



Department of Mathematics

Module Handbook

Mathematics

Bachelor of Education

Lehramt Gymnasium*

Winter Semester 2026

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*This is a secondary school teaching degree with a major in mathematics. The module handbook is valid for the 2026 study and examination regulations.

Contents

1	Description of the Study Programme	3
1.1	Qualification Objectives	3
1.2	Structure of the Study Programme	3
1.3	Examination Regulations	4
2	Study Plans	5
2.1	Overview by Modules	5
2.2	Overview by the Course of Studies	6
2.3	Overview of Programme Structure with Semester Assignment	10
3	Module Descriptions	14
	Section 1: Foundations of Mathematics	14
	Section 2: Compulsory Intermediate Modules	17
	Section 3: Didactics of Mathematics	28
	Section 4: Bachelor Thesis	31
	Section 5: Transferable Credits for the Master Degree	33
4	Courses for the Module Specialisation	38
4.1	Course Catalogue	38

1 Description of the Study Programme

1.1 Qualification Objectives

As part of the teacher training Bachelor's degree programme (B.Ed.) in Mathematics, graduates acquire basic and initial advanced subject-specific and subject-didactic knowledge and skills necessary for science-based teaching at secondary schools in Germany.

Graduates are familiar with the fundamental questions in Linear Algebra, Analysis, Geometry and Stochastics as well as Algebraic Structures and master the central techniques for solving them. In doing so, they acquire basic mathematical thought patterns such as structuring problems, creating chains of argumentation and finally the proof of mathematical theorems. Graduates are able to communicate mathematical facts, use suitable media and establish links to school mathematics. They are able to justify the educational value of mathematical content and convey the societal significance of mathematics. With the Bachelor's degree, graduates are able to apply their knowledge and skills in a teaching-related Master's study programme or, with credit for the work completed, in a science-related Bachelor's degree programme in mathematics.

1.2 Structure of the Study Programme

In Mathematics, the first year of study is filled with the large compulsory module Foundations of Mathematics, which covers the subject-specific fundamentals of Analysis and Linear Algebra from an academic point of view. The corresponding lectures are accompanied by exercise classes, where students are intensively supervised and taught basic mathematical thinking and working methods as well as the ability to present solutions. In addition, the department provides students with revision sessions as question times.

In the second and third years of the programme, students deepen their theoretical knowledge. They expand their knowledge in the areas of Algebra, Geometry, Numerical Mathematics, and Stochastics and take an introductory seminar. The content in the compulsory mathematics modules is taught through lectures and accompanying exercise classes. For each lecture there are weekly tasks, which students have to complete in paper form. In the exercise classes, the students present their solutions or create them under supervision. Through this system, which is common in mathematical study programmes, students learn to systematically work on the tasks set for them and to practise analytical and structural thinking. Furthermore, they should be able to explain complex mathematical matters and present them verbally. This requires students to be able to organise themselves and to do a lot of self-study, which is provided for and credited in the course of study. At the same time, intensive supervision and individual support options are provided.

In addition to the subject modules, students in the second and third years of study take modules in the area of subject didactic. These are designed in such a way that the subject-didactic courses in the areas of Stochastics and Geometry are to be taken in parallel with the corresponding subject modules

and are content-wise interlinked with them. The subject modules provide the academic prerequisites for the subject-didactic courses.

In the third year of the programme, students also complete a Bachelor's thesis. This can be written in one of the two chosen subject areas (including their subject didactics)

Integrating a study component at a foreign university into teacher training studies is challenging, as it involves coordinating two subjects and Educational Sciences. Whether attempting to fulfil components in all areas during the stay at the other university or adjusting the study plan at the University of Tübingen to allocate parts of the curriculum to different semesters to create flexibility, ensuring not all three areas need to be covered at the foreign university presents a challenge. Complicating matters is the fact that in the field of Mathematics, all modules are mandatory, leaving little room for content customisation. Therefore, it is essential to plan a suitable time frame for a study component at a foreign university through a personal consultation with the Faculty Course Advisor. Essentially, from the Mathematics perspective, any academic semester is suitable for this purpose. The decision will depend on the student's previous achievements and the courses offered at the chosen foreign university. **For modules consisting of several submodules, coursework completed abroad will also be credited towards individual submodules, so that course work and examination requirements need only be fulfilled for the remaining submodules, if any.**

1.3 Examination Regulations

Oral examinations are conducted in the presence of at least two examiners or one examiner, along with an observer (see also Exam Regulations General Part §12 (2)).

2 Study Plans

2.1 Overview by Modules

Here we provide an overview of the study plan as a table showing the modules to be taken.

ST	Module Number	Module Title	Type of Course	Type of Module	Course-work	Type of Exam	ECTS-Points
Section 1: Foundations of Mathematics							
1+2	MAT-10-10	Foundations of Mathematics		PM		or.	27
		- Foundations of Mathematics 1	L+E+T		EC		
		- Foundations of Mathematics 2	L+E+T		EC		
		- Exploring Mathematics	P		PC		
Section 2: Compulsory Intermediate Modules							
3-6	MAT-20-05	Compulsory Intermediate Modules Analysis		PM		or.	13.5
		- Measure and Integration Theory	L+E		EC		
		- Introduction to ODE	L+E		EC		
		- Introduction to Complex Analysis	L+E		EC		
3-6	MAT-20-06	Compulsory Intermediate Modules Algebra and Geometry		PM		or.	13.5
		- Algebraic Structures	L+E		EC		
		- Introduction to Galois Theory	L+E		EC		
		- Introduction to Geometry	L+E		EC		
5-6	MAT-20-12	Stochastics	L+E	PM	EC	wr. o. or.	9
3-4	MAT-20-13	Numerical Mathematics		PM		wr. o. or.	6
		- Introduction to Numerical Mathematics	L+E		EC		
		- Mathematical Software	P		PC		
3-4	MAT-20-20	Introductory Seminar: Presentations in Mathematics	PS	PMW	s.M.	Pr	3
Section 3: Didactics of Mathematics							
3-4	MAT-80-01	Subject Didactics Mathematics 1	LIC	PM	s.M.	wr., or., P o. T	3
5-6	MAT-80-02	Subject Didactics Mathematics 2	SLIC+SLIC	PM	-	wr., or., E, Pr o. T	6
Section 4: Bachelor Thesis							
6	MAT-30-40	Bachelor Thesis	BT	PM	s.M.	BT	6

Section 5: Transferable Credits for the Master Degree							
-	MAT-40-51	Specialisation 1	L+E	WM	EC	wr. o. or.	9
-	MAT-40-52	Specialisation 2	L+E	WM	EC	wr. o. or.	9
-	MAT-40-53	Seminar: Mathematical Specialisation	S	WM	s.M.	Pr	4
Abbreviations: Type of Module : PM=compulsory module, PMW=compulsory module with choice, WPM=elective module Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom Course Work : EC=exercise certificate, PEC=practical exercise certificate, PC=practical certificate Other : h=hours, o.=or, s.M.=see module description, ST=suggested term							

Note at the request of the student representatives on the Study Commission: For modules extending over several semesters, it is strongly recommended to review the content of each submodule thoroughly during the lecture-free periods, thereby spreading the preparation for the oral examination over a longer period of time.

2.2 Overview by the Course of Studies

Firstly, we provide an overview of the possible course of study in the form of a table both for entry in the winter semester and for entry in the summer semester. The second subject and the area of educational sciences are not broken down in detail.

Study Plan for Students Starting in the Winter Semester – Modules							
FS	CPiM	Subject Mathematics				Second Subject	ES
1	14	Foundations of Mathematics (27 CP)				Second Subject (81 CP)	Education Science and Orientation Internship (12 CP)
2	13						
3	15	Compulsory Intermediate Modules Algebra and Geometry (13,5 CP)	Numerical Mathematics (6 CP)				
4	15		Compulsory Intermediate Modules Analysis (13,5 CP)	Introductory seminar (3 CP)	SD Math 1 (3 CP)		
5	12			Subject Didactics Mathematics 2 (6 CP)			
6	12						
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points), CPiM=credit points in mathematics, ES=educational science							

Recommended Study Plan for Students Starting in the Winter Semester – Submodules			
FS	CP	List of Submodules	
1	14	Foundations of Mathematics 1 (13 CP)	Exploring Mathematics (1 CP)
2	13	Foundations of Mathematics 2 (13 CP)	
3	15	Algebraic Structures (4,5 CP) Introduction to Geometry (4,5 CP)	Introduction to Numerical Mathematics (4,5 CP) Mathematische Software (1,5 CP)
4	15	Introduction to Galoistheorie (4,5 CP)	Introductory seminar (3 CP)
		Introduction to Complex Analysis (4,5 CP)	Subject Didactics Mathematics 1 (3 CP)
5	12	Measure and Integration Theory (4,5 CP) Introduction to ODE (4,5 CP)	
			Subject Didactics Mathematics 2 - Teil 1 (3 CP)
6	12	Stochastics (9 CP)	Subject Didactics Mathematics 2 - Teil 2 (3 CP)
			possibly Bachelor Thesis (6 CP)
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points)			

Alternative Study Plan for Students Starting in the Winter Semester – Submodules			
FS	CP	List of Submodules	
1	14	Foundations of Mathematics 1 (13 CP)	Exploring Mathematics (1 CP)
2	13	Foundations of Mathematics 2 (13 CP)	
3	15	Algebraic Structures (4,5 CP)	Introduction to Numerical Mathematics (4,5 CP)
		Measure and Integration Theory (4,5 CP)	Mathematische Software (1,5 CP)
4	15	Stochastics (9 CP)	Introductory seminar (3 CP)
			Subject Didactics Mathematics 1 (3 CP)
5	12	Introduction to Geometry (4,5 CP)	
		Introduction to ODE (4,5 CP)	Subject Didactics Mathematics 2 - Teil 1 (3 CP)
6	12	Introduction to Complex Analysis (4,5 CP)	Subject Didactics Mathematics 2 - Teil 2 (3 CP)
		Introduction to Galoistheorie (4,5 CP)	possibly Bachelor Thesis (6 CP)
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points)			

Note: In the alternative study plan, two oral module examinations for multi-semester modules are scheduled after the 6th semester. If you wish to complete the practical semester in the following winter term, it will begin as early as the beginning of September, which limits the preparation time for the examinations. This can, however, be avoided by taking the Introduction to Complex Analysis or the Introduction to Galois Theory one year earlier and completing the corresponding module a semester in advance.

Study Plan for Students Starting in the Summer Semester						
FS	CPiM	Subject Mathematics		Second Subject	ES	
1	14	Foundations of Mathematics (27 CP)		Second Subject (81 CP)	Education Science and Orientation Internship (12 CP)	
2	13					
3	15	Introductory seminar (3 CP)	Compulsory Intermediate Modules Analysis (13,5 CP)			
4	15	Compulsory Intermediate Modules Algebra and Geometry (13,5 CP)				
5	12		Stochastics (9 CP)			Subject Didactics Math 1 (3 CP)
6	12	Numerical Mathematics (6 CP)	poss. Bachelor Thesis (6 CP)			
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points), CPiM=credit points in mathematics, ES=educational science						

Study Plan for Students Starting in the Summer Semester – Submodules			
FS	CP	List of Submodules	
1	14	Foundations of Mathematics 1 (13 CP) Exploring Mathematics (1 CP)	
2	13	Foundations of Mathematics 2 (13 CP)	
3	10,5	Introductory seminar (3 CP)	Subject Didactics Mathematics 1 (3 CP)
4	18	Introduction to Complex Analysis (4,5 CP)	Algebraic Structures (4,5 CP) Introduction to Geometry (4,5 CP) Introduction to Galoistheorie (4,5 CP)
		Measure and Integration Theory (4,5 CP) Introduction to ODE (4,5 CP)	
5	16,5	Stochastics (9 CP)	
6	9		Subject Didactics Mathematics 2 - Teil 1 (3 CP)
		Introduction to Numerical Mathematics (4,5 CP) Mathematische Software (1,5 CP)	Subject Didactics Mathematics 2 - Teil 2 (3 CP) possibly Bachelor Thesis (6 CP)
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points)			

