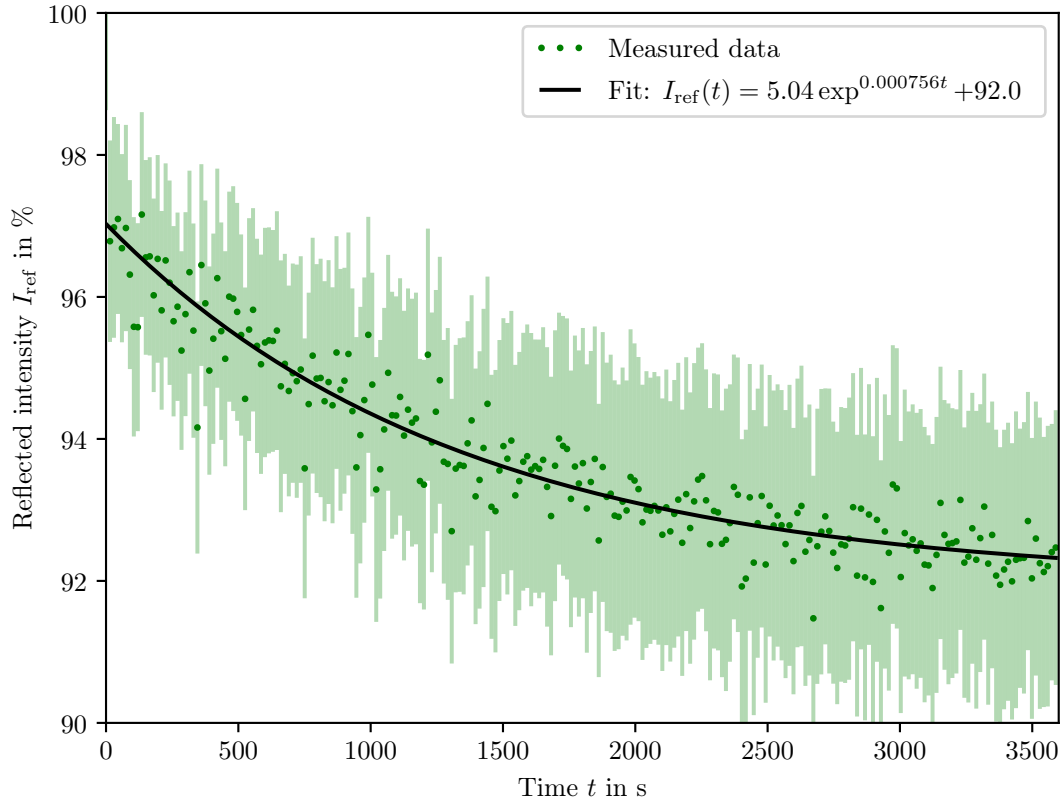


Figure III.7: Development of the reflected intensity I_{ref} from a mirror over time t . The hand-held reflectometer measured the surface reflectivity of the reference mirror STAN-SSH several times. The internal reference spectrum was also taken with STAN-SSH immediately before the measurements started. The test probe was never removed from the mirror surface and was connected with the reflectometer via the short set of optical fibers. For each individual measurement, the reflected intensities were averaged for light with wavelengths between 290 nm and 560 nm. The errorbars are the standard deviation of this set. The black line is an exponential decay fitted to the data with no respect for the standard deviation.

