EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

Mathematisch-
Naturwissenschaftliche
Fakultät

Department of Mathematics

Module Handbook

Mathematics

Bachelor of Education

Lehramt Gymnasium*

Summer Semester 2026

Last updated on 13th April 2026

*This is a secondary school teaching degree with a major in mathematics. The module handbook is valid for the 2018 study and examination regulations.

Contents

1 Description of the Study Programme

1.1 Qualification Objectives

As part of the teacher training Bachelor's degree programme (B.Ed.) in Mathematics, graduates acquire basic and initial advanced subject-specific and subject-didactic knowledge and skills necessary for science-based teaching at secondary schools in Germany.

Graduates are familiar with the fundamental questions in Linear Algebra, Analysis, Geometry and Stochastics as well as Algebraic Structures and master the central techniques for solving them. In doing so, they acquire basic mathematical thought patterns such as structuring problems, creating chains of argumentation and finally the proof of mathematical theorems. Graduates are able to communicate mathematical facts, use suitable media and establish links to school mathematics. They are able to justify the educational value of mathematical content and convey the societal significance of mathematics. With the Bachelor's degree, graduates are able to apply their knowledge and skills in a teaching-related Master's study programme or, with credit for the work completed, in a science-related Bachelor's degree programme in mathematics.

1.2 Structure of the Study Programme

In Mathematics, the first year of study is filled with the large compulsory module Foundations of Mathematics, which covers the subject-specific fundamentals of Analysis and Linear Algebra from an academic point of view. The corresponding lectures are accompanied by exercise classes, where students are intensively supervised and taught basic mathematical thinking and working methods as well as the ability to present solutions. In addition, the department provides students with revision sessions as question times.

In the second and third years of the programme, students deepen their theoretical knowledge. They expand their knowledge in the areas of Algebra, Geometry, Numerical Mathematics, and Stochastics and take a introductory seminar. The content in the compulsory mathematics modules is taught through lectures and accompanying exercise classes. For each lecture there are weekly tasks, which students have to complete in paper form. In the exercise classes, the students present their solutions or create them under supervision. Through this system, which is common in mathematical study programmes, students learn to systematically work on the tasks set for them and to practise analytical and structural thinking. Furthermore, they should be able to explain complex mathematical matters and present them verbally. This requires students to be able to organise themselves and to do a lot of self-study, which is provided for and credited in the course of study. At the same time, intensive supervision and individual support options are provided.

In addition to the subject modules, students in the second and third years of study take modules in the area of subject didactic. These are designed in such a way that the subject-didactic courses in the areas of Stochastics and Geometry are to be taken in parallel with the corresponding subject modules and are content-wise interlinked with them. The subject modules provide the academic prerequisites for the subject-didactic courses.

In the third year of the programme, students also complete a Bachelor's thesis. This can be written in one of the two chosen subject areas (including their subject didactics)

Integrating a study component at a foreign university into teacher training studies is challenging, as it involves coordinating two subjects and Educational Sciences. Whether attempting to fulfil components in all areas during the stay at the other university or adjusting the study plan at the University of Tübingen to allocate parts of the curriculum to different semesters to create flexibility, ensuring not all three areas need to be covered at the foreign university presents a challenge. Complicating matters is the fact that in the field of Mathematics, all modules are mandatory, leaving little room for content customisation. Therefore, it is essential to plan a suitable time frame for a study component at a foreign university through a personal consultation with the Faculty Course Advisor. Essentially, from the Mathematics perspective, any academic semester is suitable for this purpose. The decision will depend on the student's previous achievements and the courses offered at the chosen foreign university.

1.3 Examination Regulations

Oral examinations are conducted in the presence of at least two examiners or one examiner, along with an observer (see also Exam Regulations General Part §12 (2)).

2 Study Plans

2.1 Overview by Modules

Here we provide an overview of the study plan as a table showing the modules to be taken.

ST	Module Number	Module Title	Type of Course	Type of Module	Course-work	Type of Exam	ECTS-Points
Section 1: Foundations of Mathematics							
1+2	MAT-10-10	Foundations of Mathematics		PM		or.	27
		- Linear Algebra 1	L+E+T		EC		
		- Analysis 1	L+E+T		EC		
		- Analysis 2	L+E+T		EC		
3-4	MAT-10-11	Consolidation of the Foundations of Mathematics		PM		wr. o. or.	6
		- Algebraic Structures	L+E		EC		
		- Mathematical Software	P		PC		
Section 2: Compulsory Intermediate Modules							
5-6	MAT-20-03	Algebra	L+E	PM	EC	wr. o. or.	9
3-4	MAT-20-11	Numerical Mathematics	L+E	PM	EC	wr. o. or.	9
3-4	MAT-20-12	Stochastics	L+E	PM	EC	wr. o. or.	9
3-4	MAT-20-20	Introductory Seminar: Presentations in Mathematics	PS	PMW	s.M.	Pr	3
5-6	MAT-50-01	Geometry	L+E	PM	EC	wr. o. or.	9
Section 3: Didactics of Mathematics							
3-4	MAT-80-01	Subject Didactics Mathematics 1	LIC	PM	s.M.	K o. mP o. P	3
5-6	MAT-80-02	Subject Didactics Mathematics 2	SLIC+SLIC	PM	-	K o. mP o. R o. H o. P.	6
Section 4: Bachelor Thesis							
6	MAT-30-40	Bachelor Thesis	BT	PM	s.M.	BT	6
Section 5: Transferable Credits for the Master Degree							
-	MAT-20-02	Introduction to Complex Analysis and Ordinary Differential Equations	L+E	WM	EC	wr. o. or.	9
-	MAT-40-51	Specialisation	L+E	WM	EC	wr. o. or.	9
-	MAT-40-52	Seminar: Mathematical Specialisation	S	WM	s.M.	Pr	4

Abbreviations:

Type of Module	: PM=compulsory module, PMW=compulsory module with choice, WPM=elective module
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Course Work	: EC=exercise certificate, PEC=practical exercise certificate, PC=practical certificate
Other	: h=hours, o.=or, s.M.=see module description, ST=suggested term

Note at the request of the student representatives on the Study Commission: For modules extending over several semesters, it is strongly recommended to review the content of each submodule thoroughly during the lecture-free periods, thereby spreading the preparation for the oral examination over a longer period of time.

2.2 Overview by the Course of Studies

Firstly, we provide an overview of the possible course of study in the form of a table both for entry in the winter semester and for entry in the summer semester. The second subject and the area of educational sciences are not broken down in detail.

<table>
<tr><th colspan="7">Study Plan for Students Starting in the Winter Semester</th></tr>
<tr><th>FS</th><th>CPiM</th><th colspan="3">Subject Mathematics</th><th>Second Subject</th><th>ES</th></tr>
<tr><td>1</td><td>15</td><td colspan="2" rowspan="2">Foundations of Mathematics
(27 CP)</td><td rowspan="3"></td><td rowspan="6">Second Subject
(81 CP)</td><td rowspan="6">Education Science and Orientation Internship
(12 CP)</td></tr>
<tr><td>2</td><td>12</td></tr>
<tr><td>3</td><td>15</td><td>Numerical Mathematics
(9 CP)</td><td>Consolidation of the Foundations of Mathematics
(6 CP)</td></tr>
<tr><td>4</td><td>15</td><td>Stochastics
(9 CP)</td><td>Introductory seminar
(3 CP)</td><td>Subject Didactics Mathematics 1
(3 CP)</td></tr>
<tr><td>5</td><td>12</td><td>Geometry
(9 CP)</td><td></td><td rowspan="2">Subject Didactics Mathematics 2
(6 CP)</td></tr>
<tr><td>6</td><td>12</td><td>Algebra
(9 CP)</td><td>possibly Bachelor Thesis
(6 CP)</td></tr>
<tr><td colspan="7">Explanation of the Abbreviations:
FS=semester, CP=credit points (ECTS points), CPiM=credit points in mathematics,
ES=educational science</td></tr>
</table>