Alternative Study Plan for Students Starting in the Summer Semester – Submodules			
FS	CP	List of Submodules	
1	14	Foundations of Mathematics 1 (13 CP) Exploring Mathematics (1 CP)	
2	13	Foundations of Mathematics 2 (13 CP)	
3	15	Introductory seminar (3 CP)	Subject Didactics Mathematics 1 (3 CP)
4	13,5	Introduction to Complex Analysis (4,5 CP)	Algebraic Structures (4,5 CP)
		Measure and Integration Theory (4,5 CP)	
		Introduction to ODE (4,5 CP)	
5	16,5	Stochastics (9 CP)	Introduction to Geometry (4,5 CP)
6	9		Introduction to Galoistheorie (4,5 CP)
		Introduction to Numerical Mathematics (4,5 CP)	Subject Didactics Mathematics 2 - Teil 1 (3 CP)
		Mathematische Software (1,5 CP)	Subject Didactics Mathematics 2 - Teil 2 (3 CP)
			possibly Bachelor Thesis (6 CP)
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points)			

2.3 Overview of Programme Structure with Semester Assignment

Overview of Programme Structure with Semester Assignment for Students Starting in the Winter Semester															
		Exam				Teaching				Term					
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points (CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.					
										1. CP	2. CP	3. CP	4. CP	5. CP	6. CP
Section 1: Foundations of Mathematics									27						
Foundations of Mathematics								26	27						
1.	Lecture	or.	30-40	g	27	L	o	12		9	9				
2.	Exercise class					E	o	6		4	4				
3.	Revision course					R	o	6		0	0				
4.	Practical course					P	o	2		1					
Section 2: Compulsory Intermediate Modules									45						
Compulsory Intermediate Modules Algebra and Geometry								9	13,5						
1.	Lecture	or.	20-30	g	13,5	L	o	6				6	3		
2.	Exercise class					E	o	3				3	1,5		
Compulsory Intermediate Modules Analysis								9	13,5						
1.	Lecture	or.	20-30	g	13,5	L	o	6					3	6	
2.	Exercise class					E	o	3					1,5	3	
Numerical Mathematics								5	6						
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	6	L	o	2				3			
2.	Exercise class					E	o	1				1,5			
3.	Practical course	-		ng		P	o	1				1,5			
Stochastics								6	9						
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	9	L	o	4							6
2.	Exercise class					E	o	2							3
Introductory seminar								2	3						
1.	Introductory seminar	Pr		g	3	PS	o	2					3		
Section 3: Subject Didactics Mathematics									9						
Subject Didactics Mathematics 1								2	3						
1.	Subject Didactics Mathematics 1	wr., or., P o. T	90-180 o. 20-30	g	3	LIC	o	2					3		

Overview of Programme Structure with Semester Assignment for Students Starting in the Winter Semester																
		Exam				Teaching				Term						
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points (CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.						
										1. CP	2. CP	3. CP	4. CP	5. CP	6. CP	
Subject Didactics Mathematics 2								4	6							
1.	Subject Didactics Mathematics 2 – Part 1	wr., or., Pr, E, P o. T	90-180 o. 20-30	g	3	SLIC o		2						3		
2.	Subject Didactics Mathematics 2 – Part 2	wr., or., Pr, E, P o. T	90-180 o. 20-30	g	3	SLIC o		2							3	
Section 4: Bachelor Thesis									6							
Bachelor Thesis									6							
1.	Bachelor thesis	BA		g		BA o									6	
Explanation of the abbreviations: Marking system : g=graded, ng=non graded Form of examination : BA=bachelor thesis, Or.=oral exam, Wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Form of teaching : L=lecture, SL=seminar or lecture, E= exercise class, R=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : o.=or, SWS=hours in class per week, CP=credit points=ECTS points																

Overview of Programme Structure with Semester Assignment for Students Starting in the Summer Semester																
		Exam				Teaching				Term						
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points (CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.						
										1. CP	2. CP	3. CP	4. CP	5. CP	6. CP	
Section 1: Foundations of Mathematics									27							
Foundations of Mathematics									26	27						
1.	Lecture	or.	30-40	g	27	L	o	12		9	9					
2.	Exercise class					E	o	6		4	4					
3.	Revision course					R	o	6		0	0					
4.	Practical course					P	o	2		1						
Section 2: Compulsory Intermediate Modules									45							
Compulsory Intermediate Modules Algebra and Geometry									9	13,5						
1.	Lecture	or.	20-30	g	13,5	L	o	6					6	3		
2.	Exercise class					E	o	3					3	1,5		
Compulsory Intermediate Modules Analysis									9	13,5						
1.	Lecture	or.	20-30	g	13,5	L	o	6				3	6			
2.	Exercise class					E	o	3				1,5	3			
Numerical Mathematics									5	6						
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	6	L	o	2							3	
2.	Exercise class					E	o	1							1,5	
3.	Practical course	-		ng		P	o	1							1,5	
Stochastics									6	9						
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	9	L	o	4						6		
2.	Exercise class					E	o	2						3		
Introductory seminar									2	3						
1.	Introductory seminar	Pr		g	3	PS	o	2				3				
Section 3: Subject Didactics Mathematics									9							
Subject Didactics Mathematics 1									2	3						
1.	Subject Didactics Mathematics 1	wr., or., P o. T	90-180 o. 20-30	g	3	LIC	o	2				3				

3 Module Descriptions

Section 1: Foundations of Mathematics

Module Number: MAT-10-10	Module Title: Foundations of Mathematics		Type of Module: Compulsory Module
ECTS-Points	27		
Workload - Time in Class - Self-Study	Workload: 810 h	Time in Class: 300 h	Self-Study: 510 h
Duration	2 Semester		
Frequency	every Semester		
Term	1+2		
Language of Instruction	German		
Forms of Teaching and Learning	1. Semester: Foundations of Mathematics 1, Lecture 6 SWS + Ex.cl. 3 SWS + Rev.c. 2 SWS 2. Semester: Foundations of Mathematics 2, Lecture 6 SWS + Ex.cl. 3 SWS + Rev.c. 2 SWS 1. Semester: Exploring Mathematics, Practical course 2 SWS		
Higher Objectives	In the Foundations of Mathematics module, students learn the essential conceptual and methodological foundations of linear algebra as well as single-variable and multivariable calculus, exploring their interconnections with particular emphasis on the similarities and differences in their approaches. In the oral exam, students demonstrate that they have recognised these relationships and are capable of contextualising the core results of the lectures within these frameworks. The duration of the module supports these objectives while also accounting for the acquisition of a new language - the language of mathematics - and the development of a precise and rigorously logical working methodology. This provides students with the necessary time to make the significant transition from school-level mathematics to university-level mathematics. By demonstrating a deeper and more integrated understanding in the oral exams, students establish a strong foundation for successful participation in all subsequent modules in their academic programme.		

Content	• Basic logic and sets. • Structure of real and complex numbers. • Sequences, convergence and series; criteria for convergence; power series, sequences of functions; pointwise and uniform convergence. • Continuous functions in one dimension and between metric spaces and their properties. • One- and multidimensional differential calculus (especially: intermediate value theorem, Taylor expansion, implicit function theorem, inverse function theorem, extrema under constraints). • Onedimensional Riemann integral. • Basic concepts of topology in metric and normed spaces. • Vector spaces and linear maps. • Matrices and systems of linear equations. • Determinants, eigenvalues and diagonalisability. • Jordan canonical form. • Euclidean and unitary vector spaces, spectral theorems. • Basics of analytical geometry. • The lecture Foundations of Mathematics 1 focuses primarily on topics in one-dimensional analysis and introduces basic concepts of linear algebra up to systems of linear equations. The lecture Foundations of Mathematics 2 covers further aspects of linear algebra as well as multivariable analysis. • The submodule Exploring Mathematics serves as an introduction to mathematical working methods, mathematical modelling, and abstraction.
Objectives	The students are familiar with and understand the fundamental concepts, statements, and methods of single-variable and multivariable calculus as well as linear algebra. Their capacity for abstraction has been enhanced, they have been trained in analytical thinking, and their mathematical imagination has been stimulated. Through a proof- and structure-oriented approach, they have learned to comprehend mathematical proofs in calculus and linear algebra and to independently prove or disprove mathematical statements in simple examples. They have recognised the essential relationships within the theory of single-variable and multivariable calculus, their similarities and differences, as well as their connections to linear algebra, and are able to contextualise the core results of the lectures within these frameworks. Students are in principle able to derive a mathematical problem and approaches to its solution from a question that may not be formulated in mathematical terms by means of mathematical modelling, and to relate these back to the original question. In the exercises, they have developed a confident, precise, and independent approach to the concepts, statements, and methods covered in the lectures. Additionally, their presentation and communication skills have been cultivated through written assignments and presenting their own solutions. The students are capable of acquiring knowledge through self-study, while their teamwork abilities have been fostered through collaboration in small groups.

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade				
	Foundations of Mathematics 1	L	o	6	9	yes	or.	30-40	g	100				
		E	o	3	4									
		T	o	2	0									
	Foundations of Mathematics 2	L	o	6	9	yes								
		E	o	3	4									
		T	o	2	0									
	Exploring Mathematics	P	o	2	1	yes								
	In each of the two submodules Foundations of Mathematics 1 and 2, an exercise certificate is required as course work. The exercise certificate is awarded on the basis of regular participation in the exercises together with successful completion of a test related to the exercises. Both components must be completed within the same semester. In the submodule Exploring Mathematics, a practical certificate is required as course work and is obtained through regular active participation. The assessment of the module takes the form of an oral examination covering the submodules Foundations of Mathematics 1 and 2. Admission to the oral examination requires at least one of the two exercise certificates in the submodules Foundations of Mathematics 1 or 2. The module is deemed completed only once both exercise certificates and the practical certificate have been obtained and the oral examination has been passed. For modules extending over several semesters, it is strongly recommended to review the content of each submodule thoroughly during the lecture-free periods, thereby spreading the preparation for the oral examination over a longer period of time.													
	Literature	Possible References : • Anton Deitmar: Analysis. Springer 2016. • Otto Forster: Analysis 1. Springer Spektrum 2013. • Otto Forster: Analysis 2. Vieweg+Teubner 2011. • Theodor Bröcker: Lineare Algebra und analytische Geometrie. Birkhäuser 2013. • Gerd Fischer: Lineare Algebra. Springer Spektrum 2014. • Daniel Grieser: Mathematisches Problemlösen und Beweisen. Springer Spektrum 2017.												
Transfer	The successful participation in the module Foundations of Mathematics is a prerequisite for the participation in the module Bachelor Thesis. The exercise certificates of the module are prerequisite for all modules of the Sections 2-4.													
Prerequisites	There are no prerequisites.													
Responsible Persons	Daniele Agostini, Anton Deitmar, Frank Loose, Hannah Markwig, Thomas Markwig, Stefan Teufel													
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week														

Section 2: Compulsory Intermediate Modules

Module Number: MAT-20-05	Module Title: Compulsory Intermediate Modules Analysis		Type of Module: Compulsory Module
ECTS-Points	13.5		
Workload - Time in Class - Self-Study	Workload: 405 h	Time in Class: 135 h	Self-Study: 270 h
Duration	2 Semester		
Frequency	• Measure and Integration Theory, regularly in Winter Semester • Introduction to ODE, regularly in Winter Semester • Introduction to Complex Analysis, regularly in Summer Semester		
Term	3-6		
Language of Instruction	German		
Forms of Teaching and Learning	• Measure and Integration Theory, Lecture 2 SWS + Ex.cl. 1 SWS • Introduction to ODE, Lecture 2 SWS + Ex.cl. 1 SWS • Introduction to Complex Analysis, Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Measure and Integration Theory: – Measures, in particular the Lebesgue measure. – Lebesgue integral, L^p-spaces, Fubini's theorem, convergence theorems. – Integration on areas of $\mathbb{R}^n$, transformation formula. – Convergence theorems. • Ordinary differential equations, a choice of the following: – Picard-Lindelöf existence and uniqueness theorem. – Linear ordinary differential equations, Gronwall's lemma. – Continuous and differential dependence on initial conditions. – Basics of dynamical systems, stability of equilibrium positions. – Ordinary differential equations over the complex numbers. • Complex Analysis: – Holomorphic functions, Cauchy-Riemann equations. – Antiderivatives, Cauchy's integral formula, Cauchy's integral theorem. – Compact convergence of families of functions, power series, identity theorem. – Liouville's theorem, inverse function theorem for holomorphic functions, open mapping theorem, maximum principle. – Laurent series. – Residue theorem and applications.		

