EBERHARD KARLS UNIVERSITÄT TÜBINGEN

Mathematisch-Naturwissenschaftliche Fakultät

Department of Mathematics

Module Handbook

Mathematics

Master of Science*

Winter Semester 2026

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*Valid for the 2019 study and examination regulations.

Contents

1 Description of the Study Programme

1.1 Programme Concept and Qualification Objectives

The Master of Science Mathematics programme is a research-oriented study programme that prepares students for a career in research and university teaching as well as for a career in business and industry. Graduates are highly capable of familiarising themselves with specific fields of work analysing complex problems and developing solution strategies. They are trained in structured and conceptual thinking and their ability to abstract is particularly well developed. They also have important general qualifications such as creativity, communication, teamwork skills and perseverance.

The degree programme is designed for four semesters and is a consecutive study programme that requires the successful completion of the six-semester Bachelor of Science study programme in mathematics at the University of Tübingen or an equivalent qualification. A specific feature of the Master's degree programme is the combination of mathematical breadth with a simultaneous specialisation in a selected mathematical field. Graduates acquire sound knowledge in several different areas of mathematics and mathematical modelling, and they specialise in a selected area to such an extent that they are able to deal with current research questions. The analytical skills they acquired in the Bachelor's study programme are significantly expanded and they are enabled to carry out scientific work. They learn to critically scrutinise and evaluate theories and to develop their own initial approaches. The study programme enables them to adapt openly and creatively to new conditions in their professional life and to critically classify and apply scientific findings in a targeted manner.

When studying maths, students learn to see the world through mathematical glasses. Recognising mathematical structures in the environment and depicting the environment in simplified mathematical models is an essential skill that is taught in mathematics studies and makes the graduates attractive for a variety of professional fields. Typical employers for our graduates are research and development departments in industry, service companies such as banks or insurance companies, management consultancies, software development companies and opinion research institutes; less obvious fields of activity such as science journalism or museum education are also open to our graduates. In addition, some of them will go into academia at a national or international university or research institution after completing a doctorate.

1.2 Structure of the Study Programme

The courses offered by the Department of Mathematics in the Master of Science Mathematics programme are each assigned to one or more of the following five areas of specialisation, which are naturally derived from the research areas represented in the department:

- Algebra and Geometry,
- Analysis and Differential Geometry,

- Mathematical Physics,
- Numerical Mathematics and Optimisation
- Stochastics.

On beginning their Master's programme, each student selects a personal field of specialisation, in which they collect 18 credit points by attending specialised lectures and associated exercise classes. They attend a seminar and the modules in the Scientific Work section, which all are related to their chosen field. In this area of mathematics, the student is introduced to questions of current research. It is assumed that the student has already acquired basic knowledge in the area of the specialisation during their Bachelor's degree, as provided in the introductory modules of the respective specialisation (see list of introductory modules, p. 20). Therefore, if possible, in the *Study Focus* section, only specialisation modules (see list of specialisation modules, p. 164) and no introductory modules should be included.

The components of the Master's programme are divided into four sections:

- Study Focus,
- Advanced Knowledge in Mathematics,
- Elective Specialisation,
- Scientific Work.

As explained, the sections *Study Focus* and *Scientific Work* summarise the achievements in the student's chosen personal study specialisation. In addition, in the section *Advanced Knowledge in Mathematics* 30-33 credit points in at least two other specialisations in the department must also be achieved, whereby the exact rules for this allow students to enter a new mathematical field, but also ensure the desired depth (see section 2, p. 14). In the remaining section *Elective Specialisation*, students can give their degree programme a very individual profile by incorporating selected achievements from courses in other subject areas and thus expanding the expertise they may have acquired in their Bachelor's degree programme in a minor subject or by deepening their skills in certain areas of mathematics, possibly also in their personal study specialisation, by taking further modules from the Master's degree programme (see section 3, p. 14). In order to ensure that the choice can be made solely from the point of view of defining an individual profile of the degree programme, the grades of the modules in the *Elective Specialisation* are not included in the calculation of the final grade, unlike the grades in the modules of the other three sections.