<table>
<tr><th colspan="7">Study Plan for Students Starting in the Summer Semester</th></tr>
<tr><th>FS</th><th>CPiM</th><th colspan="3">Subject Mathematics</th><th>Second Subject</th><th>ES</th></tr>
<tr><td>1</td><td>15</td><td colspan="2" rowspan="2">Foundations of Mathematics
(27 CP)</td><td rowspan="2"></td><td rowspan="6">Second Subject
(81 CP)</td><td rowspan="6">Education Science and Orientation Internship
(12 CP)</td></tr>
<tr><td>2</td><td>12</td></tr>
<tr><td>3</td><td>15</td><td>Stochastics
(9 CP)</td><td>Introductory seminar
(3 CP)</td><td>Subject Didactics Mathematics 1
(3 CP)</td></tr>
<tr><td>4</td><td>15</td><td>Numerical Mathematics
(9 CP)</td><td>Consolidation of Foundations of Mathematics
(6 CP)</td><td></td></tr>
<tr><td>5</td><td>12</td><td>Algebra
(9 CP)</td><td></td><td rowspan="2">Subject Didactics Mathematics 2
(6 CP)</td></tr>
<tr><td>6</td><td>12</td><td>Geometry
(9 CP)</td><td>possibly Bachelor Thesis
(6 CP)</td></tr>
<tr><td colspan="7">Explanation of the Abbreviations:
FS=semester, CP=credit points (ECTS points), CPiM=credit points in mathematics,
ES=educational science</td></tr>
</table>

2.3 Overview of Programme Structure with Semester Assignment

Overview of Programme Structure with Semester Assignment for Students Starting in the Winter Semester															
		Exam				**Teaching**				**Term**					
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points (CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.					
										1. CP	2. CP	3. CP	4. CP	5. CP	6. CP
Section 1: Foundations of Mathematics									**33**						
Foundations of Mathematics								24	27						
1.	Lecture	Or.	30-40	g	27	L	o	12		9	9				
2.	Exercise class					E	o	6		6	3				
3.	Revision course					r	o	6		0	0				
Consolidation of the Foundations of Mathematics								4	6						
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	6	L	o	2				3			
2.	Exercise class					E	o	1				1,5			
3.	Practical training	-		ng		P	o	1				1,5			
Section 2: Compulsory Advanced Modules									**39**						
Numerical Mathematics								6	9						
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4				6			
2.	Exercise class					E	o	2				3			
Stochastics								6	9						
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4					6		
2.	Exercise class					e	o	2					3		
Geometry								6	9						
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4						6	
2.	Exercise class					E	o	2						3	
Algebra								6	9						
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4							6
2.	Exercise class					E	o	2							3
Introductory seminar								2	3						
1.	Introductory seminar	Pres		g	3	PS	o	2					3		

Overview of Programme Structure with Semester Assignment for Students Starting in the Winter Semester															
		Exam				Teaching				Term					
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points (CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.					
										1. CP	2. CP	3. CP	4. CP	5. CP	6. CP
Section 3: Subject Didactics Mathematics									**9**						
Subject Didactics Mathematics 1								2	3						
1.	Subject Didactics Mathematics 1	Wr. o. Or.	90-180 o. 20-30	g	3	SL	o	2					3		
Subject Didactics Mathematics 2								4	6						
1.	Subject Didactics Mathematics 2 – Part 1	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	SL	o	2						3	
2.	Subject Didactics Mathematics 2 – Part 2	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	SL	o	2							3
Sectioon 4: Bachelor Thesis									**6**						
Bachelor thesis									6						
1.	Bachelor thesis	BA		g		BA	o								6

Explanation of the abbreviations:

Marking system : g=graded, ng=non graded
Form of examination : BA=bachelor thesis, Or.=oral exam, Wr.=written exam, Pres=presentation, TP=term paper
Form of teaching : L=lecture, SL=seminar or lecture, E= exercise class, r=revision course, P=practical course, PS=introductory seminar
Status : o=obligatory, f=facultative
Other : o.=or, SWS=hours in class per week, CP=credit points=ECTS points

Overview of Programme Structure with Semester Assignment for Students Starting in the Summer Semester															
		Exam				Teaching				Term					
										The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.					
		Type of Exam	Duration(min)	Grading	Weight in the final Grade	Type of Course	Status	SWS	ECTS Points (CP)	1. CP	2. CP	3. CP	4. CP	5. CP	6. CP
Section 1: Foundations of Mathematics									**33**						
Foundations of Mathematics								24	27						
1.	Lecture	Or.	30-40	g	27	L	o	12		9	9				
2.	Exercise class					E	o	6		6	3				
3.	Revision course					r	o	6		0	0				
Consolidation of the Foundations of Mathematics								4	6						
1.	Lecture	Or.	20-30	g	6	L	o	2					3		
2.	Exercise class					E	o	1					1,5		
3.	Practical training	-		ng		P	o	1					1,5		
Section 2: Compulsory Advanced Modules									**39**						
Stochastic								6	9						
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4				6			
2.	Exercise class					E	o	2				3			
Numerical Mathematics								6	9						
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4					6		
2.	Exercise class					E	o	2					3		
Algebra								6	9						
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4						6	
2.	Exercise class					E	o	2						3	
Geometry								6	9						
1.	Lecture	Wr. o. Or.	90-180 o. 20-30	g	9	L	o	4							6
2.	Exercise class					E	o	2							3
Introductory seminar								2	3						
1.	Introductory seminar	Pres		g	3	PS	o	2				3			
Section 3: Subject Didactics Mathematics									**9**						
Subject Didactics Mathematics 1								2	3						
1.	Subject Didactics 1	Wr. o. Or.	90-180 o. 20-30	g	3	SL	o	2				3			

Overview of Programme Structure with Semester Assignment for Students Starting in the Summer Semester															
		Exam				**Teaching**				**Term**					
		Type of Exam	Duration(min)	Grading	Weight in the final Grade	Type of Course	Status	SWS	ECTS Points (CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.					
										1. CP	2. CP	3. CP	4. CP	5. CP	6. CP
Subject Didactics Mathematics 2								4	6						
1.	Subject Didactics 2 – Part 1	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	SL	o	2						3	
2.	Subject Didactics 2 – Part 2	Wr. o. Or. o. Pres o. TP	90-180 o. 20-30	g	3	SL	o	2							3
Section 4: Bachelor Thesis									**6**						
Bachelor thesis									6						
1.	Bachelor thesis	BA		g		BA	o								6

Explanation of the abbreviations:

Marking system : g=graded, ng=non graded
Form of examination : BA=bachelor thesis, Or.=oral exam, Wr.=written exam, Pres=presentation, TP=term paper
Form of teaching : L=lecture, SL=seminar or lecture, E=exercise class, r=revision course, P=practical course, PS=introductory seminar
Status : o=obligatory, f=facultative
Other : o.=or, SWS=hours in class per week, CP=credit points=ECTS points

3 Module Descriptions

Section 1: Foundations of Mathematics

Module Number: **MAT-10-10**	**Module Title:** Foundations of Mathematics		**Type of Module:** Compulsory Module
ECTS-Points	27		
Workload **- Time in Class** **- Self-Study**	Workload: 810 h	Time in Class: 270 h	Self-Study: 540 h
Duration	2 Semester		
Frequency	every Semester		
Term	1+2		
Language of Instruction	German		
Forms of Teaching and Learning	1. Semester: Linear Algebra 1, Lecture 4 SWS + Ex.cl. 2 SWS + Rev.c. 2 SWS 1. Semester: Analysis 1, Lecture 4 SWS + Ex.cl. 2 SWS + Rev.c. 2 SWS 2. Semester: Analysis 2, Lecture 4 SWS + Ex.cl. 2 SWS + Rev.c. 2 SWS		
Higher Objectives	In the Foundations of Mathematics module, students learn the essential conceptual and methodological foundations of linear algebra as well as single-variable and multivariable calculus, exploring their interconnections with particular emphasis on the similarities and differences in their approaches. In the oral exam, students demonstrate that they have recognised these relationships and are capable of contextualising the core results of the lectures within these frameworks. The duration of the module supports these objectives while also accounting for the acquisition of a new language - the language of mathematics - and the development of a precise and rigorously logical working methodology. This provides students with the necessary time to make the significant transition from school-level mathematics to university-level mathematics. By demonstrating a deeper and more integrated understanding in the oral exams, students establish a strong foundation for successful participation in all subsequent modules in their academic programme.		

