Homework 2

Phylogenetic Networks

Due date: 24/04/2014

1. (2 points) Show that a tree with exactly two leaves is a path graph.

2. (2 points) Let T be a tree, and assume that the largest degree of a vertex of T is k. Show that T has at least k leaves.

3. (3 points) Let $b(n)$ be the number of rooted binary trees on X, where $|X| = n$. Show that

$$b(n) = \frac{1}{2} \sum_{i=1}^{n-1} \binom{n}{i} b(i)b(n-i).$$

4. (3 points) By disregarding the labels of the leaves of a phylogenetic tree, we obtain a tree shape. Conversely, given a tree shape, we can label its leaves so as to obtain phylogenetic tree. It is clear that in general there is more than one way to label the leaves of a tree shape. Prove that for all $n \geq 1$, there is a binary tree shape with n leaves for which the number of leaf-labelings is an odd number.