1.3 Mentoring

The exact rules for the selection of courses in the four sections of the study programme can be found at the beginning of the respective section in chapter 3 on module descriptions. To ensure that students take a well-organised approach to their studies right from the outset and adhere to all rules, each student is assigned a mentor, preferably from their chosen study specialisation, when they start the degree programme. The student meets with this mentor at the beginning of the degree programme in order to draw up a personal study and examination plan that includes all the modules planned for the degree programme. The study and examination plan must be submitted to the head of the examination board for approval; in the case of participation in a double-degree programme (see chapter 4), other persons or committees may be involved in the approval process. In the following semesters, the student meets with their mentor at least once to adjust the study and examination plan. The adjusted study and examination plans must be resubmitted for approval. A change to the chosen personal study specialisation is also possible as part of the change to the study and examination plan upon application.

When creating or adapting the study and examination plan, a suitable time slot can also be planned for a study component at a foreign university. In principle, every semester is suitable for this. The decision will depend on the student's previous achievements and the programme offered at the chosen foreign university.

2 Study Plans

2.1 Overview by Modules

Here we provide an overview of the study plan as a table showing the modules to be taken.

ST	Module Number	Module Title	Type of Course	Type of Module	Course-work	Type of Exam	ECTS-Points
Section 1: Study Focus							
1-3		In addition to the module Seminar Study Focus modules totalling 18 LP in accordance with the more detailed regulations in 3 Module Descriptions		PMW			18
2-3	MAT-40-01	Seminar: Study Focus	S	PMW	s.M.	Pr	3
Section 2: Advanced Knowledge in Mathematics							
1-3		Modules amounting to 30-33 LP according to the more detailed regulations in 3 Module Descriptions		PMW			30-33
Section 3: Elective Specialisation							
1-3		Modules from the degree programmes of the Department of Mathematics or other departments (for more detailed regulations see below. 3 Module Descriptions)		WPM			27-30
Section 4: Scientific Work							
3	MAT-40-02	Introduction to Scientific Work	P	PM	s.M.	-	9
4	MAT-40-03	Master Thesis M.Sc. Mathematics	MT	PM		MA+mP	30

Abbreviations:

Type of Module	: PM=compulsory module, PMW=compulsory module with choice, WPM=elective module
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Course Work	: EC=exercise certificate
Other	: h=hours, o.=or, s.M.=see module description, ST=suggested term

2.2 Overview by the Course of Studies

First we provide a general study plan that shows the distribution of credit points by section. On the following pages you will find examples of study plans for different personal study specialisations.

<table>
<tr><th colspan="5">Schematic Study Plan</th></tr>
<tr><th>FS</th><th>CP</th><th colspan="2">Core Area of Mathematics</th><th>Elective Specialisation</th></tr>
<tr><td>1</td><td>30</td><td rowspan="2">Study Focus
(21 CP)</td><td rowspan="3">Advanced Knowledge
in Mathematics
(30-33 CP)</td><td rowspan="3">Elective Specialisation
(27-30 CP)</td></tr>
<tr><td>2</td><td>30</td></tr>
<tr><td>3</td><td>30</td><td rowspan="2">Scientific Work
(39 CP)</td></tr>
<tr><td>4</td><td>30</td><td colspan="2"></td></tr>
<tr><td colspan="5">Explanation of the Abbreviations:
FS=semester, CP=credit points (ECTS points)</td></tr>
</table>

2.3 Selection of Possible Courses of Studies

Unspecific Exemplary Study Plan

The following exemplary study plan shows how the coursework to be completed could be distributed over the four semesters without naming specific modules.

<table>
<tr><th colspan="6">Unspecific Exemplary Study Plan</th></tr>
<tr><th>FS</th><th>CP</th><th colspan="3">Core Area of Mathematics</th><th>Elective Specialisation</th></tr>
<tr><td>1</td><td>30</td><td>Study Focus:
Advanced Lecture
(9 CP)</td><td>Advanced Knowledge
in Mathematics:
Introductory Lecture
(9 CP)</td><td>Advanced Knowledge
in Mathematics:
Advanced Lecture
(3 CP)</td><td rowspan="3">Free
Elective
Area
(27-33
CP)</td></tr>
<tr><td>2</td><td>30</td><td>Study Focus:
Advanced Lecture
(9 CP)</td><td>Study Focus:
Seminar
(3 CP)</td><td>Advanced Knowledge
in Mathematics:
Advanced Lecture
(9 CP)</td></tr>
<tr><td>3</td><td>30</td><td>Introduction to
Scientific Work
(9 CP)</td><td>Advanced Knowledge
in Mathematics:
Advanced Lecture
(9 CP)</td><td>Advanced Knowledge
in Mathematics:
Advanced Lecture
(3 CP)</td></tr>
<tr><td>4</td><td>30</td><td colspan="4">Master Thesis
(30 CP)</td></tr>
</table>

Exemplary Study Programme Plans for the Personal Study Specialisation

We would now like to list an exemplary study plan for each of the study focusses to show what such a study programme could look like. However, as the choice of modules depends on a large number of factors (previous knowledge from the Bachelor's degree programme, current range of courses, personal interests), these should not be seen as recommendations and and generally cannot be studied in exactly this combination.