Content	• Basic logic and sets. • Structure of real and complex numbers. • Sequences, convergence and series; criteria for convergence; power series, sequences of functions; pointwise and uniform covergence. • Continous functions in one dimension and between metric spaces and their properties. • One- and multidimensional differential calculus (especially: intermediate value theorem, Taylor expansion, implicit function theorem, inverse function theorem, extrema under constraints). • One- and multidimensional Riemann integral (especially Fubini's theorem, transformation formula). • Basic concepts of topology in metric and normed spaces. • Basic concepts of the theory of ordinary differential equations (Picard-Lindelöf theorem, linear ordinary differential equations, flows). • Vector spaces and linear maps. • Matrices and systems of linear equations. • Determinants, eigenvalues and diagonalisability. • Jordan canonical form. • Euclidean and unitary vector spaces, spectral theorems. • Basics of analytical geometry. • The lecture Analysis 1 focuses predominately on contents from one-dimensional analysis, the lecture Analysis 2 on multidimensional analysis. The lecture Linear Algebra 1 covers the contents of linear algebra.
Objectives	The students are familiar with and understand the fundamental concepts, statements, and methods of single-variable and multivariable calculus as well as linear algebra. They have also developed a foundational awareness of ordinary differential equations and initial value problems. Their capacity for abstraction has been enhanced, they have been trained in analytical thinking, and their mathematical imagination has been stimulated. Through a proof- and structure-oriented approach, they have learned to comprehend mathematical proofs in calculus and linear algebra and to independently prove or disprove mathematical statements in simple examples. They have recognised the essential relationships within the theory of single-variable and multivariable calculus, their similarities and differences, as well as their connections to linear algebra, and are able to contextualise the core results of the lectures within these frameworks. In the exercises, they have developed a confident, precise, and independent approach to the concepts, statements, and methods covered in the lectures. Additionally, their presentation and communication skills have been cultivated through written assignments and presenting their own solutions. The students are capable of acquiring knowledge through self-study, while their teamwork abilities have been fostered through collaboration in small groups.

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Linear Algebra 1	L	o	4	6	yes	or.	30-40	g	100
	E	o	2	3					
	T	o	2	0					
Analysis 1	L	o	4	6	yes				
	E	o	2	3					
	T	o	2	0					
Analysis 2	L	o	4	6	yes				
	E	o	2	3					
	T	o	2	0					

In each of the three parts of the module an exercise certificate is to be acquired as coursework. The exercise certificate is acquired after regular participation in the exercise classes by taking part in a written test. Both partial assessments must be completed in the same semester.

The examination of the module consists of an oral exam covering all three parts of the module. To be eligible for the oral exam, students must have obtained at least one of the two exercise certificates for the Analysis 1 and Analysis 2 module parts, as well as the exercise certificate for the Linear Algebra 1 module part. The module is considered complete only when all three exercise certificates have been obtained and the oral exam has been successfully passed.
Of the 27 credit points for the module, 15 are allocated to the first semester and 12 to the second semester. The relatively higher share of credit points in the second semester, compared to the actual teaching hours, is due to the preparation required for the oral exam, which takes place after the second semester.

Literature

Possible References :

- Anton Deitmar: Analysis. Springer 2016.
- Otto Forster: Analysis 1. Springer Spektrum 2013.
- Otto Forster: Analysis 2. Vieweg+Teubner 2011.
- Theodor Bröcker: Lineare Algebra und analytische Geometrie. Birkhäuser 2013.
- Gerd Fischer: Lineare Algebra. Springer Spektrum 2014.

Transfer

The successful participation in the module Foundations of Mathematics is a prerequisite for the participation in the module Bachelor Thesis. The exercise certificates of the module are prerequisite for all modules of the Sections 2-4.

Prerequisites

There are no prerequisites for participation in the module.

Responsible Persons

Victor Batyrev, Anton Deitmar, Christian Hainzl, Jürgen Hausen, Frank Loose, Hannah Markwig, Thomas Markwig, Reiner Schätzle, Stefan Teufel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-10-11**	**Module Title:** Consolidation of the Foundations of Mathematics		**Type of Module:** Compulsory Module
ECTS-Points	6		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	every Semester		
Term	3-4		
Language of Instruction	German		
Forms of Teaching and Learning	• Algebraic Structures, Lecture 2 SWS + Ex.cl. 1 SWS • Mathematical Software, Practical course 1 SWS		
Comment	The coursework and the examination in the module part algebraic structures can be replaced by the module Linear Algebra from the study programme Bachelor of Science Mathematics. The Mathematical Software sub-module is usually provided to students in the Bachelor of Education Lehramt Gymnasium by participating in the practical exercises in the module Numerical Mathematics. Further courses which could be taken instead will be listed in the course catalogue.		
Content	• Algebraic structures: – Groups, subgroups, group homomorphisms, normal subgroups, quotient group. – Cyclic groups and the symmetric group. – Commutative rings with one, divisibility. – Euclidean rings, principal ideal domains, factorial rings. – The ring of integers and the polynomial ring. • Mathematical software: – Getting to know one or more subject-specific software packages. – Implementation of simple algorithms, e.g. of linear algebra, in subject-typical software.		
Objectives	Students have learnt and understood essential aspects of linear algebra based on the Foundations of Mathematics module: the algebraic structures group and ring, which are essential for all areas of mathematics. They have deepened their structural skills acquired in the Foundations of Mathematics module. They are familiar with the most fundamental statements and methods in the field. Their capacity for abstraction has been enhanced, they have been trained in analytical thinking and their mathematical imagination has been stimulated. Using a proof- and structure-orientated approach, they have learnt to understand mathematical proofs of algebra and to independently prove or disprove mathematical statements using simple examples. They are able to place the structures they have learnt in linear algebra in a larger context and understand them better. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. In addition, the students' presentation and communication skills were trained through written work and presenting their own solutions. The students are able to acquire knowledge through self-study and at the same time their ability to work in a team has been promoted by working in smaller groups. In the practical course on mathematical software, students have familiarised themselves with one or more subject-specific software packages or computer algebra systems. They are trained to work out selected problems, e.g. linear algebra, algorithmically and to implement the developed algorithms in a subject-specific software package. In doing so, they have expanded and deepened the algorithmic skills they acquired in the Foundations of Mathematics.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Algebraic Structures	L	o	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	o	1	1,5					
	Mathematical Software	P	o	1	1,5	yes	-	-	nb	0
	In the sub-module Algebraic Structures an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	**Possible References :** • Serge Lang: Algebraische Strukturen. Vandenhoek & Ruprecht 1979. • Gerd Fischer: Lineare Algebra und Analytische Geometrie. Springer 2010.									
Transfer	The module is a prerequisite for the module Bachelor Thesis.									
Prerequisites	There are no prerequisites.									
Responsible Persons	Jürgen Hausen, Hannah Markwig, Thomas Markwig, Walther Paravicini									

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 2: Compulsory Intermediate Modules