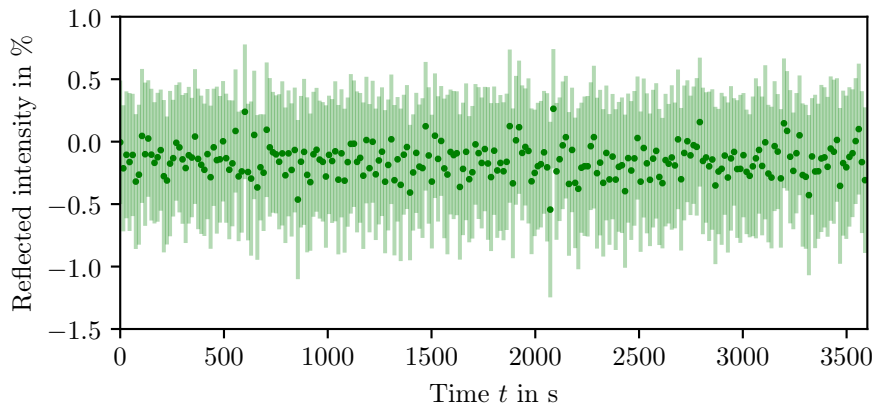


Figure III.8: Development of the measured reflectivity over time. The reflectometer was set up as it was for the experiment shown in Figure III.7, but the light source was not on during the measurement.

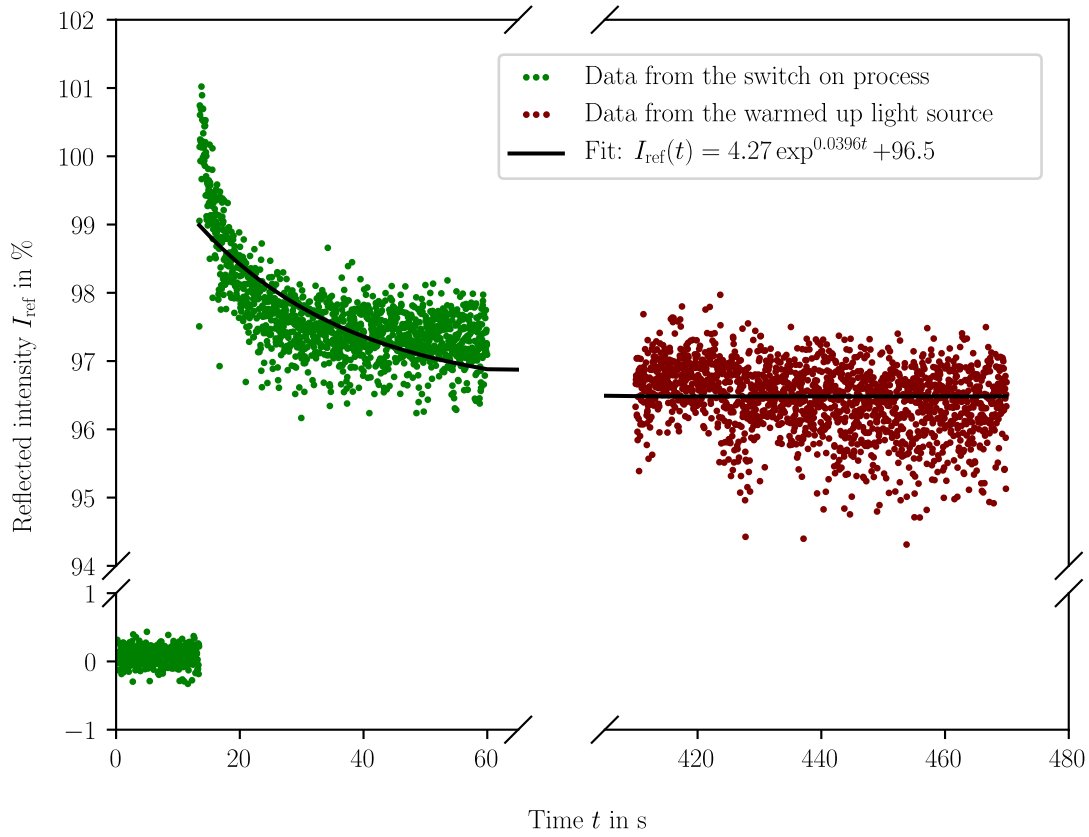


Figure III.9: Development of the measured reflectivity I_{ref} over time t . The reference mirror was attached to the test probe the whole time. The reference spectrum was taken, shortly after the light source was powered up for the first time in over two hours. Later, the reflectometer was set up to integrate intensity over $t_{\text{scan}} = 20$ ms for one minute and save every scan. At about $t = 15$ s the light source was switched on. This data is shown in green. The lamp continued running. After about five minutes the reflectometer began to save the scans again. This data is shown in red. There are no errorbars shown in this plot since they all are between 1.5 % and 2 % and therefore add no information to the figure. The black line is an exponential decay fitted to both sets of data.

from the effects that the detector could produce. The reflectometer was already reading out the detector and saving the data before the light source was lit. The data from about the first 15 s is similar to the data from Figure III.8. After the light source was switched on, a sharp peak in reflected intensity was recorded. The decay of the measured intensity is comparable to the one in Figure III.7 and both can be approximated with an exponential function. The light source warming up and transitioning into thermal equilibrium could explain this behavior of the hand-held reflectometer.