Study plan with Personal Study Specialisation Algebra and Geometry					
FS	**CP**	**Core Area of Mathematics**			**Elective Specialisation**
1	33	Algebraic Geometry (9 CP)	Probability Theory (9 CP)	Geometry in Physics (9 CP)	Advanced Module: Written Communication and Translation (B.A. Anglistik) (6 CP)
2	30	Computer Algebra (9 CP)	Study Focus: Seminar (3 CP)	Partial Differential Equations (9 CP)	Algebraic Transformation Groups (9 CP)
3	27	Introduction to Scientific Work (9 CP)	Percolation Theory (3 CP)	Geometric Evolution Equations (3 CP)	Descriptive Linguistics (M.A. English Linguistics) (12 CP)
4	30	Master Thesis (30 CP)			

Study Plan with the Personal Study Specialisation Analysis and Differential Geometry					
FS	**CP**	**Core Area of Mathematics**			**Elective Specialisation**
1	30	Harmonic Analysis on Abelian Groups (9 CP)	Commutative Algebra (9 CP)	Seminar Advanced Knowledge in Mathematics (3 CP)	Nuclear and Particle Physics (B.Sc. Physics) (9 CP)
2	30	Harmonic Analysis on General Groups (9 CP)	Study Focus: Seminar (3 CP)	Algebraic Geometry (9 CP)	Physics of Nanostructures (B.Sc. Physics) (9 CP)
3	30	Introduction to Scientific Work (9 CP)	Mathematical Quantum Theory (9 CP)	Numerics of Stochastic Differential Equations (3 CP)	Mathematical Relativity (9 CP)
4	30	Master Thesis (30 CP)			

Study Plan with the Personal Study Specialisation Mathematical Physics					
FS	**CP**	**Core Area of Mathematics**			**Elective Specialisation**
1	30	Mathematical Quantum Theory (9 CP)	Convex Geometry (9 CP)	Seminar Advanced Knowledge in Mathematics (3 CP)	Condensed Matter (B.Sc. Physics) (9 CP)
2	30	Mathematical Relativity (9 CP)	Study Focus: Seminar (3 CP)	Numerics of Stationary Differential Equations (9 CP)	Classical Field Theory (B.Sc. Physics) (9 CP)
3	30	Introduction to Scientific Work (9 CP)	Numerics of Instationary Differential Equations (9 CP)	Selected Chapters from the Theory of Dynamical Systems (3 CP)	Probability Theory (9 CP)
4	30	Master Thesis (30 CP)			

Study Plan in the Personal Study Specialisation Numerical Mathematics and Optimisation					
FS	**CP**	**Core Area of Mathematics**			**Elective Specialisation**
1	30	Optimisation with Differential Equations (9 CP)	Introduction to the Partial Differential Equations (9 CP)	$SL_2(\mathbb{R})$ (3 CP)	Algorithms (B.Sc. Computer Science) (9 CP)
2	30	Stochastic Differential Equations (9 CP)	Study Focus: Seminar (3 CP)	Algebraic Number Theory (9 CP)	Machine Learning: Algorithms and Theory (M.Sc. Computer Science) (9 CP)
3	30	Introduction to Scientific Work (9 CP)	Mathematical Statistics (9 CP)	Elastic Curves (3 CP)	Modelling and Simulation (M.Sc. Computer Science) (9 CP)
4	30	Master Thesis (30 CP)			

Study Plan with the Personal Study Specialisation Stochastics					
FS	**CP**	**Core Area of Mathematics**			**Elective Specialisation**
1	30	Stochastic Processes (9 CP)	Functional Analysis (9 CP)	$SL_2(\mathbb{R})$ (3 CP)	Introduction to the Partial Differential Equations (9 CP)
2	30	Mathematical Statistics (9 CP)	Study Focus: Seminar (3 CP)	Computer Algebra (9 CP)	Microeconomics (B.Sc. Economics and Business Administration) (9 CP)
3	30	Introduction to Scientific Work (9 CP)	Operator Algebras (9 CP)	Geometric Variation Problems (3 CP)	Microeconomics (B.Sc. Economics and Business Administration) (9 CP)
4	30	Master Thesis (30 CP)			