Module Number: **MAT-20-03**	**Module Title:** Algebra		**Type of Module:** Compulsory Module
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Summer Semester		
Term	5-6		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Groups and structure theory of finite groups. • Rings, ideals, polynomial rings, divisibility theory. • Fields and field extensions. • Geometric and algebraic applications of field theory.		
Objectives	The students deepen their structural thinking, know basic algebraic concepts and can apply them on other mathematical disciplines. They understand, in particular, through the example of field theory, how the interaction of different branches of algebra leads to new insights, e.g. answers to classical problems from antiquity. In the process they have experienced, that the coaction of different areas of mathematics can be essential for solving concrete problems. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. Furthermore the presentation and communication skills of the students was trained by written assignments and presenting their own solutions. The students are capable of adopting knowledge by self-study and at the same time their capacity for teamwork was enhanced by working in small groups.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebra	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Siegfried Bosch: Algebra. Springer 2009. • Gerd Fischer, Reinhard Sacher: Einführung in die Algebra. Teubner 1983. • Christian Karpfinger, Kurt Meyberg: Algebra: Gruppen-Ringe-Körper. Springer Spektrum 2010. • Kurt Meyberg: Algebra 1. Hanser 1980. • Kurt Meyberg: Algebra 2. Hanser 1976. • Hans-Jörg Reiffen, Günter Scheja, Udo Vetter: Algebra. Bibliographisches Institut 1984.
Transfer	If applicable, the module is requirement for the module Bachelor Thesis.
Prerequisites	At least two of the exercise certificates from the Foundations of Mathematics module must have been acquired, one of which must be the exercise certificate for Linear Algebra 1. Content-wise, knowledge from the submodule Algebraic Structures is assumed.
Responsible Persons	Jürgen Hausen, Hannah Markwig, Thomas Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-20-11	**Module Title:** Numerical Mathematics		**Type of Module:** Compulsory Module
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	3-4		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Interpolation and approximation of functions. • Numeric integration and differentiation. • Systems of linear equations and linear curve fitting. • Systems of non-linear equations and non-linear curve fitting. • Initial value problems for ordinary differential equations.		
Objectives	The students know the foundations of numerical mathematics and are capable of performing basic calculation techniques. They understand to bring the knowledge gathered in the modules Analysis and Linear Algbra in the analysis of numerical methods and to use the methods for specific problems. Their algorithmic thinking was enhanced and they are acquainted to the analysis of algorithms with a view to questions of efficiency and complexity. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. Furthermore the presentation and communication skills of the students were trained by written assignments and presenting their own solutions. The students are capable of adopting knowledge by self-study and at the same time their capacity for teamwork was enhanced by working in small groups.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Numerical Mathematics	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Peter Deuflhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008. • Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg+Teubner 2009.		
Transfer	If applicable, the module is prerequisite for the module Bachelor Thesis.		

Prerequisites	At least two of the exercise certificates from the module Foundations in Mathematics must have been acquired. One of these must be the certificate for Linear Algebra 1. Furthermore, before admission to the examination, the practical certificate for the practical course in Numerical Analysis from the module Introduction to Scientific Programming must have been obtained.
Responsible Persons	Christian Lubich, Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-20-12</td><td colspan="2">Module Title:
Stochastics</td><td>Type of Module:
Compulsory Module</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">regularly in Summer Semester</td></tr>
<tr><td>Term</td><td colspan="3">3-4</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">

- Introduction to probability theory and statistics.
- Topics from probability theory: Probability spaces, simple conditional probabilities, urn models, random variables, distribution functions, discret and continous distributions, expectation and variance, inequalities, independence, joint probability distribution, notions of convergence, laws of lagre numbers, central limit theorem.
- Topics from statistics: Point estimators, hypothesis testing, standard testing methods.

</td></tr>
<tr><td>Objectives</td><td colspan="3">The students know the basic principles of stochastics. They have the ability to abstract stochastic questions and are capable of using their knowledge on specific problems.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. Furthermore the presentation and communication skills of the students were trained by written assignments and presenting their own solutions. The students are capable of adopting knowledge by self-study and at the same time their capacity for teamwork was enhanced by working in small groups.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Stochastics</td><td>L</td><td>o</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>o</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :

- Hans-Otto Georgii: Stochastik. De Gruyter 2015.
- Ulrich Krengel: Einführung in die Wahrscheinlichkeitstheorie und Statistik. Vieweg 2005.

</td></tr>
<tr><td>Transfer</td><td colspan="3">If applicable, the module is prerequisite for the module Bachelor Thesis.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">At least two of the exercise certificates from the module Foundations in Mathematics must have been acquired. One of these must be the certificate for Linear Algebra 1.</td></tr>
<tr><td>Responsible Persons</td><td colspan="3">Martin Möhle, Martin Zerner</td></tr>
</table>

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-20-20	**Module Title:** Introductory Seminar: Presentations in Mathematics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	every Semester		
Term	3-4		
Language of Instruction	German		
Forms of Teaching and Learning	Introductory seminar, talk, presentation, e-learning, blended learning		
Content	Various topics from the foundations of mathematics.		
Objectives	The students independently work on a coherent mathematical topic and prepare it in a didactical appealing form. They learn how to present their work to a group, how to be responsive to questions regarding the content and how to lead a professional discussion.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introductory Seminar	PS	o	2	3	yes	Pr	60-90	g	100

The acquisition of the credit points requires alongside with a successfull presentation the regular active participation in the course, like by asking questions, contributing to a discussion or working on problem tasks. Additionally a written elaboration of the own talk or the issue of a handout for the participants may be required. This further work constitutes the coursework of the module.

Transfer	The module Introductory Seminar Presentation in Mathematics is, if applicable, prerequisite for the module Bachelor Thesis.
Prerequisites	At least two of the exercise certificates from the module Foundations in Mathematics must have been acquired. One of these must be the certificate for linear algebra 1.
Responsible Persons	The dean of studies at the Department of Mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-01	**Module Title:** Geometry		**Type of Module:** Compulsory Module
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	5-6		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Axiomatic foundation of planar geometry. • Euclidean and non-Euclidean geometry. • Parametrised curves and surfaces.		
Objectives	The students deepen their axiomatic way of thinking and are capable of giving correct proofs. They know the basic principles of geometry, are able to solve concrete problems and know the fundamental links between geometry and topology. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Geometry	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Michele Audin: Geometry. Springer 2003. • Marcel Berger: Geometry Revealed: A Jacob's Ladder to Modern Higher Geometry. Springer 2010. • David A. Brannan, Matthew F. Esplen, Jeremy J. Gray: Geometry. Cambridge University Press 2012. • John Stillwell: The four pillars of geometry. Springer 2005.
Transfer	If applicable, the module is a prerequisite for the module bachelor thesis.

Prerequisites	At least two of the exercise certificates from the module Foundations of Mathematics must have been acquired. One of these must be the exercise certificate of Linear Algebra 1.
Responsible Persons	Christoph Bohle, Carla Cederbaum, Hannah Markwig, Ivo Radloff

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 3: Didactics of Mathematics

Module Number: MAT-80-01	**Module Title:** Subject Didactics Mathematics 1		**Type of Module:** Compulsory Module
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	regularly in Summer Semester		
Term	3-4		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture, exercise class, proseminar, talk, presentation, e-learning, blended learning, project work, case studies		
Content	**Didactics of Algebra and Arithmetic:** This course deals with the foundations of the didactics of mathematics in the educational plans and in particular the didactic reduction of important basic concepts of algebra and arithmetic to school level, various ways of introducing important concepts of algebra and arithmetic at school and ways of motivating basic algebraic and arithmetic ideas.		
Objectives	Students know the basic didactic principles of teaching concepts and can orientate themselves in the educational plans. They are able to compare and evaluate subject-specific approaches to central concepts in algebra and arithmetic. They have the ability to convey algebraic and arithmetic content in a way that is both student- and subject-orientated.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see table below		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Subject Didactics Mathematics 1	LIC	o	2	3	no	K o. mP o. P	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Transfer	The module Didactics of Mathematics 1 is compulsory for the module Bachelor Thesis, if the bachelor thesis is written in mathematics.
Prerequisites	At least two of the exercise certificates from the module Foundations of Mathematics must have been acquired. One of these must be the exercise certificate for Linear Algebra 1.
Responsible Persons	Frank Loose, Walther Paravicini

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests

Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-80-02	**Module Title:** Subject Didactics Mathematics 2		**Type of Module:** Compulsory Module
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	2 Semester		
Frequency	every Semester		
Term	5-6		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture, exercise, proseminar, talk, presentation, e-learning, blended learning, project work, case studies		
Content	The module consists of the two parts • **didactics of geometry and linear algebra,** • **didactics of analysis and stochastics.** It deals with the didactic reduction of important basic concepts of analysis, linear algebra, geometry or stochastics at school level, various options for introducing important terms in analysis, linear algebra, geometry or stochastics at school as well as motivational options for analytic, geometric and stochastic basic ideas.		
Objectives	Students are familiar with the basic didactic principles of teaching concepts. They are able to compare and evaluate subject-specific approaches to central concepts in analysis, linear algebra, geometry and stochastics. They have the ability to convey geometric and algebraic content in a way that is both student- and subject-orientated.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Subject Didactics Mathematics 2 – Part 1	SLIC	o	2	3	yes	K o. mP o. R o. H o. P.	90-180 o. 20-30	g	50
Subject Didactics Mathematics 2 – Part 2	SLIC	o	2	3	yes	K o. mP o. R o. H o. P.	90-180 o. 20-30	g	50

The module consists of two parts, for which the teaching learning methods (lecture, exercise class or seminar) as well as the type of examination (written or oral exam, presentation or paper) are usually different. This is taken into account by the fact that, the examination in this module consists of two equally weighted parts.