It is obvious that this effect is not taken into account by the internal analysis-software of the hand-held reflectometer. The internal reference spectrum that the reflectometer takes once after it is powered on, is therefore not a reliable reference for continued testing. The data, shown in Figure III.9 in red, was taken after the light source was running for about five minutes. This data is stable with regards for its spread. The procedure described in Paragraph 4 on Page 84 is therefore valid: A new reference measurement may differ from the internally saved reference spectrum but its difference is stable over about a minute and this is how long it takes to conduct a couple of surface reflectivity measurements. Letting the light source warm up for five minutes is indeed helpful.

Appendix IV

Features Visible on the CCD

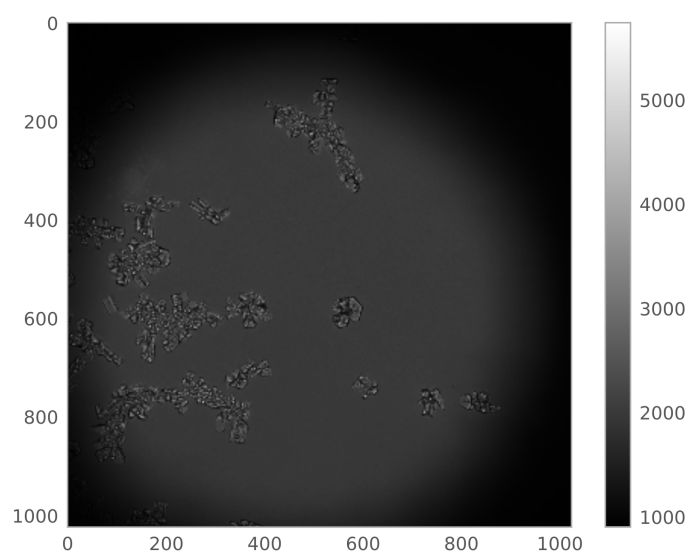


Figure IV.1: Exemplary reference picture, taken with the 405 nm LED. This picture showcases some of the more edgy structures that may appear on the CCD pictures and bear a crystal-like appearance.

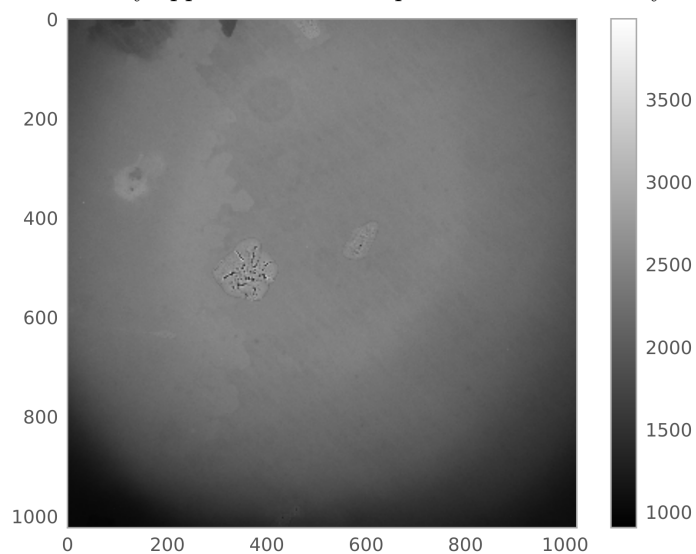


Figure IV.2: Exemplary reference picture, taken with the 310 nm LED. This picture showcases some of the rounder structures that may appear on the CCD pictures and bear a droplet-like appearance.