Literature	Possible References : • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 1978. • Anton Deitmar: Analysis. Springer Spektrum 2017. • Jürgen Elstrodt: Maß- und Integrationstheorie. Springer 2011. • Otto Forster: Analysis 3. Friedr. Vieweg+Teubner 2011. • Lars Valerian Ahlfors: Complex analysis. McGraw-Hill 1979. • John B. Conway: Functions of one complex variable. Springer 1996. • Wolfgang Fischer, Ingo Lieb: Einführung in die Komplexe Analysis. Springer 2010. • Walter Rudin: Reelle und komplexe Analysis. Oldenbourg 2009. • Earl A. Coddington, Norman Levinson: Theory of ordinary differential equations. McGraw-Hill 1955. • William T. Reid: Ordinary differential equations. John Wiley & Sons 1971. • Hille, Einar: Ordinary differential equations in the complex domain. Dover Publications 1997. • Wasow, Wolfgang: Asymptotic expansions for ordinary differential equations. John Wiley 1965.
Transfer	-
Prerequisites	At least one of the exercise certificates from the module Foundations of Mathematics must have been acquired.
Responsible Persons	Anton Deitmar, Peter Pickl, Reiner Schätzle, Stefan Teufel
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-20-06	Module Title: Compulsory Intermediate Modules Algebra and Geometry		Type of Module: Compulsory Module
ECTS-Points	13.5		
Workload - Time in Class - Self-Study	Workload: 405 h	Time in Class: 135 h	Self-Study: 270 h
Duration	2 Semester		
Frequency	• Algebraic Structures, regularly every Semester • Introduction to Galois Theory, regularly in Summer Semester • Introduction to Geometry, regularly in Winter Semester		
Term	3-6		
Language of Instruction	German		
Forms of Teaching and Learning	• Algebraic Structures, Lecture 2 SWS + Ex.cl. 1 SWS • Introduction to Galois Theory, Lecture 2 SWS + Ex.cl. 1 SWS • Introduction to Geometry, Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Algebraic Structures – Groups, subgroups, group homomorphisms, normal subgroups, quotient group. – Cyclic groups and the symmetric group. – Commutative rings with one, divisibility. – Euclidean rings, principal ideal domains, factorial rings. – The ring of integers and the polynomial ring. • Introduction to Galois Theory – Fields and field extensions. – Fundamental theorem of Galois theory and applications, in particular: * Constructions with ruler and compass. * Solvability of equations. • Introduction to Geometry – Selected topics in geometry from the following canon are covered: * Axiomatic foundation of planar geometry. * Euclidean geometry. * Non-Euclidean geometry. * Analytic geometry. * Parametrised curves and surfaces.		

Literature	Possible References : • Serge Lang: Algebraische Strukturen. Vandenhoeck & Ruprecht 1979. • Siegfried Bosch: Algebra. Springer 2009. • Gerd Fischer, Reinhard Sacher: Einführung in die Algebra. Teubner 1983. • Christian Karpfinger, Kurt Meyberg: Algebra: Gruppen-Ringe-Körper. Springer Spektrum 2010. • Hans-Jörg Reiffen, Günter Scheja, Udo Vetter: Algebra. Bibliographisches Institut 1984. • Michele Audin: Geometry. Springer 2003. • Marcel Berger: Geometry Revealed: A Jacob's Ladder to Modern Higher Geometry. Springer 2010. • David A. Brannan, Matthew F. Esplen, Jeremy J. Gray: Geometry. Cambridge University Press 2012. • John Stillwell: The four pillars of geometry. Springer 2005.
Transfer	-
Prerequisites	At least one of the exercise certificates from the module Foundations of Mathematics must have been acquired.
Responsible Persons	Daniele Agostini, Carla Cederbaum, Jürgen Hausen, Hannah Markwig, Thomas Markwig, Walther Paravicini
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Responsible Persons	Martin Möhle, Elmar Teufl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-20-13	Module Title: Numerical Mathematics					Type of Module: Compulsory Module				
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	1 Semester									
Frequency	regularly in Winter Semester									
Term	3-4									
Language of Instruction	German									
Forms of Teaching and Learning	• Introduction to Numerical Mathematics, Lecture 2 SWS + Ex.cl. 1 SWS • Mathematical Software, Practical course 1 SWS									
Content	• Interpolation and approximation of functions. • Numeric integration and differentiation. • Systems of linear equations and linear curve fitting.									
Objectives	The students know the foundations of numerical mathematics and are capable of performing basic calculation techniques. They understand to bring the knowledge gathered in the modules Analysis and Linear Algebra in the analysis of numerical methods and to use the methods for specific problems. Their algorithmic thinking was enhanced and they are acquainted to the analysis of algorithms with a view to questions of efficiency and complexity. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. Furthermore the presentation and communication skills of the students were trained by written assignments and presenting their own solutions. The students are capable of adopting knowledge by self-study and at the same time their capacity for teamwork was enhanced by working in small groups.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introduction to Numerical Mathematics	L	o	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	o	1	1,5					
	Mathematical Software	P	o	1	1,5	yes	-	-	nb	0
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Peter Deuflhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008. • Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg+Teubner 2009.									
Transfer	If applicable, the module is prerequisite for the module Bachelor Thesis.									

Prerequisites	At least one of the exercise certificates from the module Foundations in Mathematics must have been acquired. Furthermore, before admission to the examination, the practical certificate for the submodule Mathematical Software must have been obtained.
Responsible Persons	Christian Lubich, Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-20-20	Module Title: Introductory Seminar: Presentations in Mathematics					Type of Module: Compulsory Module with Choice				
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	every Semester									
Term	3-4									
Language of Instruction	German									
Forms of Teaching and Learning	Introductory seminar, talk, presentation, e-learning, blended learning									
Content	Various topics from the foundations of mathematics.									
Objectives	The students independently work on a coherent mathematical topic and prepare it in a didactical appealing form. They learn how to present their work to a group, how to be responsive to questions regarding the content and how to lead a professional discussion.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introductory Seminar	PS	o	2	3	yes	Pr	60-90	g	100
	The acquisition of the credit points requires alongside with a successful presentation the regular active participation in the course, like by asking questions, contributing to a discussion or working on problem tasks. Additionally a written elaboration of the own talk or the issue of a handout for the participants may be required. This further work constitutes the coursework of the module.									
Transfer	The module Introductory Seminar Presentation in Mathematics is, if applicable, prerequisite for the module Bachelor Thesis.									
Prerequisites	At least two of the exercise certificates from the module Foundations in Mathematics must have been acquired. One of these must be the certificate for linear algebra 1.									
Responsible Persons	The dean of studies at the Department of Mathematics									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

Section 3: Didactics of Mathematics

Module Number: MAT-80-01	Module Title: Subject Didactics Mathematics 1						Type of Module: Compulsory Module			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	regularly in Summer Semester									
Term	3-4									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture, exercise class, seminar, talk, presentation, e-learning, blended learning, project work, case studies									
Content	Didactics of Algebra and Arithmetic: This course deals with the foundations of the didactics of mathematics in the educational plans and in particular the didactic reduction of important basic concepts of algebra and arithmetic to school level, various ways of introducing important concepts of algebra and arithmetic at school and ways of motivating basic algebraic and arithmetic ideas.									
Objectives	Students know the basic didactic principles of teaching concepts and can orientate themselves in the educational plans. They are able to compare and evaluate subject-specific approaches to central concepts in algebra and arithmetic. They have the ability to convey algebraic and arithmetic content in a way that is both student- and subject-orientated.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Subject Didactics Mathematics 1	LIC	o	2	3	no	wr., or., P o. T	90-180 o. 20-30	g	100
	Whether the form of the examination is decided by the instructor with approval by the head of the examination board.									
Transfer	The module Didactics of Mathematics 1 is compulsory for the module Bachelor Thesis, if the bachelor thesis is written in mathematics.									
Prerequisites	At least two of the exercise certificates from the module Foundations of Mathematics must have been acquired. One of these must be the exercise certificate for Linear Algebra 1.									
Responsible Persons	Frank Loose, Walther Paravicini									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests

Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio,
T=continuous assessment tests

Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course,
PS=introductory seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 4: Bachelor Thesis

Module Number: MAT-30-40	Module Title: Bachelor Thesis						Type of Module: Compulsory Module			
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: regularly, as required			Self-Study: 180 h			
Duration	1 Semester									
Frequency	every Semester									
Term	6									
Language of Instruction	German									
Forms of Teaching and Learning	Bachelor thesis									
Content	The students have to work under instruction of an advisor on a defined task from mathematics or subject didactics mathematics with scientific methods and present the results in written form. In detail this includes: • the formulation of a scientific question in accordance with the advisor; • the independent search for and the study of relevant scientific literature; • the formulation of suited questions and methodical approaches for their solution; • the independent realisation of the project, the written presentation of the project and the results in the context of the current state of research.									
Objectives	The students • can work independently and with scientific methods on an assigned topic, • operate a literature research for scientific sources, • choose scientific methods and techniques or develop them further to solve a problem, • communicate the results in a clearly structured fashion and in academically suited form in their thesis.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Bachelor Thesis	BT	f	-	6	yes	BT	-	g	100
	The coursework consists of an examination interview on the contents of the bachelor thesis. The coursework is graded as pass or fail. The module grade is the grade for the bachelor thesis. The module is only passed if the examination and coursework have been successfully completed.									
Transfer	Bachelorarbeit									

Prerequisites	Subject specific prerequisite for admission to the module Bachelor Thesis is besides the general part of the examination regulations the acquisition of the credit points from the modules of Section 1 Foundations of Mathematics as well as of at least 21 credit points from the modules of the Section 2 and at least 3 credit points from the modules of Section 3.
Responsible Persons	The dean of studies at the Department of Mathematics
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Section 5: Transferable Credits for the Master Degree

In anticipation of a prospective Master's programme in the Master of Education for Secondary Schools at the University of Tübingen, certain achievements can be made during the Bachelor's programme under specific conditions, which can be credited towards the Master's programme. This aims to offer flexibility in individual study planning during the transition from the Bachelor's to the Master of Education.

Conditions and Scope

Up to a total of 24 ECTS credits for the Master's programme can be acquired in the Bachelor of Education if all of the following conditions are met:

- There is an enrolment (matriculation) in and an examination entitlement in the Bachelor of Education for Secondary Schools;
- A total of at least 150 ECTS credits have already been acquired in the two main subjects and in educational sciences;
- There is an enrolment in and an examination entitlement in the subject in which credits for the Master's programme are to be acquired.

It can be freely chosen how many ECTS credits are earned in which of the studied subjects. For example, all 24 ECTS credits can be earned in one subject if modules are offered in the required extent. Master's modules of a subject taken as a third subject cannot be advanced. Module examinations within the framework of Master's credits can only be repeated once. For further regulations concerning Master's credits, please refer to the study and examination regulations.

In the subject of Mathematics, the following modules can be advanced within the framework of Transferable Credits for the Master Degree.

Module Number: MAT-40-51	Module Title: Specialisation 1		Type of Module: Elective Module
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	every Semester		
Term	-		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Comment	A course must be selected from the catalogue of courses in Section 4.1 of the module handbook, comprising 4 hours of lectures and 2 hours of exercises per week. The approval of additional courses or alternative course formats (e.g., two courses with 2 hours of lectures and 1 hour of exercises each) is at the discretion of the head of the examination board, upon a written request by the student.		

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests

Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-53	Module Title: Seminar: Mathematical Specialisation					Type of Module: Elective Module				
ECTS-Points	4									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	every Semester									
Term	-									
Language of Instruction	German									
Forms of Teaching and Learning	Seminar, talk, presentation, e-learning, blended learning									
Content	Various topics from the advanced fields of mathematics.									
Objectives	The students independently work on a coherent mathematical topic and prepare it in a didactical appealing fashion. They learn how to present their work to a group, how to be responsive to questions regarding the content and how to lead a professional discussion. The work and the presentation may be the foundation or a deepened study in the scope of a master thesis.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Seminar	S	o	2	4	yes	Pr	60-90	g	100
	The acquisition of the credit points requires alongside with a successful presentation the regular active participation in the course, like by asking questions, contributing to a discussion or working on problem tasks. Additionally a written elaboration of the own talk or the issue of a handout for the participants may be required. These further efforts constitute the coursework of the module.									
Transfer	It is to be transferred to the consecutive master's programme.									
Prerequisites	The participation in the module requires the successful completion of at least one of the modules Specialisation 1 or 2.									
Responsible Persons	The dean of studies at the Department of Mathematics									

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Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status : o=obligatory, f=facultative
Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

4 Courses for the Module Specialisation

4.1 Course Catalogue

The following lists the courses that can be included in the module Specialisation (MAT-40-51). Additional courses can be approved upon written request by the head of the examination board.