Transfer	If applicable, part of the module Subject Didactics of Mathematics 2 is a prerequisitefor the module Bachelor Thesis, if the bachelor thesis is written in mathematics.
Prerequisites	The module Foundations of Mathematics must be completed. The module Geometry should be taken parallel to the Didactics of Geometry or should have been taken beforehand, as knowledge from the module Geometry is required in the Didactics of Geometry module.
Responsible Persons	Frank Loose, Walther Paravicini

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 4: Bachelor Thesis

Module Number: **MAT-30-40**	**Module Title:** Bachelor Thesis		**Type of Module:** Compulsory Module
ECTS-Points	6		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: regularly, as required	Self-Study: 180 h
Duration	1 Semester		
Frequency	every Semester		
Term	6		
Language of Instruction	German		
Forms of Teaching and Learning	Bachelor thesis		
Content	The students have to work under instruction of an advisor on a defined task from mathematics or subject didactics mathematics with scientific methods and present the results in written form. In detail this includes: • the formulation of a scientific question in accordance with the advisor; • the independent search for and the study of relevant scientific literature; • the formulation of suited questions and methodical approaches for their solution; • the independent realisation of the project, the written presentation of the project and the reults in the context of the current state of research.		
Objectives	The students • can work independently and with scientific methods on an assigned topic, • operate a literature research for scientific sources, • choose scientific methods and techniques or develop them further to solve a problem, • communicate the results in a clearly structured fashion and in academically suited form in their thesis.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see below		
Transfer	Bachelorarbeit		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Bachelor Thesis	BT	f	-	6	yes	BT	-	g	100

The coursework consists of an examination interview on the contents of the bachelor thesis. The coursework is graded as pass or fail. The module grade is the grade for the bachelor thesis. The module is only passed if the examination and coursework have been successfully completed.

Prerequisites	Subject specific prerequisite for admission to the module Bachelor Thesis is besides the general part of the examination regulations the acquisition of the credit points from the modules of Section 1 Foundations of ;athematics as well as of at least 21 credit points from the modules of the Section 2 and at least 3 credit points from the modules of Section 3.
Responsible Persons	The dean of studies at the Department of Mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 5: Transferable Credits for the Master Degree

In anticipation of a prospective Master's programme in the Master of Education for Secondary Schools at the University of Tübingen, certain achievements can be made during the Bachelor's programme under specific conditions, which can be credited towards the Master's programme. This aims to offer flexibility in individual study planning during the transition from the Bachelor's to the Master of Education.

Conditions and Scope

Up to a total of 24 ECTS credits for the Master's programme can be acquired in the Bachelor of Education if all of the following conditions are met:

- There is an enrolment (matriculation) in and an examination entitlement in the Bachelor of Education for Secondary Schools;
- A total of at least 150 ECTS credits have already been acquired in the two main subjects and in educational sciences;
- There is an enrolment in and an examination entitlement in the subject in which credits for the Master's programme are to be acquired.

It can be freely chosen how many ECTS credits are earned in which of the studied subjects. For example, all 24 ECTS credits can be earned in one subject if modules are offered in the required extent. Master's modules of a subject taken as a third subject cannot be advanced. Module examinations within the framework of Master's credits can only be repeated once. For further regulations concerning Master's credits, please refer to the study and examination regulations.

In the subject of Mathematics, the following modules can be advanced within the framework of Transferable Credits for the Master Degree.

Module Number: **MAT-20-02**	**Module Title:** Introduction to Complex Analysis and Ordinary Differential Equations		**Type of Module:** Elective Module
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Summer Semester		
Term	-		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		

<table>
<tr><td>Content</td><td>

- Complex Analysis:
  - Holomorphic functions, Cauchy-Riemann equations.
  - Antiderivatives, Cauchy's integral formula, Cauchy's integral theorem.
  - Compact convergence of families of functions, formal and convergent power series, complex-analytical functions, identity theorem.
  - Liouville's theorem, inverse function theorem for holomorphic functions, open mapping theorem, maximum principle.
  - Laurent series, holomorphic functions with isolated singularities, Casorati-Weierstrass theorem.
  - Residue theorem and applications.
- Ordinary differential equations, a choice of the following:
  - Picard-Lindelöf existence and uniqueness theorem.
  - Linear ordinary differential equations, Gronwall's lemma.
  - Continous dependence on initial conditions, differential dependence on initial conditions.
  - Basics of dynamical systems, stability of equilibrium positions, characteristic exponents, first integrals, Liapunov-functions.
  - Ordinary differential equations over the complex numbers.
  - Regularity, the criterion of Fuchs.
  - The method of Frobenius.

</td></tr>
<tr><td>Objectives</td><td>The students know the foundations of the theory of complex analysis and ordinary differential equations. The are acquainted to essential calculation techniques and can calculate line integrals as well as explicitly solve simple differential equations. They know fundamental applications of the theory like e.g. the fundamental theorem of algebra and the Newtonian equations of motion. They also have the ability to transfer abstract questions into concrete problems of complex analysis or respectively of ordinary differential equations and solve them this way.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. Furthermore the presentation and communication skills of the students was trained by written assignments and presenting their own solutions. The students are capable of adopting knowledge by self-study and at the same time their capacity for teamwork was enhanced by working in small groups.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td>

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Complex Analysis and ODEs.	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

</td></tr>
</table>

Literature	**Possible References :** • Lars Valerian Ahlfors: Complex analysis. McGraw-Hill 1979. • John B. Conway: Functions of one complex variable. Springer 1996. • Wolfgang Fischer, Ingo Lieb: Einführung in die Komplexe Analysis. Springer 2010. • Walter Rudin: Reelle und komplexe Analysis. Oldenbourg 2009. • Earl A. Coddington, Norman Levinson: Theory of ordinary differential equations. McGraw-Hill 1955. • William T. Reid: Ordinary differential equations. John Wiley & Sons 1971. • Hille, Einar: Ordinary differential equations in the complex domain. Dover Publications 1997. • Wasow, Wolfgang: Asymptotic expansions for ordinary differential equations. John Wiley 1965.
Transfer	It is to be transferred to the consecutive master's programme.
Prerequisites	The examination in the module Algebraic Structures and Mathematical Software must be passed and the exercise certificate for Linear Algebra 1 must be acquired.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-51	**Module Title:** Specialisation		**Type of Module:** Elective Module
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	every Semester		
Term	-		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Comment	A course must be selected from the catalogue of courses in Section 4.1 of the module handbook, comprising 4 hours of lectures and 2 hours of exercises per week. The approval of additional courses or alternative course formats (e.g., two courses with 2 hours of lectures and 1 hour of exercises each) is at the discretion of the head of the examination board, upon a written request by the student.		
Content	The content is determined by the choice of a course.		
Objectives	The students have acquired in-depth knowledge in a specific area of mathematics and gained further experience in presenting and communicating mathematical topics. They are capable of identifying the key statements of the lecture, reproducing the techniques used for their derivation and proof, and critically evaluating them. Additionally, they can integrate the methodological and theoretical foundations of the chosen mathematical subfield and place them within the broader mathematical context. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see table below		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
see Comment	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Transfer	It is to be transferred to the consecutive master's programme.
Prerequisites	The modules of Section 1 Foundations of Mathematics must have been completed successfully.
Responsible Persons	The dean of studies at the Department of Mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests
Teaching Format	: L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-52	**Module Title:** Seminar: Mathematical Specialisation		**Type of Module:** Elective Module
ECTS-Points	4		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	every Semester		
Term	-		
Language of Instruction	German		
Forms of Teaching and Learning	Seminar, talk, presentation, e-learning, blended learning		
Content	Various topics from the advanced fields of mathematics.		
Objectives	The students independently work on a coherent mathematical topic and prepare it in a didactical appealing fashion. They learn how to present their work to a group, how to be responsive to questions regarding the content and how to lead a professional discussion. The work and the presentation may be the foundation or a deepened study in the scope of a master thesis.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Seminar	S	o	2	4	yes	Pr	60-90	g	100