2.4 Overview by Course of Study and Examination Requirements

Overview by Course of Study and Examination Requirements													
		Exam				**Teaching**				**Semester**			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points(CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
Section 1: Study Focus									**21**				
In addition to the module Seminar Study Focus, modules totalling 18 credit points from the selected Personal Study Specialisation in compliance with the in accordance with the requirements on page 12 must be included. Below, a possible schematic distribution of credit points across the academic semesters is provided.													
First Advanced Module from the Personal Study Specialisation								6	9				
1.	Lecture	Wr. o. Or.	90–120 o. 20–30	g	9	L	o	4		6			
2.	Exercise class					E	o	2		3			
Second Advanced module from the Personal Study Specialisation								6	9				
1.	Lecture	Wr. o. Or.	90–120 o. 20–30	g	9	L	f	4			6		
2.	Exercise class					E	f	2			3		
Seminar Study Focus								2	3				
1.	Seminar	R	45–90 3	S	o	2			3				
Section 2: Advanced Knowledge in Mathematics									**30–33**				
Here, modules amounting to 30–33 credit points can be selected from the list of modules offered by the Department of Mathematics, taking into account the guidelines on page 14 . Below, a possible schematic distribution of credit points across the academic semesters is provided.													
Module from the Introductory Modules selection								6	9				
1.	Lecture	Wr. o. Or.	90–120 o. 20–30	g	9	L	o	4		6			
2.	Exercise class					E	o	2		3			
First specialisation module (not from the Personal Study Specialisation								2	3				
1.	Lecture	Or.	20–30	g	3	L	o	2		3			
Second specialisation module (not from the Personal Study Specialisation)								6	9				
1.	Lecture	Wr. o. Or.	90–120 o. 20–30	g	9	L	o	4			6		
2.	Exercise class					E	o	2			3		
Third specialisation module (not from the Personal Study Specialisation)								6	9				
1.	Lecture	Wr. o. Or.	90–120 o. 20–30	g	9	L	o	4				6	
2.	Exercise class					E	o	2				3	

Overview by Course of Study and Examination Requirements													
		Exam				**Teaching**				**Semester**			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points(CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module. 1. CP	2. CP	3. CP	4. CP
Fourth specialisation module (not from the Personal Study Specialisation)								2	3				
1.	Lecture	Or.	20–30	g	3	L	o	2				3	
Section 3: Elective Specialisation									**27–30**				
Here you can choose modules totalling 27–30 credit points from the list of modules offered by the Department of Mathematics and other departments in accordance with the restrictive requirements on page 14.													
Section 4: Scientific Work									**39**				
Introduction to Scientific Work									9				
1.	Project	Proj.		ng	9		o					9	
Final Module									30				
1.	Master Thesis	MA + Or.		g	30		o						30

Explanation of the Abbreviations:

Marking system : g=graded, ng=non graded
Form of examination : MA=master thesis, Or.=oral exam, Wr.=written exam, Pres=presentation, TP=term paper
Form of teaching : L=lecture, SL=Seminar or lecture, E=exercise class, r=revision course, P=practical course, PS=introductory seminar, S=seminar
Status : o=obligatory, f=facultative
Other : o.=or, SWS=hours in class per week, CP=credit points=ECTS Points

3 Module Descriptions

Section 1: Study Focus

In the section *Study Focus*, a total of 21 credit points must be earned. Of these, 3 credit points are allocated to the module *Seminar Study Focus*. Within this module, a seminar must be attended that is related to the personal study specialisation.

The remaining 18 credit points are to be earned through modules involving lectures with or without exercises, concluding with oral or written examinations. The modules must be related to the chosen personal study specialisation (see module description), and should preferably be specialisation modules No further seminars or modules that conclude without examination can be included.