- Algebraic Curves and Riemann Surfaces 41
- Algebraic Number Theory 43
- Algebraic Topology 1 42
- Algorithms of Numerical Mathematics 43
- Automorphic Forms 44
- Bayesian Networks and Causality 45
- Calculus of Variations 93
- Commutative Algebra 71
- Consistency Proofs 95
- Convex Geometry 72
- Cryptography 73
- Elementary Number Theory 58
- Elliptic Curves and Cryptography 59
- Explicit Mathematics 60
- Financial Mathematics and Numerics 61
- Functional Analysis 62
- Game Theory 87
- Geometric Group Theory 64
- Geometry of Manifolds 1 63
- Graph Theory 65
- Groups and Representations 66

• Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic	67
• Information Geometry	68
• Information Theory, Pattern Recognition, and Neural Networks	69
• Interacting Many-Body Quantum Systems	70
• Introduction to Analytic Number Theory	47
• Introduction to Berkovich Geometry	48
• Introduction to Commutative Algebra and Algebraic Geometry	54
• Introduction to Dynamical Systems	51
• Introduction to Geometric Measure Theory	52
• Introduction to Geometric Measure Theory – Measure Theoretic Methods	53
• Introduction to Geometric Measure Theory – Varifolds	53
• Introduction to Mathematical Logic	49
• Introduction to Optimisation	50
• Introduction to Partial Differential Equations	55
• Introduction to Partial Differential Equations – Part 1	56
• Introduction to Riemann Surfaces	57
• Introduction to Stochastic Differential Equations - Part 1	58
• Introduction to set theory	50
• Lie Groups	74
• Linear Control Theory	74
• Mathematical Aspects of 3d Computer Vision	75
• Mathematical Aspects of Deep Learning 1	76
• Mathematical Introduction to Data Science	77
• Mathematical Statistics	78
• Nonparametric Estimation of Distributions	79
• Number Theory and Cryptography	95
• Numerics of Stationary Differential Equations	80
• Numerics of Stochastic Differential Equations	81
• Optimal Control Theory with Ordinary Differential Equations	81

• Optimal Transport	82
• Optimisation with Differential Equations	83
• Ordinary Differential Equations - Analysis and Numerics	65
• Probability Theory	94
• Problem Solving Strategies	84
• Propagation of Chaos	85
• Representation Theory of Finite Groups	46
• Software Obfuscation: Mixed Boolean-Arithmetic Expressions	86
• Statistical Learning 1	87
• Stochastic Differential Equations	88
• Theoretical Aspects of Machine Learning	89
• Theory and Numerics for Constrained Optimisation Problems	90
• Theory of Mathematical Proofs	77
• Topology	91
• Tropical Enumerative Geometry	91
• Tropical Enumerative Geometry - Part 1	92

Course Title:	Algebraic Curves and Riemann Surfaces		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Compact Riemann surfaces. • Normalisation of plane curves. • Topological genus. • Coverings. • Forms and integration. • Sheaves and cohomology. • Hodge theory. • Arithmetic and geometric genus. • Abel's theorem. • Riemann-Roch theorem. • Serre duality. • Jacobian and Abelian varieties. • Riemann bilinear relations. • Jacobi inverse problem. • Elliptic curves and functions. • j-Invariant. • Uniformisation. • Topology of non-compact Riemann surfaces.		
Special Objectives	Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. By example of the concept of sheaves, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	Possible References : • Frederice Mangolte: Real Algebraic Varieties. Springer 2020. • Robert Silhol: Real Algebraic Surfaces. Springer 1989. • Riccardo Benedetti, Jean-Jacques Risler: Real Algebraic and Semi-algebraic Sets. Editions Hermann 1990. • Alex Degtyarev, Viatcheslav Kharlamov: Topological properties of real algebraic varieties: du côté de chez Rokhlin. arXiv:math/0004134.
Responsible Persons	Daniele Agostini, Ivo Radloff

Course Title:	Algebraic Topology 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Set theoretical topology. • Basic concepts of category theory. • The fundamental group of a punctured topological space. • Theory of covering spaces. • Basic concepts of singular homology theory. • Applications.		
Special Objectives	The students learn how to realise ideas in topology, e.g. the detection of holes in topological spaces, into a precise theory, even with a sophisticated technique. In particular, they recognise how abstract concepts, e.g. from category theory and homological algebra, provide effective ways of speaking that enable the formation of ideas to be adequately implemented.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	Possible References : • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.		
Responsible Persons	Anton Deitmar, Frank Loose		

Course Title:	Algebraic Number Theory		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Rings of integers. • Class numbers. • Dirichlet's unit theorem. • Extension of Dedekind rings. • Valuation theory. • Local fields. • Adeles and ideles.		
Special Objectives	The students have learned the central terms, results and methods of algebraic number theory.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.		
Literature	Possible References : • Jürgen Neukirch: Algebraische Zahlentheorie. Springer 2007. • Alexander Schmidt: Einführung in die algebraische Zahlentheorie. Springer 2007. • Andre Weil: Basic number theory. Springer 1995.		
Responsible Persons	Daniele Agostini, Anton Deitmar		

Course Title:	Algorithms of Numerical Mathematics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		

Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS
Content	Advanced, important algorithms of numerics (without differential equations) such as: • Fast Fourier transformation; • QR algorithms for the calculation of eigenvalues; • Method of conjugated gradients and more general Krylov space methods as iterative methods in numeric linear algebra and in non-linear optimisation; • Simplex method and interior point methods in linear optimisation.
Special Objectives	The students have learned the key concepts, results, and methods of algorithmic numerical mathematics.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Literature	Possible References : • Peter Deufhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008. • Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg 2009.
Responsible Persons	Christian Lubich, Andreas Prohl

Course Title:	Automorphic Forms		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Modular forms for the module group and its congruence subgroups. • Examples: Eisenstein series, Ramanujan delta function, theta series. • Modular curves. • Arithmetic applications and conjectures. • Hecke operators. • The L-function of a modular form and its connections with elliptic curves.		
Special Objectives	Students have familiarised themselves with the central concepts and methods of the theory of automorphic forms in examples and are able to use them. They have understood the connection between modular, real representation theory and adelic L-functions.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Literature	Possible References : • Deitmar, Anton: Automorphic Forms. Springer 2012. • Goldfeld, Dorian: Automorphic forms and L-functions for the group $GL(n, \mathbb{R})$. Cambridge University Press 2015. • Serre, Jean-Pierre: A course in arithmetic. Springer 1973.
Responsible Persons	Anton Deitmar

Course Title:	Bayesian Networks and Causality		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	Uncertainty is a fact of life, and robust artificial intelligence for real life application has to be able to deal with it. Therefore, the development of mathematical representations that effectively incorporate probabilities was a key step in the development of artificial intelligence. However, human understanding goes further than that: We go beyond observing events, such as <i>Whenever the sprinkler is on in my greenhouse, the plants are wet</i> or <i>Whenever the plants are wet, the sprinkler is on</i>, to postulating a relationship of cause and effect: <i>If I turn on the sprinkler, the plants will be wet, but if I water the plants, that will certainly not activate the sprinkler.</i> In this course, Bayesian networks are studied, which are a widely used representation for probability distributions. In particular, commonly used inference and learning algorithms are discussed. Moreover, it will be shown that Bayesian networks do not only represent probability distributions, but are also able to express causal relationships. Finally the causal expressivity of Bayesian networks will be looked at, aiming to learn causal structure from observational data. • Part I: Bayesian Networks as an Efficient Representation of Probability Distributions: – Computing probabilities using Bayesian networks. – d-Separation: A graphical criterion for probabilistic independence. – Parameter and structure learning in Bayesian networks. • Part II: Bayesian Networks as a Representation for Causal Knowledge: – Functional causal models: A representation of causal knowledge. – Pearl's causal ladder: Predicting the effects of external interventions and reasoning with counterfactuals. – Causal Bayesian networks. – Causal structure discovery: Learning causal relationships from data. – Counterfactual identifiability: Answering counterfactual. questions using causal Bayesian networks.		
Special Objectives	In the first part of the course, students have learned how Bayesian networks represent probability distributions and how this representation can be used to efficiently compute probabilities or determine whether two random variables are independent. In the second part, students have learned to distinguish inference tasks in artificial intelligence according to Pearl's causal ladder: probabilistic, interventional, and counterfactual reasoning, which generally require increasingly detailed knowledge. They are familiar with identifiability results, which provide assumptions under which certain queries on the causal ladder can be answered using only knowledge from lower levels. For instance, they know how to answer queries about the effects of external interventions using only knowledge of the correct probability distribution, which can often be estimated from observational data.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.		
Literature	Possible References : • Judea Pearl: Causality: Models, Reasoning and Inference. Cambridge University Press 2009.		
Responsible Persons	Stephan Eckstein		

Course Title:	Representation Theory of Finite Groups		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h

Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	• Groups and group actions. • Representations, irreducibility, Schursch's lemma. • Semisimplicity, Maschke's theorem. • Characters, orthogonality relations. • Isotypical decomposition, character tables. • Representations of the symmetric group. • Semi-simple Artinian algebras.		
Special Objectives	In the lecture, students learn the basic principles of of representation theory and develop an understanding for the interaction of geometric and algebraic methods. methods.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		
Literature	Possible References : • William Fulton, Joe Harris: Representation theory. Springer 1991. • Bertram Huppert: Character theory of finite groups. De Gruyter 1998. • Serge Lang: Algebra. Springer 2002. • Jean-Pierre Serre: Linear representations of finite groups. Springer 1977.		
Responsible Persons	Jürgen Hausen, Thomas Markwig		

Course Title:	Introduction to Analytic Number Theory		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		

Content	• Arithmetic functions and Dirichlet series, • Prime number theorem and Dirichlet's prime number theorem, • Zeros of the Riemannian zeta function, Riemann hypothesis and the explicit formula.
Special Objectives	The students understand the interplay between analysis and number theory. They can apply analytical methods to number theoretic problems. They understand the mechanism of analytical continuation through integral representation and have learned to independently transfer it to other cases, such as automorphic L-functions. They have gained an understanding of the Riemann hypothesis, which is considered the most difficult problem of all math, and understand its depth.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.
Literature	Possible References : • Komaravolu Chandrasekharan: Introduction to Analytic Number Theory. Springer 1968.
Responsible Persons	Anton Deitmar

Course Title:	Introduction to Berkovich Geometry		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Non-Archimedean fields, valuations, and absolute value functions. • Ultrametric triangle inequality and induced topology. • Affinoid domains. • Berkovich affine and projective line. • Analytification of algebraic varieties.		
Special Objectives	The students have become familiar with the most important examples of non-Archimedean fields and their induced topology. They have gained an understanding of the fundamental challenges in developing a theory of analytic geometry over these fields and have studied Berkovich's approach to addressing these issues. The students have examined the projective line in Berkovich's framework in detail, both set-theoretically and topologically. In doing so, they have encountered a type of geometric space fundamentally different from other examples encountered in their studies (such as vector spaces, varieties, or manifolds). They are also familiar with the connections to algebraic geometry through the analytification functor.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Literature	Possible References : • Annette Werner: Nichtarchimedische Geometrie. Vorlesungsskript.
Responsible Persons	Hannah Markwig

Course Title:	Introduction to Mathematical Logic		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Propositional logic. • Languages of the first order: – Completeness and compactness. • Theory of computations: – Register machines; – Gödelisation. • Incompleteness of arithmetic: – First and second incompleteness theorem. • Set theory: – Ordinal- and cardinal numbers; – Incompleteness of set theory.		
Special Objectives	Students are able to understand mathematical theorems and theories in the context of mathematical logic. They understand the limits of possible mathematical knowledge, recognise the difference between truth and provability and can apply basic theoretical model thinking to mathematical content.		
Prerequisites	There are no prerequisites.		
Literature	Possible References : • Rautenberg, Wolfgang: Einführung in die Mathematische Logik. Vieweg+Teubner 2008. • Ziegler, Martin: Mathematische Logik. Birkhäuser 2016.		

Responsible Persons	Anton Deitmar
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Course Title:	Introduction to set theory		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	Content: •		
Special Objectives	-		
Prerequisites	There are no further prerequisites.		
Literature	Possible References : •		
Responsible Persons	Frank Loose		

Course Title:	Introduction to Optimisation		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 3 SWS + Exercise class 1 SWS		
Content	• Optimality theory for smooth, convex and linear optimisation problems optimisation problems with constraints. • Foundations of the theory of convex sets and functions. • Duality theory for convex and linear optimisation problems. • Solution methods for linear optimisation problems.		