The acquisition of the credit points requires alongside with a successful presentation the regular active participation in the course, like by asking questions, contributing to a discussion or working on problem tasks. Additionally a written elaboration of the own talk or the issue of a handout for the participants may be required. These further efforts constitute the coursework of the module.

Transfer	It is to be transferred to the consecutive master's programme.
Prerequisites	The participation in the module requires the successful completion of at least one of the modules Introduction to Complex Analysis and Ordinary Differential Equations or Specialistion.
Responsible Persons	The dean of studies at the Department of Mathematics

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : BT=bachelor's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests

Teaching Format : L=lecture, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

4 Courses for the Module Specialisation

4.1 Course Catalogue

The following lists the courses that can be included in the module Specialsation (MAT-40-51). Additional courses can be approved upon written request by the head of the examination board.

Course Title:	Algebraic Topology 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Set theoretical topology. • Basic concepts of category theory. • The fundamental group of a punctured topological space. • Theory of covering spaces. • Basic concepts of singular homology theory. • Applications.		
Special Objectives	The students learn how to realise ideas in topology, e.g. the detection of holes in topological spaces, into a precise theory, even with a sophisticated technique. In particular, they recognise how abstract concepts, e.g. from category theory and homological algebra, provide effective ways of speaking that enable the formation of ideas to be adequately implemented.		
Literature	**Possible References :** • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.		
Responsible Persons	Anton Deitmar, Frank Loose		

Course Title:	Algorithms of Numerical Mathematics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	Advanced, important algorithms of numerics (without differential equations) such as: • Fast Fourier transformation; • QR algorithms for the calulation of eigenvalues; • Method of conjugated gradients and more general Krylov space methods as iterative methods in numeric linear algebra and in non-linear optimisation; • Simplex method and interior point methods in linear optimisation.
Special Objectives	The students have learned the key concepts, results, and methods of algorithmic numerical mathematics.
Literature	**Possible References :** • Peter Deuflhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008. • Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg 2009.
Responsible Persons	Christian Lubich, Andreas Prohl

Course Title:	Introduction to Mathematical Logic		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Propositional logic. • Languages of the first order: – Completeness and compactness. • Theory of computations: – Register machines; – Gödelisation. • Incompleteness of arithmetic: – First and second incompleteness theorem. • Set theory: – Ordinal- and cardinal numbers; – Incompleteness of set theory.		
Special Objectives	Students are able to understand mathematical theorems and theories in the context of mathematical logic. They understand the limits of possible mathematical knowledge, recognise the difference between truth and provability and can apply basic theoretical model thinking to mathematical content.		

Literature	**Possible References :** • Rautenberg, Wolfgang: Einführung in die Mathematische Logik. Vieweg+Teubner 2008. • Ziegler, Martin: Mathematische Logik. Birkhäuser 2016.
Responsible Persons	Anton Deitmar

Course Title:	Introduction to set theory		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	**Content:** •		
Special Objectives	-		
Literature	**Possible References :** •		
Responsible Persons	Frank Loose		

Course Title:	Introduction to Optimisation		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 3 SWS + Exercise class 1 SWS		

Content	• Optimality theory for smooth, convex and linear optimisation problems optimisation problems with constraints. • Foundations of the theory of convex sets and functions. • Duality theory for convex and linear optimisation problems. • Solution methods for linear optimisation problems.
Special Objectives	Students know and understand methods and algorithms for solving convex and linear optimisation problems. They have learnt to apply the methods to simple problems related to economics, technology or physics. They will be able to critically assess the possibilities and limitations of using the methods.
Literature	**Possible References :** • Florian Jarre, Joseph Stoer: Optimierung: Einführung in mathematische Theorie und Methoden. Springer 2019. • Jorge Nocedal, Stephen J. Wright: Numerical optimization. Springer 2006.
Responsible Persons	Christian Lubich

Course Title:	Introduction to Dynamical Systems		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Kepler's laws. • Equilibrium positions. • Stability. • Predator-prey model. • Poincaré-Bendixson theorem. • Limit sets. • Periodic orbits. • Celestial mechanics.		
Special Objectives	The students can ask and examine qualitative questions about the solutions of ordinary differential equations, like e.g.: How long do mathematical solutions exist? Are there equilibrium states or periodic orbits? When are trajectories stable?		

Literature	**Possible References :** • Morris W. Hirsch, Stephen Smale: Differential equations, dynamical systems, and linear algebra. Academic Press 1974. • Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 2010. • Carl Ludwig Siegel, Jürgen Moser: Lectures on celestial mechanics. Springer 1995.
Responsible Persons	Frank Loose

Course Title:	Introduction to Geometric Measure Theory		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem. • Surface- and cosurface formula. • Countable rectifiable sets, rectifiable varifolds.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		
Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.		
Responsible Persons	Reiner Schätzle		

Course Title:	Introduction to Geometric Measure Theory – Measure Theoretic Methods
Specialisation	Analysis

Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		
Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.		
Responsible Persons	Reiner Schätzle		

Course Title:	Introduction to Geometric Measure Theory – Varifolds		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Surface- and cosurface formula. • Countable rectifiable sets, rectifiable varifolds.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		

Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Responsible Persons	Reiner Schätzle

Course Title:	Introduction to Commutative Algebra and Algebraic Geometry		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Rings and ideals. • Gröbner bases. • Localization. • Noetherian rings and modules. • Integral ring extensions. • Krull's principal ideal theorem and dimension theory. • Hilbert's Nullstellensatz and Noether normalisation. • Affine varieties, Zariski topology, morphisms.		
Special Objectives	The students have become familiar with the central concepts, results, and methods of commutative algebra and affine algebraic geometry. They have experienced the profound interplay between algebra and geometry through the example of affine varieties. Furthermore, the students understand how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions.		

Literature	**Possible References :** • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Responsible Persons	Jürgen Hausen

Course Title:	Introduction to Partial Differential Equations		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces. • L^2 theory. • Important examples (Laplace equation, wave equation, heat equation). • Fundamental solutions (elliptic situation). • Weak solutions of elliptic equations.		
Special Objectives	The students got to know a central branch of analysis, whose terms and methods are fundamental for many fields, like numerics or stochastics. Also evolutionary equations, who have strong connections to geometry, are issue of the lecture. The students are acquainted with central terms, results and methods of linear partial differential equations and are able to use these methods in advanced courses.		
Literature	**Possible References :** • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.		