<table>
<tr><td>Module Number:
MAT-40-01</td><td colspan="2">Module Title:
Seminar: Study Focus</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">3</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
90 h</td><td>Time in Class:
30 h</td><td>Self-Study:
60 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">every Semester</td></tr>
<tr><td>Term</td><td colspan="3">2-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Seminar, talk, presentation, e-learning, blended learning</td></tr>
<tr><td>Content</td><td colspan="3">Various topics from the chosen Personal Study Specialisation.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students work independently on in-depth questions from the area of their Personal Study Specialisation and prepare them in a didactically appealing and scientifically sound form. They train their presentation techniques and sharpen their professional discussion style. Depending on the topic chosen, they are introduced to current research questions and prepared for their Master's thesis.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3"><table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td>Seminar</td><td>S</td><td>o</td><td>2</td><td>3</td><td>yes</td><td>Pr</td><td>60-90</td><td>g</td><td>100</td></tr>
</table>
In addition to a successful talk, the acquisition of credit points also requires a regular active participation in the course, for example in the form of questions and contributions to the discussion or by completing assignments. In addition, a written elaboration of one's own presentation or the preparation of a handout for the participants may be part of the work to be done. This additional work constitutes the coursework for the module.</td></tr>
<tr><td>Transfer</td><td colspan="3">The successful completion of the module is a prerequisite for the module Master Thesis M.Sc. Mathematik.</td></tr>
</table>

Prerequisites	Participation in the module requires the successful acquisition of at least 9 ECTS points from modules in the Study Specialisation.
Responsible Persons	The dean of studies at the Department of Mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 2: Advanced Knowledge in Mathematics

In the section *Advanced Knowledge in Mathematics*, credit points ranging from 30 to 33 must be earned through modules from the lists of introductory or specialisation modules. The following requirements must be observed:

- The modules must be assignable to areas other than the personal study specialisation (see module description);
- At least 27 credit points must be earned through modules involving lectures with or without exercises, concluding with an exam;
- Modules worth at least 9 credit points each must be included from at least two specialisations, which are different from both each other and from the chosen personal study specialisation.

These requirements ensure that the programme profile reflects both the necessary breadth in terms of the mathematical areas studied and the necessary depth within each area studied. In addition, this approach allows students to design their studies flexibly and individually, while also enabling the department to continually adjust to current developments in mathematics by offering new modules.

Section 3: Elective Specialisation

In the section *Elective Specialisation*, credit points ranging from 27 to 30 must be earned. Upon request, modules from all degree programmes of the University of Tübingen, including the Master of Science in Mathematics programme, can be included. When designing the study and examination plan, it is essential to ensure that the planned choices contribute to a meaningful overall profile of the programme. Additionally, the following notes must be observed:

- At least 9 credit points should come from modules offered by departments other than mathematics;
- Modules from a maximum of two other departments may be included; exceptions can be approved by the examination board in justified cases;
- The modules from other departments should build on those included in the Bachelor of Science;
- Modules from the Master of Science in Mathematics programme can also be included from the list of modules for Mathematical General Education and from the chosen personal study specialisation;
- The grades of graded modules in the Elective Specialisation are not included in the calculation of the final grade.

These requirements are intended to ensure that students, throughout their combined studies, comprising both the Bachelor and Master of Science, acquire competencies beyond their core subject of mathematics in other scientific fields and potential applications of mathematics, while also allowing them to set meaningful focal points. Depending on individual objectives, students should also have the opportunity to expand their intra-mathematical knowledge and make up for any missing prerequisites in the chosen personal study specialisation from the Bachelor's programme, which may arise, for example, from transferring from another university, during the Master's programme.

Which modules are offered in other degree programmes can generally be found in the module handbook of the respective programme, along with the information related to the specific module. If modules from subjects that have not yet been studied as part of the Bachelor's programme are selected, the module handbook for the Bachelor of Science in Mathematics provides recommendations for the most frequently chosen subjects, indicating which modules may be suitable as an introduction to the respective field. Further information can be obtained from the Faculty Course Advisors of the respective programmes; see also:

https://www.uni-tuebingen.de/studium/beratung-und-info/studienfachberatung.html

The following is a list of some of the modules offered by the Department of Mathematics that can only be taken in this section.

Module Number: MAT-00-14	**Module Title:** Science Communication in STEM Fields		**Type of Module:** Elective Module
ECTS-Points	1		
Workload - Time in Class - Self-Study	Workload: 30 h	Time in Class: 15 h	Self-Study: 15 h

Duration	1 Semester
Frequency	not regularly
Term	3-4
Language of Instruction	English
Forms of Teaching and Learning	Presentation, group work, blended learning, practical exercises
Content	• Introduction to vocational fields in science communication. • Practical training in science communication.
Objectives	The students know the current situation of science communication and different career paths in this field. They are familiar with the challenges of different formats and media (print media, museum pedagogy, interactive formats, etc.). Moreover, the students have learned about the practical aspects which are relevant for science communicators, like free licensing and employment arrangements. They have trained science communication skills that are relevant to different science communication formats by means of oral and written practical exercises.
Requirements for obtaining Credits / Grading (Weighting if applicable)	see table below

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Science communication in STEM Ffields	S	f	1	1	yes	none	-	ng	-

Within the module a coursework in the form of active participation and a written essay on a project is required. There is no exam.