Special Objectives	Students know and understand methods and algorithms for solving convex and linear optimisation problems. They have learnt to apply the methods to simple problems related to economics, technology or physics. They will be able to critically assess the possibilities and limitations of using the methods.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	Possible References : • Florian Jarre, Joseph Stoer: Optimierung: Einführung in mathematische Theorie und Methoden. Springer 2019. • Jorge Nocedal, Stephen J. Wright: Numerical optimization. Springer 2006.
Responsible Persons	Christian Lubich

Course Title:	Introduction to Dynamical Systems		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Kepler's laws. • Equilibrium positions. • Stability. • Predator-prey model. • Poincaré-Bendixson theorem. • Limit sets. • Periodic orbits. • Celestial mechanics.		
Special Objectives	The students can ask and examine qualitative questions about the solutions of ordinary differential equations, like e.g.: How long do mathematical solutions exist? Are there equilibrium states or periodic orbits? When are trajectories stable?		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations.		

Literature	Possible References : • Morris W. Hirsch, Stephen Smale: Differential equations, dynamical systems, and linear algebra. Academic Press 1974. • Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 2010. • Carl Ludwig Siegel, Jürgen Moser: Lectures on celestial mechanics. Springer 1995.
Responsible Persons	Frank Loose

Course Title:	Introduction to Geometric Measure Theory		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem. • Surface- and cosurface formula. • Countable rectifiable sets, rectifiable varifolds.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	Possible References : • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.		
Responsible Persons	Reiner Schätzle		

Course Title:	Introduction to Geometric Measure Theory – Measure Theoretic Methods		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	Possible References : • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.		
Responsible Persons	Reiner Schätzle		

Course Title:	Introduction to Geometric Measure Theory – Varifolds		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	• Surface- and cosurface formula. • Countable rectifiable sets, rectifiable varifolds.
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Literature	Possible References : • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Responsible Persons	Reiner Schätzle

Course Title:	Introduction to Commutative Algebra and Algebraic Geometry		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Rings and ideals. • Gröbner bases. • Localization. • Noetherian rings and modules. • Integral ring extensions. • Krull's principal ideal theorem and dimension theory. • Hilbert's Nullstellensatz and Noether normalisation. • Affine varieties, Zariski topology, morphisms.		

Special Objectives	The students have become familiar with the central concepts, results, and methods of commutative algebra and affine algebraic geometry. They have experienced the profound interplay between algebra and geometry through the example of affine varieties. Furthermore, the students understand how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Literature	Possible References : • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Responsible Persons	Daniele Agostini, Jürgen Hausen

Course Title:	Introduction to Partial Differential Equations		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces. • L^2 theory. • Important examples (Laplace equation, wave equation, heat equation). • Fundamental solutions (elliptic situation). • Weak solutions of elliptic equations.		

Special Objectives	The students got to know a central branch of analysis, whose terms and methods are fundamental for many fields, like numerics or stochastics. Also evolutionary equations, who have strong connections to geometry, are issue of the lecture. The students are acquainted with central terms, results and methods of linear partial differential equations and are able to use these methods in advanced courses.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Literature	Possible References : • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Course Title:	Introduction to Partial Differential Equations – Part 1		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces.		
Special Objectives	The students have familiarised themselves with the first basic features of a central area of analysis, the concepts and methods of which are fundamental for many other areas, such as numerics and stochastics. Students are familiar with the central concepts, results and methods of linear partial differential equations and can successfully apply these methods in the more advanced courses.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	Possible References : • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Course Title:	Introduction to Riemann Surfaces		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Coverings and fundamental groups. • Topological classification of the surfaces. • Theorem of Riemann-Hurwitz. • Differential forms and integration. • Sheaves and cohomology. • Theorem of Riemann-Roch. • Serre duality. • Kobayashi metric. • Theorem of Picard.		
Special Objectives	Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. Using the sheaf concept, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.		

Literature	Possible References : • Hershel M. Farkas, Irwin Kra: Riemann Surfaces. Springer 1992. • Otto Forster: Riemannsche Flächen. Springer 1977. • Klaus Lamotke: Riemannsche Flächen. Springer 2009. • Jürgen Jost: Compact Riemann surfaces. Springer 2006.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Course Title:	Introduction to Stochastic Differential Equations - Part 1		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Introduction to Brownian motion and stochastic integration. • Solution concepts for stochastic differential equations. • Stability of stochastic differential equations. • Numerical approximation of stochastic differential equations.		
Special Objectives	Students master the basic principles and techniques for constructing solutions of stochastic differential equations.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.		
Literature	Possible References : • Bernt Oksendal: Stochastic differential equations. Springer 2000.		
Responsible Persons	Andreas Prohl		

Course Title:	Elementary Number Theory		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h

Frequency	not regularly
Language of Instruction	German
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS
Content	• Divisibility in the integers. • Prime numbers. • Congruences. • Quadratic residues. • Arithmetic functions. • Multiplicative functions. • Classical theorems. • Applications.
Special Objectives	Students deepen their basic knowledge of integers and experience applying this knowledge to mathematical problems of various kinds.
Prerequisites	Knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	Possible References : • Friedhelm Padberg: Elementare Zahlentheorie. Spektrum Akademischer Verlag 2001. • Stefan Mueller-Stach, J. Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg 2006.
Responsible Persons	Thomas Markwig

Course Title:	Elliptic Curves and Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Basic concepts of cryptography. • Symmetric cryptosystems, public key systems, discrete logarithm, RSA. • Factorisation into primes, attacks on cryptosystems. • Basic concepts of plane projective geometry. • Elliptic curves as Abelian groups. • Curves over finite fields, Frobenius morphism, endomorphism ring. • Counting points, Hasse bound, Weil conjectures, Schoof's algorithm. • Cryptosystems on elliptic curves, algorithms and attacks.		
Special Objectives	Students are familiar with the basic tasks and concepts of cryptography. They understand the cryptographically motivated questions relating to elliptic curves and have an insight into the advanced algebraic and geometric techniques for answering them. They understand the challenges of algorithmic implementation and are familiar with standard algorithms.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory, • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.		
Literature	Possible References : • Albrecht Beutelspacher, Jörg Schwenk, Klaus-Dieter Wolfenstetter: Moderne Verfahren in der Kryptographie. Springer 2015. • Joseph H. Silverman: The arithmetic of elliptic curves. Springer 2009. • Ian Blake, Gadiel Seroussi, Nigel Smart: Elliptic curves in cryptography. CUP 1999.		
Responsible Persons	Thomas Markwig		

Course Title:	Explicit Mathematics		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		

Content	• Applicative theories. • Explicit mathematics. • Universes in explicit mathematics. • Applications in proof theory.
Special Objectives	Students are familiar with an alternative logical theoretical framework for arithmetic and subsystems of analysis and are familiar with their function in proof-theoretical investigations.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	Possible References : • Solomon Feferman: A language and axioms for explicit mathematics, in Algebra and Logic. Lecture Notes in Mathematics, 450, pp. 87-139, Springer-Verlag, Berlin, 1975. • Solomon Feferman: Constructive theories of functions and classes. In Logic Colloquium '78, (Proc. Mons Colloq.), pp. 159-224, North-Holland, Amsterdam, 1979. • Gerhard Jäger, Reinhard Kahle, Thomas Strahm: On applicative theories. In Andrea Cantini, Ettore Casari, and Pierluigi Minari, editors, Logic and Foundations of Mathematics, pages 83–92, Kluwer, 1999. • Reinhard Kahle: The applicative realm. Textos de Matemática, 40, Departamento de Matemática, Universidade de Coimbra, 2007. • Gerhard Jäger, Reinhard Kahle, Thomas Studer: Universes in explicit mathematics. Annals of Pure and Applied Logic, 109(3), 141-162, 2001.
Responsible Persons	Reinhard Kahle

Course Title:	Financial Mathematics and Numerics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	The course is designed to provide a comprehensive introduction to the mathematical models and numerical techniques that are essential for understanding and solving problems in modern finance. The course will delve into stochastic processes, modeling random phenomena in financial markets, and the numerical methods necessary for approximating solutions to complex financial equations. Key topics include the mathematics behind derivative pricing models, such as the Black-Scholes framework, and the use of stochastic calculus (SDE theory) in risk management and portfolio optimisation. By integrating theory with numerical practice, this course bridges the gap between financial mathematics and real-world applications, equipping students with the quantitative skills required for careers in quantitative finance, risk analysis, and financial engineering.		
Special Objectives	The students know important mathematical models for the description of problems in financial mathematics and can apply numerical approaches to their solutions in a targeted manner.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-11 / MAT-20-13 Numerical Mathematics, • MAT-20-12 Stochastics.
Literature	Possible References : • Steven Shreve: Stochastic Calculus for Finance. Springer 2005.
Responsible Persons	Andreas Prohl

Course Title:	Functional Analysis		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Normed spaces, Banach spaces, dual spaces. • Hahn-Banach theorem, uniform boundedness principle. • Closed graph theorem, open mapping theorem, Banach-Alaoglu theorem. • Compact operators, normal operators, spectral theorems.		
Special Objectives	The students are acquainted with the basic principles and techniques of the theory of infinite dimensional spaces and can apply them to problems in analysis and geometry. They understand the complexity of problems of spectral theory and can use its results for the solution of analytical problems.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	Possible References : • Nicolas Bourbaki: Topological vector spaces. Springer 1987. • Adam Bowers, Nigel Dalton: An introductory course in functional analysis. Springer 2014. • Harro Heuser: Funktionalanalysis. Teubner 2006. • Markus Haase: Functional analysis. American Mathematical Society 2014. • Peter D. Lax: Functional analysis. Wiley 2002. • Gert Kjaergaard Pedersen: Analysis now. Springer 1995. • Walter Rudin: Functional analysis. McGraw-Hill 1991. • Dirk Werner: Funktionalanalysis. Springer 2011. • Kosaku Yosida: Functional analysis. Springer 1995. • Hans Wilhelm Alt: Lineare Funktionalanalysis. Springer 2012.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Course Title:	Geometry of Manifolds 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Manifolds and submanifolds. • Vector fields and flows. • Metrics, foundations of Riemannian geometry. • Complex structures. • Theorem of Gauß-Bonnet on surfaces.		
Special Objectives	The students know and understand the fundamental concepts of real and complex differential geometry and the basic techniques for handling them. Especially they have deepened their understanding of differential and integral calculus and have exemplarily experienced how mathematical concepts are used in a natural way in geometry.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	Possible References : • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011.
Responsible Persons	Christoph Bohle, Frank Loose

Course Title:	Geometric Group Theory		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Group actions on graphs, free groups. • Quasi isometries. • Growth types. • Hyperbolic groups. • Ends.		
Special Objectives	Students learn to explore properties of finitely generated groups using geometric tools, starting from the Cayley graph of the group. They are able to investigate the geometric properties of the Cayley graphs with the help of analytical methods and to work out their connections to the underlying group. Students understand how algebra and analysis can work together to develop a new theory at the interface of algebra and geometry that leads to interesting statements about groups.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		
Literature	Possible References : • Clara Löh: Geometric Group Theory - an Introduction. Springer 2017. • Thorsten Camps, Volkmar Große Rebel, Gerhard Rosenberger: Einführung in die kombinatorische und die geometrische Gruppentheorie. Heldermann Verlag 2008.		
Responsible Persons	Hannah Markwig		

Course Title:	Ordinary Differential Equations - Analysis and Numerics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Non-linear ordinary differential equations: Theorems of Hartman-Grobman and Poincare-Bendixson, bifurcation theory. • Numerical approximation: linear multi-step processes, adaptive processes, geometric integration.		
Special Objectives	Students are familiar with the basic methods for studying qualitative behavior and for simulating solutions of non-linear ordinary differential equations. They have learned constructive methods for solving them and are in principle able to implement these with the help of the computer.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics. Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.		
Literature	Possible References : • Lawrence Perko: Differential equations and dynamical systems. Springer 1993. • David Griffiths, Desmond J. Higham: Numerical methods for ordinary differential equations. Springer 2010.		
Responsible Persons	Andreas Prohl		

Course Title:	Graph Theory		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Basic concepts in graph theory, • Basic graph theory algorithms, • Flows, cuts, connectedness, matchings, • Cycle and cut space (cohomology theory), • Spectral graph theory, matrix tree theorem, • Planar graphs, theorem of Kuratowski and Wagner, • Planar embeddings, • Graph colorings, • Theory of minors.		
Special Objectives	Students know the basic concepts of graph theory, can analyse the structure of graphs and use graph theory methods in practice. They will also recognise connections to geometry and algebra and be able to benefit from them.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	Possible References : • Bela Bollobas: Modern graph theory, Springer, 1998. • John Adrian Bondy, Uppaluri Siva Ramachandra Murty: Graph theory, Springer, 2008. • Reinhard Diestel: Graph theory, Springer, 2018. • Jonathan L. Gross, Jay Yellen, Mark Anderson: Graph theory and its applications, CRC Press, 2019.		
Responsible Persons	Elmar Teufl		