Responsible Persons	Gerhard Huisken, Reiner Schätzle

Course Title:	Introduction to Partial Differential Equations – Part 1		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces.		
Special Objectives	The students have familiarised themselves with the first basic features of a central area of analysis, the concepts and methods of which are fundamental for many other areas, such as numerics and stochastics. Students are familiar with the central concepts, results and methods of linear partial differential equations and can successfully apply these methods in the more advanced courses.		
Literature	**Possible References :** • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.		
Responsible Persons	Gerhard Huisken, Reiner Schätzle		

Course Title:	Elementary Number Theory		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		

Content	• Divisibility in the integers. • Prime numbers. • Congruences. • Quadratic residues. • Arithmetic functions. • Multiplicative functions. • Classical theorems. • Applications.
Special Objectives	Students deepen their basic knowledge of integers and experience applying this knowledge to mathematical problems of various kinds.
Literature	**Possible References :** • Friedhelm Padberg: Elementare Zahlentheorie. Spektrum Akademischer Verlag 2001. • Stefan Mueller-Stach, J. Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg 2006.
Responsible Persons	Victor Batyrev, Thomas Markwig

Course Title:	Functional Analysis		
Specialisation	Analysis		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Normed spaces, Banach spaces, dual spaces. • Hahn-Banach theorem, uniform boundedness principle. • Closed graph theorem, open mapping theorem, Banach-Alaoglu theorem. • Compact operators, normal operators, spectral theorems.		
Special Objectives	The students are aquainted with the basic principles and techniques of the theory of infinte dimensional spaces and can apply them to problems in analysis and geometry. They understand the complexity of problems of spectral theory and can use its results for the solution of analytical problems.		

Literature	**Possible References :** • Nicolas Bourbaki: Topological vector spaces. Springer 1987. • Adam Bowers, Nigel Dalton: An introductory course in functional analysis. Springer 2014. • Harro Heuser: Funktionalanalysis. Teubner 2006. • Markus Haase: Functional analysis. American Mathematical Society 2014. • Peter D. Lax: Functional analysis. Wiley 2002. • Gert Kjaergaard Pedersen: Analysis now. Springer 1995. • Walter Rudin: Functional analysis. McGraw-Hill 1991. • Dirk Werner: Funktionalanalysis. Springer 2011. • Kosaku Yosida: Functional analysis. Springer 1995. • Hans Wilhelm Alt: Lineare Funktionalanalysis. Springer 2012.
Responsible Persons	Carla Cederbaum, Anton Deitmar, Gerhard Huisken, Reiner Schätzle

Course Title:	Geometry of Manifolds 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Manifolds and submanifolds. • Vector fields and flows. • Metrics, foundations of Riemannian geometry. • Complex structures. • Theorem of Gauß-Bonnet on surfaces.		
Special Objectives	The students know and understand the fundamental concepts of real and complex differential geometry and the basic techniques for handling them. Especially they have deepened their understanding of differential and integral calculus and have exemplarily experienced how mathematical concepts are used in a natural way in geometry.		

Literature	**Possible References :** • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011.
Responsible Persons	Christoph Bohle, Frank Loose

Course Title:	Foundations of Discrete Mathematics		
Specialisation	Stochastics		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Logic. • Sets, relations, functions. • Partial orders. • Combinatorics. • Number theory. • Graph theory. • Algorithms and formal languages. • Discrete optimization.		
Special Objectives	Students have learned how to use basic methods of discrete mathematics. They can analyze discrete structures and identify discrete structures in different contexts.		
Literature	**Possible References :** • Ronald Graham, Donald Knuth, Oren Patashnik: Concrete Mathematics. Addison-Wesley 1994. • Kenneth H. Rosen: Discrete Mathematics and Its Application. McGraw-Hill 2019. • Ralph P. Grimaldi: Discrete and Combinatorial Mathematics. Addison-Wesley 2004. • Norman L. Biggs: Discrete Mathematics. Oxford University Press 2002.		
Responsible Persons	Martin Möhle, Martin Zerner, Elmar Teufl		

Course Title:	Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	Starting from a system of axioms for plane absolute geometry with the basic concepts of incidence and congruence, the associated Bachmann reflection geometry is developed. After the introduction of the hyperbolic axiom, this is continued with reflection-geometric end theory. A Euclidean field is created from the rotations around an end and the translations along a straight line, with the help of which the hyperbolic plane under consideration is described algebraically.		
Special Objectives	The students have learnt to look at one and the same mathematical object (in this case absolute and hyperbolic planes) from completely different perspectives and to link them together. In particular, they have learnt about Bachmann's group-theoretically oriented reflection geometry, which rarely appears in the curriculum, and thus deepen their knowledge of groups. They also deepened their knowledge about the interweaving of geometry and algebra.		
Literature	**Possible References :** • Friedrich Bachmann: Aufbau der Geometrie aus dem Spiegelungsbegriff. Springer 1959. • Robin Hartshorne: Geometry: Euclid and beyond. Springer 2000. • Helmut Karzel, Kay Sörensen, Dirk Windelberg: Einführung in die Geometrie. Vandenhoeck und Ruprecht 1973.		
Responsible Persons	Hermann Hähl, Hannah Markwig		

Course Title:	Commutative Algebra		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Rings and Ideals. • Localisation and local rings. • Noetherian and Artinian rings and modules. • Integral ring extensions and Cohen-Seidenberg theorems. • Krull's principal ideal theorem and dimension theory. • Primary decomposition. • Normality, regularity and discrete valuation rings. • Hilbert's Nullstellensatz and Noether normalisation.
Special Objectives	The students are familiar with and understand the language and methods of commutative algebra, which are essential for studying the fields of algebra, geometry, and number theory. They recognise how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions.
Literature	**Possible References :** • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Responsible Persons	Victor Batyrev, Thomas Markwig

Course Title:	Convex Geometry		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Cones, polytopes, polyhedra, fans, polyedral complexes. • Normal fans of polygons. • Triangulations, subdivisions, secondary fans, discriminants.		

Special Objectives	In the lecture the students learn basic terms, results and methods of convex geometry. They develope a deepened understanding for the concept of duality of mathematical objects on the example of polytopes and fans. Furthermore they enhance their geometric view and their spatial sense.
Literature	**Possible References :** • Günter M. Ziegler: Lectures on Polytopes. Springer 1998.
Responsible Persons	Hannah Markwig

Course Title:	Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Brief review of key concepts and results from algebra and number theory. • Historical ciphers and their cryptanalysis (Caesar, Vigenere, substitution); encryption schemes. • Diffie-Hellman protocol and fast exponentiation. • Discrete logarithms: Shanks' algorithm and Pollard's rho method. • RSA: correctness, security, and attacks. • Signature schemes.		
Special Objectives	Students are familiar with the fundamental concepts and results of elementary number theory and algebra, as well as their application in cryptography. They can implement the methods covered in Python or SageMath in an exemplary manner and know what to pay attention to. Using classical ciphers, they understand typical strengths and weaknesses; they master the Diffie-Hellman protocol and are familiar with the man-in-the-middle attack. They can compute discrete logarithms in cyclic groups, understand the RSA scheme, and are able to interpret the recommendations of the Federal Office for Information Security (BSI). In various attack scenarios, they can identify weaknesses of RSA when the requisite conditions are not met. By engaging with numerous open problems in cryptography – whose solution approaches can, perhaps surprisingly, stem from very different areas of mathematics – students practise critical thinking. The exercises are central and support students in working independently and in a practice-oriented way, especially with CAS systems such as SageMath.		