Appendix V

Irregular Warm-Up Characteristic

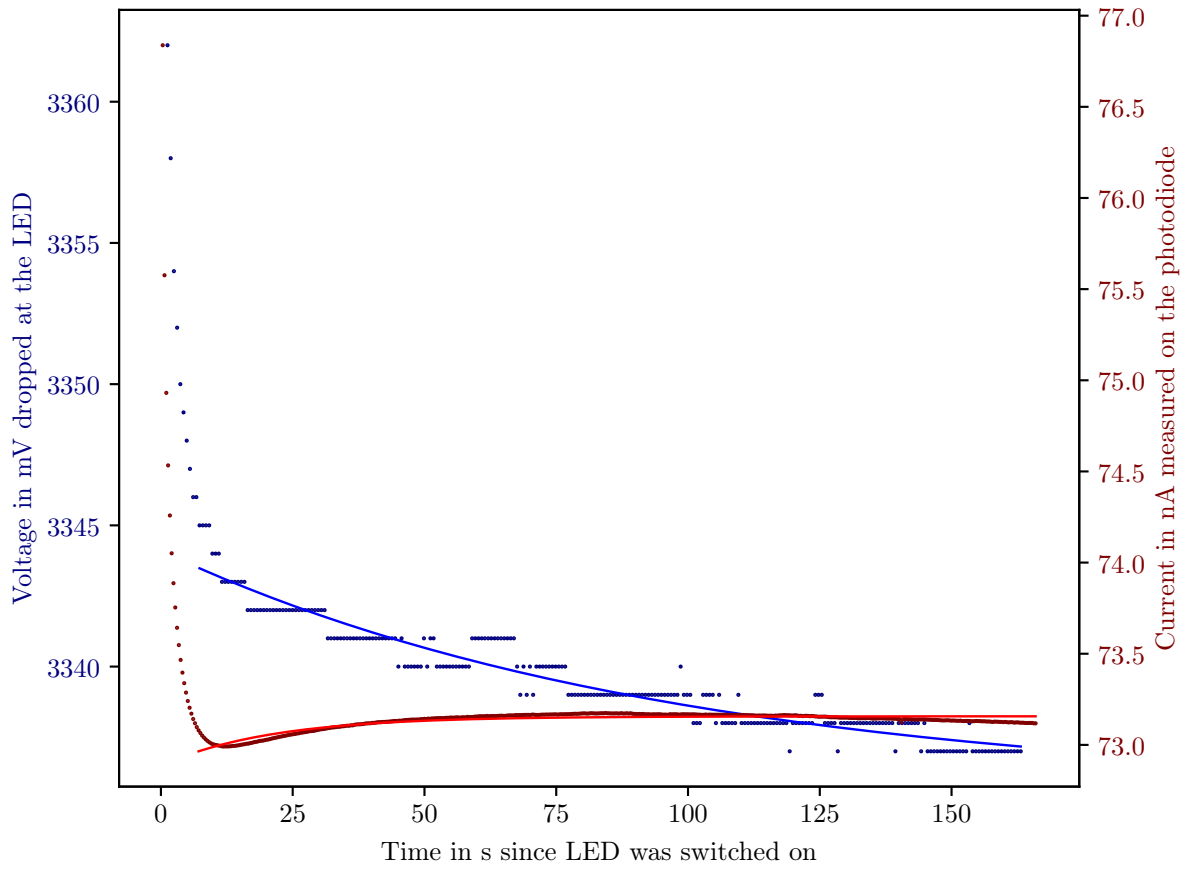


Figure V.1: The development of the forward voltage of an LED U_{LED} and the current generated by a photodiode J_{PD} that is illuminated by the LED with time. The LED in test was the SMB1N-365V 365 nm LED.

The current J_{PD} and probably the intensity of the LED I_{LED} do not follow an unmodified exponential decay like in Equation 4.11. This behavior is to some extent reproducible with the 365 nm LED. It was not observed at any other LED, yet.