Course Title:	Groups and Representations		
Specialisation	Mathematical Physics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Groups: subgroups, homomorphisms, isomorphisms, group actions, orbits, stabilisers, equivalence classes, normal subgroups, cosets, factor groups. • Representations: faithful, unitary and irreducible representations, reducibility, characters, Schur's lemma(s), orthogonality of irreducible representations. • Applications: symmetries and degeneracies in quantum mechanics, selection rules. • Representations of finite groups: group algebra, regular representation, ideals, idempotents. • Symmetric groups: Young tableaux, Young operators, dimensions and characters. • Applications: identical particles in quantum theories. • Lie groups: Haar measure, representations, Lie algebras. • Tensor representations of classical groups: symmetry classes, Young tableaux. • Applications: SU(2) and SU(3) in particle physics (spin, isospin, flavour) • Moreover a selection of the following: – Irreducible representations of the Lorentz and Poincaré groups. – Applications: notion of particles in quantum theories. – Roots and weights; Killing-Cartan classification of semi-simple Lie algebras		
Special Objectives	The students know the basic concepts of group and representation theory. They are able to apply these abstract algebraic concepts in the context of theoretical physics and have, thus, developed a deepened understanding for the connections between mathematics and physics. The students are familiar with a number of complex examples of applications of the representation theory of groups in physics.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		
Literature	Possible References : • Irene Verona Schensted: A course on the Application of Group Theory to Quantum Mechanics. NEO Press 1976. • Barry Simon: Representations of Finite and Compact Groups. AMS 1996. • Wu-Ki Tung: Group Theory and Physics. World Scientific 1985.		
Responsible Persons	Stefan Keppeler		

Course Title:	Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		

Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS
Content	Starting from a system of axioms for plane absolute geometry with the basic concepts of incidence and congruence, the associated Bachmann reflection geometry is developed. After the introduction of the hyperbolic axiom, this is continued with reflection-geometric end theory. A Euclidean field is created from the rotations around an end and the translations along a straight line, with the help of which the hyperbolic plane under consideration is described algebraically.
Special Objectives	The students have learnt to look at one and the same mathematical object (in this case absolute and hyperbolic planes) from completely different perspectives and to link them together. In particular, they have learnt about Bachmann's group-theoretically oriented reflection geometry, which rarely appears in the curriculum, and thus deepen their knowledge of groups. They also deepened their knowledge about the interweaving of geometry and algebra.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures. Knowledge from the following modules is helpful but not strictly required: • MAT-20-06 Introduction to Geometry.
Literature	Possible References : • Friedrich Bachmann: Aufbau der Geometrie aus dem Spiegelungsbegriff. Springer 1959. • Robin Hartshorne: Geometry: Euclid and beyond. Springer 2000. • Helmut Karzel, Kay Sörensen, Dirk Windelberg: Einführung in die Geometrie. Vandenhoeck und Ruprecht 1973.
Responsible Persons	Hermann Hähl, Hannah Markwig

Course Title:	Information Geometry		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Foundations of Information Geometry (e.g. Fisher information metrics and dual relationships for parametric statistical models, Kullback-Leibler divergence, natural gradient). • Application to neural data processing (in particular supervised learning in artificial neural networks).		
Special Objectives	Students have an elementary understanding of how to apply concepts of differential geometry to problems in information theory and statistics.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		

Literature	Possible References : • Shun-Ichi Amari, Hiroshi Nagaoka: Methods of Information Geometry. AMS 2001. • Anthony C. C. Coolen, Reimer Kuehn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005. • Shun-Ichi Amari: Natural Gradient works Efficiently in Learning. Neural Computation 1998. • Yann Ollivier: Riemannian Metrics for Neural Networks I - Feedforward Networks. Information and Inference, IMA 2015.
Responsible Persons	Christoph Bohle

Course Title:	Information Theory, Pattern Recognition, and Neural Networks		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Introduction to information theory: – The possibility of reliable communication over unreliable channels. The Hamming code and repetition codes. • Entropy and data compression: – Entropy, conditional entropy, mutual information, Shannon information content. The idea of typicality and the use of typical sets for source coding. Shannon's source coding theorem. Codes for data compression. Uniquely decodeable codes and the Kraft-MacMillan inequality. Completeness of a symbol code. Prefix codes. Huffman codes. Arithmetic coding. • Communication over noisy channels: – Definition of channel capacity. Capacity of binary symmetric channel; of binary erasure channel; of Z channel. Joint typicality, random codes, and Shannon's noisy channel coding theorem. Real channels and practical error-correcting codes. Hash codes. • Statistical inference, data modelling and pattern recognition: – The likelihood function and Bayes' theorem. Clustering as an example. • Approximation of probability distributions: – Laplace's method. (Approximation of probability distributions by Gaussian distributions.) Monte Carlo methods: Importance sampling, rejection sampling, Gibbs sampling, Metropolis method. (Slice sampling, Hybrid Monte Carlo, Overrelaxation, exact sampling.) Variational methods and mean field theory. Ising models. • Neural networks and content-addressable memories: – The Hopfield network.		

Special Objectives	Students are familiar with fundamental concepts of information theory – such as entropy and noisy channel coding – and how these are applied in computing and approximating probabilities.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Literature	Possible References : • David J.C. MacKay: Information Theory, Inference, and Learning Algorithms. CUP 2003.
Responsible Persons	Stephan Eckstein

Course Title:	Interacting Many-Body Quantum Systems		
Specialisation	Mathematical Physics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Bosons and fermions. • Bose-Einstein condensation. • Mean-field theory. • Nonlinear Schrödinger equation. • Hartree- and Hartree-Fock theory. • Bogoliubov approximations.		
Special Objectives	Students have learned about the difference between Fermions and Boson and its impact on many body systems. They understand that, at cold temperatures, the fluctuations of the particle number in a certain area is significantly suppressed for Fermions. They understand how the Bosonic nature leads to the famous effect of Bose Einstein condensation and that, in certain scaling regimes, interacting many body systems can in good approximation be described by nonlinear evolution equations. Finally they are able to give higher order corrections and to calculate the leading order correlations between the particles.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	Possible References : • Elliott H. Lieb, Robert Seiringer, Jan Philip Solovej, Jakob Yngvason: The Mathematics of the Bose Gas and its Condensation. Birkhäuser 2000. • Niels Benedikter , Marcello Porta , Benjamin Schlein: Effective Evolution Equations from Quantum Dynamics. Springer 2016.
Responsible Persons	Peter Pickl

Course Title:	Commutative Algebra		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Rings and Ideals. • Localisation and local rings. • Noetherian and Artinian rings and modules. • Integral ring extensions and Cohen-Seidenberg theorems. • Krull's principal ideal theorem and dimension theory. • Primary decomposition. • Normality, regularity and discrete valuation rings. • Hilbert's Nullstellensatz and Noether normalisation.		
Special Objectives	The students are familiar with and understand the language and methods of commutative algebra, which are essential for studying the fields of algebra, geometry, and number theory. They recognise how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.		

Literature	Possible References : • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Responsible Persons	Daniele Agostini, Thomas Markwig

Course Title:	Convex Geometry		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Cones, polytopes, polyhedra, fans, polyedral complexes. • Normal fans of polygons. • Triangulations, subdivisions, secondary fans, discriminants.		
Special Objectives	In the lecture the students learn basic terms, results and methods of convex geometry. They develop a deepened understanding for the concept of duality of mathematical objects on the example of polytopes and fans. Furthermore they enhance their geometric view and their spatial sense.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		
Literature	Possible References : • Günter M. Ziegler: Lectures on Polytopes. Springer 1998.		
Responsible Persons	Hannah Markwig		

Course Title:	Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Brief review of key concepts and results from algebra and number theory. • Historical ciphers and their cryptanalysis (Caesar, Vigenere, substitution); encryption schemes. • Diffie-Hellman protocol and fast exponentiation. • Discrete logarithms: Shanks' algorithm and Pollard's rho method. • RSA: correctness, security, and attacks. • Signature schemes.		
Special Objectives	Students are familiar with the fundamental concepts and results of elementary number theory and algebra, as well as their application in cryptography. They can implement the methods covered in Python or SageMath in an exemplary manner and know what to pay attention to. Using classical ciphers, they understand typical strengths and weaknesses; they master the Diffie-Hellman protocol and are familiar with the man-in-the-middle attack. They can compute discrete logarithms in cyclic groups, understand the RSA scheme, and are able to interpret the recommendations of the Federal Office for Information Security (BSI). In various attack scenarios, they can identify weaknesses of RSA when the requisite conditions are not met. By engaging with numerous open problems in cryptography – whose solution approaches can, perhaps surprisingly, stem from very different areas of mathematics – students practise critical thinking. The exercises are central and support students in working independently and in a practice-oriented way, especially with CAS systems such as SageMath.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		
Literature	Possible References : • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Christian Karpfinger, Hubert Kiechle: Kryptologie, Algebraische Methoden und Algorithmen, Vieweg 2010. • Dan Boneh, Victor Shoup: A Graduate Course in Applied Cryptography. 2023 (online Version: https://toc.cryptobook.us/). • Jonathan Katz, Yehuda Lindell: Introduction to Modern Cryptography. Chapman and Hall/CRC 2020.		
Responsible Persons	Thomas Markwig		

Course Title:	Lie Groups		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Manifolds and Lie groups, • Lie algebras and exponential map, • Covering spaces and classification of Lie groups by their Lie algebras, • Classical Lie groups, • Operations of Lie groups and homogeneous spaces.		
Special Objectives	Lie groups lie at the interface between geometry, algebra and analysis. They are suitable for describing the symmetries of geometric objects, but also algebraic equations or solutions of differential equations, in particular if these symmetries form a continuous set. The students learn from a prominent example how different disciplines of mathematics can work together very successfully and how a convincing formalism is developed that can precisely describe a variety of symmetry phenomena.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-01 Measure and Integration Theory, • MAT-20-04 Integration on Submanifolds of $\mathbb{R}^n$.		
Literature	Possible References : • Joachim Hilgert, Karl-Hermann Neeb: Liegruppen und Lie-Algebren. Vieweg 1991. • Gerhard P. Hochschild: The structure of Lie groups. Holden-Day 1965. • Frank W. Warner: Foundations of differentiable manifolds and Lie groups. Springer 1983.		
Responsible Persons	Anton Deitmar, Frank Loose		

Course Title:	Linear Control Theory		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		

Language of Instruction	German
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS
Content	Mathematical methods are indispensable for the management and control of complex systems and processes. The underlying theory is not only fascinating due to its diverse applications, but also, in its abstract form, due to the clarity and elegance of its methods and results. In this lecture, finite-dimensional systems are dealt with first, for which a good knowledge of analysis and linear algebra is sufficient. The aims are Kalman's controllability criterion and the resulting criteria for stabilisability. If there is enough time, we will extend the theory to infinite-dimensional systems. In the exercise classes we will apply the theory to concrete examples.
Special Objectives	Students have learnt basic methods of linear control theory. At the same time, they have experienced and understood the interaction of various theoretical concepts from linear algebra and analysis and their benefits for specific applications.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	Possible References : • Hans Wilhelm Knobloch, Huibert Kwakernaak: Lineare Kontrolltheorie. Springer 1985. • Jerzy Zabczyk: Mathematical Control Theory. Birkhäuser 1992. • Ruth F. Curtain, Hans Zwart: An Introduction to Infinite-Dimensional Systems Theory. Springer 1995.
Responsible Persons	Rainer Nagel

Course Title:	Mathematical Aspects of 3d Computer Vision		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	• Foundations of projective geometry. • Visual odometry. • Classical Simultaneous Localisation and Mapping (SLAM) methods and deep learning variants. • Computer-based experiments using the Robot Operating System (ROS2).		
Special Objectives	The students have acquired the mathematical foundations of 3D computer vision. They are familiar with modern architectures and the mathematical questions arising from them.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics. In addition, prior knowledge of deep learning is required, such as that provided in module MAT-50-14.
Literature	Possible References : • Richard Hartley, Andrew Zisserman: Multiple View Geometry in Computer Vision. Cambridge University Press 2003. • Sebastian Thrun, Wolfram Burgard, Dieter Fox: Probabilistic Robotics. MIT Press 2005. • Luca Carlone, Ayoung Kim, Timothy D. Barfoot, Daniel Cremers, Frank Dellaert: SLAM Handbook: From Localization and Mapping to Spatial Intelligence. Cambridge University Press 2026.
Responsible Persons	Christoph Bohle

Course Title:	Mathematical Aspects of Deep Learning 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Artificial neural networks and their training using backpropagation/stochastic gradient descent. Probabilistic interpretation. Mathematical problems arising in neural network science. • Classical architectures for computer vision (CNNs) and sequential data (RNNs). (Self-)supervised learning methods. • Transformer models, a modern architecture based on the attention mechanism, and its applications in natural language processing (GPT) and computer vision. • Simple neuroscience models for neural networks. Biological plausibility of backpropagation and transformers.		
Special Objectives	The students have learned the fundamentals of information processing using artificial neural networks and biologically more plausible alternatives. They are familiar with contemporary architectures and mathematical questions arising in the framework.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.		

