

Homework 3

Phylogenetics

Due date: Thu., 07/11/2013

1. (2 points) Let $b(n)$ be the number of rooted binary trees on X , where $|X| = n$. Show that for $n \geq 2$,

$$b(n) = \sum_{i=1}^{n-1} \binom{n-1}{i-1} b(i)b(n-i).$$

2. (3 points) Let $r(n)$ be the number of rooted trees (not necessarily binary) on X , where $|X| = n$, and let $r(x) = \sum_{n=1}^{\infty} r(n)x^n/n!$. Show that

$$r(x) = x + \sum_{k=2}^{\infty} \frac{r(x)^k}{k!}.$$

3. (3 points) Let $p(n, f)$ be the number of (unrooted) trees with f internal edges and leaves labeled by elements of X , where $|X| = n$. Show that for $n \geq 3$,

$$p(n, f) = (f+1)p(n-1, f) + (n+f-2)p(n-1, f-1).$$

4. (2 points) Show that for all $n \geq 1$, there is a binary tree shape on n leaves for which the number of leaf-labelings of that shape is an odd number.