Bibliography

- [Aharonian04] AHARONIAN, F., AKHPERJANIAN, A., BEILICKE, M., BERNLÖHR, K., BÖRST, H.-G. ET AL.: *The Crab Nebula and Pulsar between 500 GeV and 80 TeV: Observations with the HEGRA Stereoscopic Air Cerenkov Telescopes*, in *The Astrophysical Journal*, October 2004, volume 614, no. 2.
- [Apo] Apogee Instruments Inc., 1020 Sundown Way, Ste 150, CA 95661 Roseville: *High Performance Cooled CCD Camera System - ALTA U47*.
- [Bigongiari16] BIGONGIARI, CIRO: *The Cherenkov Telescope Array*, in *Nuclear Physics B Proceedings Supplement*, 2016 .
- [Bouvier13] BOUVIER, A., GEBREMEDHIN, L., JOHNSON, C., KUZNETSOV, A., WILLIAMS, D. A. ET AL.: *Photosensor Characterization for the Cherenkov Telescope Array: Silicon Photomultiplier versus Multi-Anode Photomultiplier Tube*, in *Hard X-Ray, Gamma-Ray, and Neutron Detector Physics XV*, volume 8852 of *Society of Photo-Optical Instrumentation Engineers (SPIE)*, San Diego, California, 2013 .
- [Bretz13] BRETZ, THOMAS and RIBORDY, MATHIEU: *Design Constraints on Cherenkov Telescopes with Davies-Cotton Reflectors*, Technical report, High Energy Physics Laboratory, CH 1015 Lausanne, January 2013.
- [Chadwick18] CHADWICK, P.M., ARCARO, C., DAZZI, F., DORO, M., FÖRSTER, A. ET AL.: *Mirror Qualification Test Plan*, April 2018.
Is marked as a 'DRAFT'. Is not published publicly.
- [Cornils06] CORNILS, RENÉ: *Justierung und Abbildungsfunktion der HESS-Reflektoren sowie Untersuchung der ultraleuchtkräftigen Infrarot-Galaxie Arp 220 mit dem HESS-Teleskopsystem*, Ph.D. thesis, Universität Hamburg, Department Physik, Hamburg, December 2006.
- [CTA Consortium11] CTA CONSORTIUM, ACTIS, M., AGNETTA, G., AHARONIAN, F., AKHPERJANIAN, A. ET AL.: *Design Concepts for the Cherenkov Telescope Array CTA: An Advanced Facility for Ground-Based High-Energy Gamma-Ray Astronomy*, in *Experimental Astronomy*, 2011, volume 32.
- [Demtröder17] DEMTRÖDER, WOLFGANG: *Kern-, Teilchen- und Astrophysik*, number 4 in *Experimentalphysik*, Springer Spektrum, Kaiserslautern, Rhineland Palatinate, fifth edition, 2017.
- [Dick] DICK, JÜRGEN: *LED as Light Source for Mirror Testing*. Slide show, presented at internal meeting.
- [Fienberg07] FIENBERG, RICHARD TRESCH: *Sharpening the 200-Inch*, in *Sky and Telescope*, September 2007.