Literature	Possible References : • Ian Goodfellow, Yoshua Bengio, Aaron Courville: Deep Learning. MIT 2016. • Simon J. D. Price: Understanding Deep Learning. MIT Press 2023. • Christopher M. Bishop, Hugh Bishop: Deep Learning - Foundations and Concepts. Springer 2024.
Responsible Persons	Christoph Bohle

Course Title:	Theory of Mathematical Proofs		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	• Axiomatic theories, incompleteness. • Gentzen's proof of consistency for arithmetic. • Ordinal number analysis. • Provable recursive functions. • Predicative analysis. • Theories of inductive definitions.		
Special Objectives	Students are familiar with the methods and results of modern proof theory, in particular with calculations for mathematical theories and their metamathematical properties.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	Possible References : • Wolfram Pohlers. Proof Theory. Springer 2009.		
Responsible Persons	Reinhard Kahle		

Course Title:	Mathematical Introduction to Data Science		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		

Language of Instruction	English
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS
Content	The lecture provides a mathematically rigorous survey of core data-science techniques from regression and k-nearest neighbours to high-dimensional phenomena, dimensionality reduction, and modern classifiers (Perceptron, SVMs, Kernel Methods, Neural Networks). Emphasis is put on both theoretical foundations and hands-on Python implementation with real datasets.
Special Objectives	The students can formulate and solve linear, polynomial and logistic regression models. They are able to implement k-nearest neighbours classification and build and evaluate classifiers (Perceptron, SVM (kernel methods), Neural Networks). Moreover, they know how to optimise convex objectives using gradient-descent techniques.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.
Literature	Possible References : • Sven A. Wegner: Mathematical Introduction to Data Science. Springer 2024.
Responsible Persons	Andreas Prohl

Course Title:	Mathematical Statistics		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Statistical models, exponential families, sufficient statistics. • Rao-Blackwell theorem, Lehmann-Scheffe theorem, Cramer-Rao theorem. • Estimation methods, UMVU estimator, quality criteria, asymptotic behaviour of estimators. • Hypothesis testing, confidence interval, Neyman-Pearson lemma. • Testing methods, UMPU tests, 1- and 2-sample tests. • Models with growing density quotients, non parametric models. • Introduction in regression and variance analysis.		
Special Objectives	Students can model statistical relationships mathematically. They can mathematically construct, analyse, compare and apply statistical estimation and test methods and interpret their results.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics. Knowledge from the following modules is helpful but not strictly required: • MAT-75-01 Probability Theory.
Literature	Possible References : • Peter J. Bickel, Kjell A. Doksum: Mathematical Statistics: Basic Ideas and Selected Topics. Chapman & Hall 2016. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Erich L. Lehmann, Joseph P. Romano: Testing statistical hypotheses. Springer 2005. • Erich L. Lehmann, George Casella: Theory of point estimation. Springer 1998. • Wiebe R. Pestman: Mathematical Statistics. De Gruyter 2009 • Helmut Pruscha: Vorlesungen über Mathematische Statistik. Springer Vieweg 2000. • Mark J. Schervish: Theory of Statistics. Springer 1995.
Responsible Persons	Martin Möhle, Elmar Teufl

Course Title:	Nonparametric Estimation of Distributions		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	This course explores a fundamental question in statistics: Given n data points from an unknown random vector, how well can we infer the true characteristics of the random vector, i.e., its distribution? In particular, we will focus on the role of the dimension of the vector, leading to a treatment of the curse of dimensionality. Further, we will explore how prior knowledge (e.g., smoothness or structural understanding of the random vector) can help overcome the curse of dimensionality. In contrast to parametric statistics, where the distribution is assumed to belong to a finite-dimensional family (e.g., estimating mean and covariance matrix of a normal distribution), the focus is purely on nonparametric settings, where no such parametric forms are given a priori. These settings require suitably chosen evaluation criteria based on distances between probability distributions, and we will treat both density estimation with L^p-type losses and estimation in Wasserstein distance.		

Special Objectives	The statistical learning problem for probability distributions can be formulated precisely. The importance of the chosen metric and of the dimension of the distribution is well understood. Students are familiar with the fundamental proof techniques for establishing both upper bounds, using explicit estimators, and lower bounds, using information-theoretic methods. They develop a structural understanding of why $\sqrt{\frac{\log(1/\delta)}{n}}$ (Bandwidth) assumptions are necessary to achieve convergence rates that are better than those dictated by the curse of dimensionality. Through the exercise classes, students demonstrate their ability to work independently on topics covered in the lectures.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Literature	Possible References : • Alexandre B. Tsybakov: Introduction to Nonparametric Estimation. Springer 2009. • Jonathan Niles-Weed, Quentin Berthet: Minimax estimation of smooth densities in Wasserstein distance. The Annals of Statistics 50.3 (2022): 1519-1540. • Nicolas Fournier, Arnaud Guillin: On the rate of convergence in Wasserstein distance of the empirical measure. Probability Theory and Related Fields 162.3 (2015): 707-738. • Daniel Bartl, Stephan Eckstein: Fast Wasserstein rates for estimating probability distributions of probabilistic graphical models. arXiv:2510.09270 (2025).
Responsible Persons	Stephan Eckstein

Course Title:	Numerics of Stationary Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	Numerical covering of boundary value problems of stationary (i.e. time independent) ordinary and elliptic partial differential equations, with emphasis to the methods of finite elements.		
Special Objectives	The students have learned the central terms, results and methods of the numerical treatment of boundary value problems of stationary differential equations.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations, • MAT-20-11 / MAT-20-13 Numerical Mathematics. Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.		

Literature	Possible References : • Dietrich Braess: Finite Elemente. Springer Spektrum 2013. • Wolfgang Hackbusch: Theorie und Numerik elliptischer Differentialgleichungen. Teubner 1986.		
Responsible Persons	Christian Lubich		

Course Title:	Numerics of Stochastic Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Random number generator, Ito-Taylor expansion. • Strong and weak approximation, consistency. • Euler-Maruyama method, Milstein method, stochastic Runge-Kutta method. • Approximation of stopped diffusion processes.		
Special Objectives	Students master the basic principles and techniques for the numerical approximation of solutions of stochastic differential equations.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.		
Literature	Possible References : • Peter E. Kloeden, Eckhard Platen: Numerical solution of stochastic differential equations. Springer 1999.		
Responsible Persons	Andreas Prohl		

Course Title:	Optimal Control Theory with Ordinary Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h

Frequency	not regularly
Language of Instruction	German
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS
Content	• Brief overview of existence and uniqueness theory for ODEs. • Numerical solutions to ODEs. • Introduction to optimal control problems with ODEs. • Existence and uniqueness theory for linear quadratic optimal control problems (LQ problems). • Pontryagin's maximum principle. • Numerical approximation of LQ problems.
Special Objectives	Students are familiar with the problems of optimal control using ordinary differential equations and various approaches to solving the problem. They are also familiar with qualitative statements on unambiguous solvability.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations.
Literature	Possible References : • Matthias Gerds: Optimal Control of ODEs and DAEs. De Gruyter 2012.
Responsible Persons	Andreas Prohl

Course Title:	Optimal Transport		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• The course introduces the theory of optimal transport at the interface of analysis, probability, and optimisation. Starting from the classical problems of Monge and Kantorovich, the fundamental measure-theoretic and analytical tools are developed. • Building on this, central results are presented, including the existence and duality of optimal transport plans, c-cyclical monotonicity, Brenier's theorem, and the geometric structure of Wasserstein spaces. • In the second part of the course, the algorithmic perspective of optimal transport is explored. Topics include entropic regularisation, the Sinkhorn algorithm, and selected applications, for example in image processing, statistics, and machine learning.
Special Objectives	The students are familiar with the fundamental concepts, methods, and key results of the theory of optimal transport and are able to situate these within a broader mathematical context spanning analysis, probability, and optimisation. They can understand the classical formulations due to Monge and Kantorovich, as well as essential structural concepts such as duality, c-cyclical monotonicity, and the geometry of Wasserstein spaces, and apply them in simple settings. Furthermore, they are acquainted with basic algorithmic approaches to optimal transport and can assess their relevance for selected applications. In the exercise classes, they deepen their understanding of the lecture material, further develop their ability to construct precise mathematical arguments, and practise the written and oral presentation of their own solution methods.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 Submodules Measure and Integration Theory, and Integration on Submanifolds of $\mathbb{R}^n$. • MAT-20-12 Stochastics.
Literature	Possible References : • Alessio Figalli, Federico Glaudo: An Invitation to Optimal Transport, Wasserstein Distances, and Gradient Flows. EMS 2023. • Gabriel Peyré, Marco Cuturi: Computational Optimal Transport. Now Publishers 2019.
Responsible Persons	Nicolai Jork, Andreas Prohl

Course Title:	Optimisation with Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Direct method in the calculus of variations, Euler-Lagrange equation. • Brouwer-Minty theorem, non-linear evolution equations. • Gateaux and Frechet differentiability. • Proof of existence of optimal controls, necessary optimality conditions. • Adjoint, convergent optimisation methods in Banach spaces. • Variational discretisation concepts.
Special Objectives	Students master the basic principles and techniques for deriving optimality conditions for prototypical control problems with constraints in the form of partial differential equations.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Literature	Possible References : • Michael Hinze, Rene Pinnau, Michael Ulbrich, Stefan Ullrich: Optimization with PDE constraints. Springer 2009.
Responsible Persons	Andreas Prohl

Course Title:	Problem Solving Strategies		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	• Basic Techniques and Problem Reduction. • Proofs and Logic. • Sets and Functions. • Elementary Combinatorics. • Counting Techniques and Cardinality. • The Pigeonhole Principle (Box Principle). • Bounds, Estimation, and Inequalities. • The Extremal Principle. • The Invariance Principle: Invariants and Monovariants. • Proofs by Colouring. • Graph-Theoretic Modelling.
Special Objectives	Students acquire a versatile toolkit of mathematical and heuristic problem-solving strategies that they can draw upon both in their subsequent studies and, where appropriate, in competition-oriented mathematical problem solving. This includes fundamental results and proof techniques, as well as the ability to recognise characteristic structural features of a problem that indicate suitable solution approaches or allow unsuitable strategies to be ruled out at an early stage.
Prerequisites	Knowledge from Linear Algebra and the submodule Algebraic Structures are assumed.
Literature	Possible References : • George Pólya: How to Solve It: A New Aspect of Mathematical Method. Princeton University Press 1973. • Arthur Engel: Problem-Solving Strategies. Springer 1998.
Responsible Persons	Thomas Markwig

Course Title:	Propagation of Chaos		
Specialisation	Mathematical Physics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS		
Content	• Interacting many body systems (quantum and classical), importance of correlations. • Mean-field situations (e.g., Vlasov) and collisions (Boltzmann). • Explicit treatment of correlations. • Large deviations from the expected value.		