Literature	**Possible References :** • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Christian Karpfinger, Hubert Kiechle: Kryptologie, Algebraische Methoden und Algorithmen, Vieweg 2010. • Dan Boneh, Victor Shoup: A Graduate Course in Applied Cryptography. 2023 (online Version: https://toc.cryptobook.us/). • Jonathan Katz, Yehuda Lindell: Introduction to Modern Cryptography. Chapman and Hall/CRC 2020.
Responsible Persons	Thomas Markwig

Course Title:	Lie Groups		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Manifolds and Lie groups, • Lie algebras and exponential map, • Covering spaces and classification of Lie groups by their Lie algebras, • Classical Lie groups, • Operations of Lie groups and homogeneous spaces.		
Special Objectives	Lie groups lie at the interface between geometry, algebra and analysis. They are suitable for describing the symmetries of geometric objects, but also algebraic equations or solutions of differential equations, in particular if these symmetries form a continuous set. The students learn from a prominent example how different disciplines of mathematics can work together very successfully and how a convincing formalism is developed that can precisely describe a variety of symmetry phenomena.		
Literature	**Possible References :** • Joachim Hilgert, Karl-Hermann Neeb: Liegruppen und Lie-Algebren. Vieweg 1991. • Gerhard P. Hochschild: The structure of Lie groups. Holden-Day 1965. • Frank W. Warner: Foundations of differentiable manifolds and Lie groups. Springer 1983.		
Responsible Persons	Anton Deitmar, Frank Loose		

Course Title:	Linear Control Theory		
Specialisation	Analysis		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	Mathematical methods are indispensable for the management and control of complex systems and processes. The underlying theory is not only fascinating due to its diverse applications, but also, in its abstract form, due to the clarity and elegance of its methods and results. In this lecture, finite-dimensional systems are dealt with first, for which a good knowledge of analysis and linear algebra is sufficient. The aims are Kalman's controllability criterion and the resulting criteria for stabilisability. If there is enough time, we will extend the theory to infinite-dimensional systems. In the exercise classes we will apply the theory to concrete examples.		
Special Objectives	Students have learnt basic methods of linear control theory. At the same time, they have experienced and understood the interaction of various theoretical concepts from linear algebra and analysis and their benefits for specific applications.		
Literature	**Possible References :** • Hans Wilhelm Knobloch, Huibert Kwakernaak: Lineare Kontrolltheorie. Springer 1985. • Jerzy Zabczyk: Mathematical Control Theory. Birkhäuser 1992. • Ruth F. Curtain, Hans Zwart: An Introduction to Infinite-Dimensional Systems Theory. Springer 1995.		
Responsible Persons	Rainer Nagel		

Course Title:	Non-Linear Optimisation		
Specialisation	Scientific Computing		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS		

Content	• Finite-dimensional optimisation, gradient method with Armijo's rule, globalised Newton method. • Restricted optimisation, Farkas' lemma, tangent cone. • Abadie CQ, KKT conditions, Slater conditions. • Linear programme, duality, simplex method. • Penalty and barrier methods, interior point method. • Nonlinear programs, SQP methods, non-smooth optimisation.
Special Objectives	Students master the basic principles and techniques of analysis and numerics of constrained optimisation problems.
Literature	**Possible References :** • Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.
Responsible Persons	Andreas Prohl

Course Title:	Software Obfuscation: Mixed Boolean-Arithmetic Expressions		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• What is software obfuscation and what is it used for? What techniques are there, and why is deobfuscation so difficult? What does a reverse engineer do? Introductory examples. • Mathematical foundations: ideals in $\mathbb{Z}$, greatest common divisor, Euclidean algorithm, time complexity estimates, rings and fields, computations in residue class rings, permutations, matrices and determinants, linear algebra over rings. • Boolean algebras and Boolean functions. • Fundamentals of mixed Boolean-arithmetic expressions (MBA). • Obfuscation using MBAs and permutation polynomials. • Deobfuscation using MBAs; analysis and comparison of state-of-the-art tools and their weaknesses. • Deobfuscation using program synthesis.		
Special Objectives	The students are familiar with initial techniques in the field of software obfuscation and deobfuscation and understand the mathematical results required for this. They are able to apply these confidently.		

Literature	**Possible References :** • Yongxin Zhou, Andrew Main, Yuxin Gu, Harrison Johnson. Information Hiding in Software with Mixed Boolean-Arithmetic Transforms. In: International Workshop on Information Security Applications (WISA). Springer, 2007, pp. 6175. • Bernhard Reichenwallner, Peter Meerwald-Stadler. Ecient Deobfuscation of Linear Mixed Boolean-Arithmetic Expressions. In: Proceedings of the 15th ACM Workshop on Articial Intelligence and Security (AISec 2022). ACM, 2022, pp. 119130.
Responsible Persons	Thomas Markwig

Course Title:	Topology		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	• Review of metric spaces: closed sets, environment, continuity, complete metric spaces, compactness in metric spaces metric spaces. • Set-theoretic topology: topological spaces, continuity convergence, compactness, separation axioms. • Spaces of continuous functions: Urysohn's lemma and applications, Stone-Cech compactification, the theorem of Stone-Weierstraß, notions of convergence in functions, compactness in spaces of functions. • Baire's spaces and application of Baire's theory: Baire's function classes, existence theorems. • Outlook on algebraic topology.		
Special Objectives	Students have familiarised themselves with the central concepts, results and methods of set-theoretical topology and have understood that this theory can be used to describe many phenomena in different areas of mathematics. In this way, they link their knowledge of very different areas of mathematics.		
Literature	**Possible References :** • Felix Hausdorff: Grundzüge der Mengenlehre. Von Veit & Comp. 1914. • Boto von Querenburg: Mengentheoretische Topologie. Springer 2001. • Volker Runde: A Taste of Topology. Springer 2005.		
Responsible Persons	Rainer Nagel		

Course Title:	Calculus of Variations		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Direct method of calculus of variations. • Euler-Lagrange equations. • Palais-Smale condition. • Mountain-Pass Lemma according to Ambrosetti-Rabinowitz.		
Special Objectives	In the first part of the course, students have learnt the direct method of calculus of variations, which is primarily used to prove the existence of weak solutions of partial differential equations, but also has applications in e.g. differential geometry. They have also acquired the necessary basics from functional analysis and partial differential equations and can also use these in a different context, e.g. geometric analysis. In the second part of the course, students learnt about a so-called mountain-pass lemma. With its help, they can analyse non-uniqueness in the existence of solutions of partial differential equations.		
Literature	**Possible References :** • Michael Struwe: Variational Methods, Springer 2008. • David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order, Springer 1998. • Walter Rudin: Functional Analysis, Mc Graw Hill Education 1991.		
Responsible Persons	Reiner Schätzle		

Course Title:	Probability Theory		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Characteristic functions and additions to the central limit theorem. • Conditional expectations and further measure-theoretic foundations. • Markov chains and martingales in discrete time, classification, asymptotic behaviour, stopping times, stationarity, ergodicity. • Introduction to processes in contiuous time like Poisson processes and Brownian motion.
Special Objectives	The students got to know the central terms results and methods of probability theory. They can model, analyse and interprete stochastic dependency structures of random quantities in a measure theoretically founded manner. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.
Literature	**Possible References :** • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Richard Durrett: Probability, Theory and Examples. Cambridge University Press 2010. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Jean Jacod, Philip E. Protter: Probability essentials. Springer 2004. • Olav Kallenberg. Foundations of Modern Probability. Springer 2002. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2013. • David Meintrup, Stefan Schäffler: Stochastik. Springer 2005. • Albert N. Shiryaev: Probability-1. Springer 2016.
Responsible Persons	Martin Möhle, Martin Zerner

Course Title:	Number Theory and Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• RSA cryptosystem, primality tests, AKS algorithm. • Factorisation methods, number field sieve. • Quadratic reciprocity in cryptography. • Evaluation of the discrete logarithm. • Dynamical systems and Pollard's rho algorithm. • Elliptic curve cryptography. • Lattices and post-quantum cryptography. • Zero-knowledge proofs, digital signatures and hash functions.
Special Objectives	The students know the basic concepts of elementary number theory and their applications in cryptography. They have deepened and extended their knowledge about neighbouring disciplines: They encounter methods of the theory of dynamical systems and become acquainted with elliptic curves over finite fields. They understand how fundamental cryptographic protocolls are working. Through studying many open problems of crytography, whose solutions may suprisingly come from most distinct branches of mathematics, the students learn to think critically.
Literature	**Possible References :** • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Stefan Müller-Stach, Jens Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg+Teubner 2011. • Joseph H. Silverman, John T. Tate: Rational points on elliptic curves. Springer 1992. • Nigel Smart: Cryptography: An introduction. McGraw-Hill 2003. (online version: https://www.cs.bris.ac.uk/~nigel/Crypto_Book/). • Lawrence C. Washington: Elliptic curves: Number theory and cryptography. Chaman & Hall/CRC 2008.
Responsible Persons	Elena Klimenko, Thomas Markwig