- [Garczarczyk16] GARCZARCZYK, MARKUS: *Medium-Sized Telescope Technical Design Report*, Technical report, CTA Construction Project, February 2016.
- [Garczarczyk18a] GARCZARCZYK, MARKUS: *Medium Size Telescopes for the Cherenkov Telescope Array*, in *Ground-based and Airborne Telescopes VII*, volume 10700 of *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, July 2018 .
- [Garczarczyk18b] GARCZARCZYK, MARKUS: *MST Mirror Test and Specifications*, slide show, presented at CTA Consortium meeting, April 2018.
- [Giacconi79] GIACCONI, R., HOLT, S. S., CLARK, G., NOVICK, R., BRANDUARDI, G. ET AL.: *The Einstein (HEAO 2) X-Ray Observatory*, in *Astrophysical Journal*, June 1979, volume 230, no. 1: pp. 540 – 550.
- [Göggelmann17] GÖGGELMANN, STEPHANIE: *A Common Mirror Test Facility for the Cherenkov Telescope Array*, Master's thesis, Eberhard Karls Universität Tübingen, Institut für Astronomie und Astrophysik, Tübingen, Baden-Württemberg, October 2017.
- [Heitler44] HEITLER, W.: *The Quantum Theory of Radiation*, The International Series of Monographs on Physics, Oxford University Press, second edition, 1944.
- [Hillas96] HILLAS, A. M.: *Differences between Gamma-Ray and Hadronic Showers*, in *Space Science Reviews*, January 1996, volume 75: pp. 17–30.
- [JAM19] *Internal website of the cta observatory*, online, July 2019. Login required.
URL jama.cta-observatory.org
- [Kraushaar65] KRAUSHAAR, W., CLARK, G. W., GARMIRE, G., HELMKEN, H., HIGBIE, P. ET AL.: *Explorer XI Experiment on Cosmic Gamma Rays*, in *Astrophysical Journal*, April 1965, volume 141, no. 3: pp. 845 – 863.
- [Lambert92] LAMBERT, JOHANN HEINRICH: *Lamberts Photometrie*, *Lamberts Photometrie: (Photometria, sive De mensura et gradibus luminis, colorum et umbrae)* (1760), W. Engelmann, 1892.
URL books.google.de/books?id=lHoVawAAQBAJ
- [Lei13] Leica Geosystems AG, CH 9435 Heerbrugg: *Leica DISTO X310 The Original Laser Distance Meter*, 2013.
- [Mackay86] MACKAY, CRAIG D.: *Charge-Coupled Devices in Astronomy*, in *Annual Review of Astronomy and Astrophysics*, 1986, volume 24: pp. 255–283.
- [Newell66] NEWELL, HOMER E. (Editor): *Significant Achievements in Solar Physics 1958 - 1964*, U.S. Government Printing Office, Washington, D.C., July 1966.
- [Oce] Ocean Optics - A Halma Group Company, 830 Douglas Avenue, FL 34698 Dunedin: *Jaz Data Sheet*.
- [Roi14a] Roithner Laser Technik GmbH, Wiedner Hauptstraße 76, A 1040 Vienna: *SMB1N-365V*, August 2014.
- [Roi14b] Roithner Laser Technik GmbH, Wiedner Hauptstraße 76, A 1040 Vienna: *SMB1N-405V-02*, September 2014.
- [Roi14c] Roithner Laser Technik GmbH, Wiedner Hauptstraße 76, A 1040 Vienna: *SMB1N-470*, April 2014.

- [Roi14d] Roithner Laser Technik GmbH, Wiedner Hauptstraße 76, A 1040 Vienna: *SMB1N-525V*, July 2014.
- [Roi16] Roithner Laser Technik GmbH, Wiedner Hauptstraße 76, A 1040 Vienna: *DUVxxx-SD356*, June 2016.
- [Spangenberg16] SPANGENBERG, JOHANNES: *A Fast Testing Facility for CTA Mirrors*, Master's thesis, Eberhard Karls Universität Tübingen, Institut für Astronomie und Astrophysik, Tübingen, Baden-Württemberg, October 2016.
- [Sph16] Sphere Optics GmbH, Gewerbestrasse 13, D 82211 Herrsching: *Zenith Polymer Diffuse Reflectance Standards, Targets and Materials*, 2016.
- [Tayabaly15] TAYABALY, K., SPIGA, D., CANESTRARI, R., BONNOLI, G., LAVAGNA, M. ET AL.: *Roughness Tolerances for Cherenkov Telescope Mirrors*, in *Optics for EUV, X-Ray, and Gamma-Ray Astronomy VII*, volume 9603 of *Society of Photo-Optical Instrumentation Engineers (SPIE)*, 2015 .
- [Vincent05] VINCENT, P.: *H.E.S.S. Phase II*, in *29th International Cosmic Ray Conference Pune*, 2005 .
- [Weekes89] WEEKES, T. C., CAWLEY, M. F., FEGAN, D. J., GIBBS, K. G., HILLAS, A. M. ET AL.: *Observation of TeV Gamma Rays from the Crab Nebula Using the Atmospheric Cerenkov Imaging Technique*, in *Astrophysical Journal*, July 1989, volume 342: pp. 379–395.