Special Objectives	Students learn how different kinds of many-body systems can be described by effective, non-linear equations. They are able to distinguish and compare different types of convergence of microscopic many-body systems against the effective theory, both in classical and quantum mechanical situations. Based on an argument similar to the law of large numbers, they understand how the independence of particles leads to the effective equation. They learn to prove that independence is indeed preserved - at least approximately - under the evolution of time (propagation of chaos). Building on this, they understand various proof strategies adapted to the respective situation.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Literature	Possible References : • Louis-Pierre Chaintron, Antoine Diez: Propagation of chaos: a review of models, methods and applications. arXiv:2203.00446. • Francois Golse: Mean-Field Limits in Statistical Dynamics. arXiv:2201.02005.
Responsible Persons	Peter Pickl

Course Title:	Software Obfuscation: Mixed Boolean-Arithmetic Expressions		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• What is software obfuscation and what is it used for? What techniques are there, and why is deobfuscation so difficult? What does a reverse engineer do? Introductory examples. • Mathematical foundations: ideals in $\mathbb{Z}$, greatest common divisor, Euclidean algorithm, time complexity estimates, rings and fields, computations in residue class rings, permutations, matrices and determinants, linear algebra over rings. • Boolean algebras and Boolean functions. • Fundamentals of mixed Boolean-arithmetic expressions (MBA). • Obfuscation using MBAs and permutation polynomials. • Deobfuscation using MBAs; analysis and comparison of state-of-the-art tools and their weaknesses. • Deobfuscation using program synthesis.		
Special Objectives	The students are familiar with initial techniques in the field of software obfuscation and deobfuscation and understand the mathematical results required for this. They are able to apply these confidently.		

Prerequisites	Knowledge from Linear Algebra and the submodule Algebraic Structures are assumed.
Literature	Possible References : • Yongxin Zhou, Andrew Main, Yuxin Gu, Harrison Johnson. Information Hiding in Software with Mixed Boolean-Arithmetic Transforms. In: International Workshop on Information Security Applications (WISA). Springer, 2007, pp. 6175. • Bernhard Reichenwallner, Peter Meerwald-Stadler. Efficient Deobfuscation of Linear Mixed Boolean-Arithmetic Expressions. In: Proceedings of the 15th ACM Workshop on Artificial Intelligence and Security (AISec 2022). ACM, 2022, pp. 119130.
Responsible Persons	Thomas Markwig

Course Title:	Game Theory		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The focus is on Nash and generalised Nash equilibrium problems and their numerical solution.		
Special Objectives	Students are familiar with the fundamental issues of game theory. They are familiar with analytical and numerical approaches to analysing them.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	Possible References : • Christian Kanzow, Alexandra Schwartz: Spieltheorie. Birkhaeuser 2018.		
Responsible Persons	Andreas Prohl		

Course Title:	Statistical Learning 1		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Non-parametric regression, regression estimator. • (Universal) consistency. • Rate convergence. • Stone's theorem. • Kernel estimator, k-NN estimator. • Slow rate convergence, minimax convergence rates.
Special Objectives	Students are familiar with basic non-parametric regression estimators, in particular with their universal consistency and rate convergence. They are familiar with the basic principles and methods of stochastic learning as required for machine learning applications.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.
Literature	Possible References : • Laslo Györfi, Michael Kohler, Adam Krzyzak, Harro Walk: A distribution-free theory of nonparametric regression. Springer 2002.
Responsible Persons	Andreas Prohl

Course Title:	Stochastic Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Stochastic processes, filtrations, martingales. • Wiener process, random walk, Donsker's theorem. • Diffusion semigroup, Ito's integral. • Solution of a stochastic differential equation. • Markov property, Malliavin calculus, rough path theory.		
Special Objectives	Students master the basic principles and techniques for constructing solutions of stochastic differential equations.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Literature	Possible References : • Bernt Oksendal: Stochastic differential equations. Springer 2000.
Responsible Persons	Andreas Prohl

Course Title:	Theoretical Aspects of Machine Learning		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	The lecture covers some recent aspects of theoretical machine learning such as: • The theory of Reproducing Kernel Hilbert Spaces (RKHS). • Applications of RKHS theory such as SVMs, kernel regression, kernel PCA, and kernel mean embeddings. • Approximation capabilities of neural networks. • Dynamics of neural networks and the neural tangent kernel. • Recent advances in high dimensional statistics, in particular overparametrisation and generalisation.		
Special Objectives	The students learn the mathematical foundations of supervised learning theory, neural networks, support vector machines and kernel methods. They are familiar with fundamental modern topics in machine learning and with their theoretical basis, mathematical approach and conceptual tools as needed for the discussion and justification of algorithms.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.		

Literature	Possible References : • Mehryar Mohri, Afshin Rostamizadeh, Ameet Talwalkar: Foundations of Machine Learning. MIT Press 2012. • Shai Shalev-Shwartz, Shai Ben-David: Understanding Machine Learning: From Theory to Algorithms. CUP 2014. • Peter L. Bartlett, Andrea Montanari, Alexander Rakhlin: Deep learning: a statistical viewpoint. Acta Numerica 2021. • Daniel A. Roberts, Sho Yaida, Boris Hanin: The Principles of Deep Learning Theory: An Effective Theory Approach to Understanding Neural Networks. Cambridge University Press 2022.
Responsible Persons	Andreas Prohl

Course Title:	Theory and Numerics for Constrained Optimisation Problems		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	We start with the unconstrained convex minimisation problem (on spaces), and the gradient method with step size control according to Armeijo for the approximate calculation of a minimum, as well as its variants. The simplex method solves linear programmes on polyhedra. Central to this is the convex (non-linear) minimisation task on sets, and the characterisation of a minimum with (necessary) optimality conditions (tangent cone, linearised tangent cone, Abadie condition, Karush-Kuhn-Tucker conditions). In addition, numerical solution methods based on these theoretical concepts (interior points method, penalty methods, SQP method) are presented and analysed.		
Special Objectives	The participants have become familiar with relevant current algorithms for the optimisation of constrained optimisation problems: these include gradient methods with step size control, the simplex method, interior point methods, penalisation methods and the SQP method. You will be able to analyse the algorithms and compare their complexity.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-11 / MAT-20-13 Numerical Mathematics.		
Literature	Possible References : • Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.		
Responsible Persons	Andreas Prohl		

Course Title:	Topology		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Review of metric spaces: closed sets, environment, continuity, complete metric spaces, compactness in metric spaces metric spaces. • Set-theoretic topology: topological spaces, continuity convergence, compactness, separation axioms. • Spaces of continuous functions: Urysohn's lemma and applications, Stone-Cech compactification, the theorem of Stone-Weierstraß, notions of convergence in functions, compactness in spaces of functions. • Baire's spaces and application of Baire's theory: Baire's function classes, existence theorems. • Outlook on algebraic topology.		
Special Objectives	Students have familiarised themselves with the central concepts, results and methods of set-theoretical topology and have understood that this theory can be used to describe many phenomena in different areas of mathematics. In this way, they link their knowledge of very different areas of mathematics.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	Possible References : • Felix Hausdorff: Grundzüge der Mengenlehre. Von Veit & Comp. 1914. • Boto von Querenburg: Mengentheoretische Topologie. Springer 2001. • Volker Runde: A Taste of Topology. Springer 2005.		
Responsible Persons	Ulrich Groh, Rainer Nagel		

Course Title:	Tropical Enumerative Geometry		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Enumerative geometry of algebraic curves, in particular in the plane. • Tropical enumerative problems and multiplicities. • Combinatorial methods, floor diagrams and lattice paths. • Correspondence theorems for curves in the plane through given points. • Tropical and classic Gromov-Witten theory in genus 0. • Real counts, Welschinger invariants and polynomial invariants. • Hurwitz numbers. • Tropical correspondences for Hurwitz numbers.
Special Objectives	The students know basic terms, results and methods of enumerative geometry in the context of tropical geometry methods. They develop a deeper understanding of the possibilities and limitations of the tropical access in connection with more complex issues. Furthermore, they deepen their knowledge in the field of algebraic geometry towards modular spaces and Gromov-Witten theory.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	Possible References : • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.
Responsible Persons	Hannah Markwig

Course Title:	Tropical Enumerative Geometry - Part 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Enumerative geometry of algebraic curves, in particular in the plane. • Tropical enumerative problems and multiplicities. • Combinatorial methods, floor diagrams and lattice paths. • Correspondence theorems for curves in the plane through given points. • Real counts, Welschinger invariants and polynomial invariants.		

Special Objectives	The students know basic terms, results and methods of enumerative geometry in the context of tropical geometry methods. They develop a deeper understanding of the possibilities and limitations of the tropical access in connection with more complex issues. Furthermore, they deepen their knowledge in the field of algebraic geometry towards moduli spaces.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	Possible References : • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.
Responsible Persons	Hannah Markwig

Course Title:	Calculus of Variations		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Direct method of calculus of variations. • Euler-Lagrange equations. • Palais-Smale condition. • Mountain-Pass Lemma according to Ambrosetti-Rabinowitz.		
Special Objectives	In the first part of the course, students have learnt the direct method of calculus of variations, which is primarily used to prove the existence of weak solutions of partial differential equations, but also has applications in e.g. differential geometry. They have also acquired the necessary basics from functional analysis and partial differential equations and can also use these in a different context, e.g. geometric analysis. In the second part of the course, students learnt about a so-called mountain-pass lemma. With its help, they can analyse non-uniqueness in the existence of solutions of partial differential equations.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	Possible References : • Michael Struwe: Variational Methods, Springer 2008. • David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order, Springer 1998. • Walter Rudin: Functional Analysis, Mc Graw Hill Education 1991.		

Responsible Persons	Reiner Schätzle		
Course Title:	Probability Theory		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Characteristic functions and additions to the central limit theorem. • Conditional expectations and further measure-theoretic foundations. • Markov chains and martingales in discrete time, classification, asymptotic behaviour, stopping times, stationarity, ergodicity. • Introduction to processes in continuous time like Poisson processes and Brownian motion.		
Special Objectives	The students got to know the central terms results and methods of probability theory. They can model, analyse and interpret stochastic dependency structures of random quantities in a measure theoretically founded manner. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.		
Literature	Possible References : • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Richard Durrett: Probability, Theory and Examples. Cambridge University Press 2019. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Jean Jacod, Philip E. Protter: Probability essentials. Springer 2004. • Olav Kallenberg. Foundations of Modern Probability. Springer 2021. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2020. • David Meintrup, Stefan Schäffler: Stochastik. Springer 2005. • Albert N. Shiryaev: Probability-1. Springer 2016.		
Responsible Persons	Martin Möhle, Elmar Teufl		

Course Title:	Consistency Proofs		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	• Historical examples of the question of consistency (limits; parallel axiom; set-theoretic paradoxes). • Philosophical foundational programs (logicism; formalism; intuitionism). • The Hilbert program and Gödel's theorems. • Gentzen's transfinite consistency proof for number theory. • Alternative approaches to consistency (including Gödel's T). • Current situation of consistency proofs.		
Special Objectives	Students learn about the historical context from which the question of the non-contradiction of formal mathematical theories arose, as well as the relevant modern techniques for investigating this question mathematically. They are able to categorise the problem of non-contradiction in mathematics both historically and philosophically. In addition, they have acquired the mathematical tools to be able to comprehend the corresponding proofs of non-contradiction and, to a certain extent, to carry them out themselves.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	Possible References : • Kurt Gödel: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatsh. f. Mathematik und Physik 38, 173-198 (1931). • Gerhard Gentzen: Die Widerspruchsfreiheit der reinen Zahlentheorie. Math. Ann. 112, 493-565 (1936). • Reinhard Kahle, Michael Rathjen (Hrsg.): Gentzen's Centenary: The quest for consistency. Springer 2015. • Reinhard Kahle, Michael Rathjen (Hrsg.): The Legacy of Kurt Schütte. Springer 2020.		
Responsible Persons	Reinhard Kahle		

Course Title:	Number Theory and Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		

Language of Instruction	German or English
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS
Content	• RSA cryptosystem, primality tests, AKS algorithm. • Factorisation methods, number field sieve. • Quadratic reciprocity in cryptography. • Evaluation of the discrete logarithm. • Dynamical systems and Pollard's rho algorithm. • Elliptic curve cryptography. • Lattices and post-quantum cryptography. • Zero-knowledge proofs, digital signatures and hash functions.
Special Objectives	The students know the basic concepts of elementary number theory and their applications in cryptography. They have deepened and extended their knowledge about neighbouring disciplines: They encounter methods of the theory of dynamical systems and become acquainted with elliptic curves over finite fields. They understand how fundamental cryptographic protocols are working. Through studying many open problems of cryptography, whose solutions may surprisingly come from most distinct branches of mathematics, the students learn to think critically.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	Possible References : • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Stefan Müller-Stach, Jens Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg+Teubner 2011. • Joseph H. Silverman, John T. Tate: Rational points on elliptic curves. Springer 1992. • Nigel Smart: Cryptography: An introduction. McGraw-Hill 2003. (online version: https://www.cs.bris.ac.uk/~nigel/Crypto_Book/). • Lawrence C. Washington: Elliptic curves: Number theory and cryptography. Chaman & Hall/CRC 2008.
Responsible Persons	Thomas Markwig