Literature	**Possible References :** • Sam Illingworth, Grant Allen: Effective science communication IOP Publishing 2016. • Beatrice Dernbach, Christian Kleinert, Herbert Münder: Handbuch Wissenschaftskommunikation. Springer 2013.
Transfer	The module can be assigned to the Section *Elective Specialisation*.
Prerequisites	There are no prerequisites.
Responsible Persons	Carla Cederbaum

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 4: Scientific Work

The section *Scientific Work* particularly prepares students for academic work. It contains two modules.

In the module *Introduction to Scientific Work*, students are introduced to the thematic environment of their master's thesis. They become familiar with the essential principles of independent literature research in a scientific context and learn the main aspects of writing scientific mathematical texts. The project work and the reading course, which involves guided reading of scientific texts, are essential teaching and learning forms. The *Master Thesis M.Sc. Mathematics* module provides the framework for preparing the master's thesis and concludes with it as an assessment. Topics from the chosen personal study specialisation must be addressed in both modules.

Module Number: MAT-40-02	**Module Title:** Introduction to Scientific Work		**Type of Module:** Compulsory Module
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 30 h	Self-Study: 240 h
Duration	1 Semester		
Frequency	every Semester		
Term	3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Individual supervision by a mentor, study of scientific works.		
Content	• Definition of an advanced scientific project in coordination with the mentor. • Independent search and study of the relevant scientific literature. • Formulation of specific problems and methodical approach to their solution. • Written presentation of the project in conext of current state of research on 5-10 pages. This module serves generally as a preparation for the Master Thesis		
Objectives	Students • develop skills to systematically familiarise themselves with a new subject, • learn to work critically and to form a substantiated, professional and interdisciplinary judgement, • acquire qualifications in such areas as literature research, identification of relevant problems and appropriate methods, as well as in the written presentation of a research proposal.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Scientific Project	P	o	1	9	yes	-	-	ng	-

Transfer	Successful completion of this module is a prerequisite for participation in module Master Thesis.
Prerequisites	Successful completion of modules worth at least 30 ECTS in the Sections Specialisation and Advanced Knowledge in Mathematics.

Responsible Persons	The dean of studies at the Department of Mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-40-03	**Module Title:** Master Thesis M.Sc. Mathematics		**Type of Module:** Compulsory Module
ECTS-Points	30		
Workload - Time in Class - Self-Study	Workload: 900 h	Time in Class: regularly, as required	Self-Study: 900 h
Duration	1 Semester		
Frequency	every Semester		
Term	4		
Language of Instruction	German or English		
Forms of Teaching and Learning	Master thesis		
Content	The master thesis brings the master studies to a close. The students have to work under instruction of an advisor on a defined task from mathematics, which will lead up to the current state of research, with scientific methods and present the results in written form. In detail this includes: • the formulation of a scientific question in accordance with the advisor; • the independent search for and the study of relevant scientific literature; • the formulation of suited questions and methodical approaches for their solution; • the independent realisation of the project, the written presentation of the project and the results in the context of the current state of research. The results shall contribute to scientific knowledge. In the master thesis, a question should be be worked on, which belongs to the mathematical area of the chosen personal specialisation.		
Objectives	The students • are capable of familiarising themselves with a problem, which will lead up to current research topics, till a deadline and to independently develop a solution strategy, • can use suited scientific methods with growing independence and can present the results in a scientifically appropriate form, • can independently work on a scientific topic and are capable of using their mathemtical methodology knowledge, • enhance their problem solving expertise and can transfer their methodology knowledge, • are able to present the results of ther projects to a professional audience.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see below		
Transfer	-		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Master Thesis	MT	o	-	30	no	MA+mP	-	g	100

The oral examination is assessed on a pass/fail basis only. The module grade is the grade of the master thesis. The module is only deemed passed if both assessments have been passed. The oral examination covers the contents of the master thesis.

Prerequisites	Subject specific prerequisite for admission to the module Master Thesis is besides the general part of the examination regulations the successful completion of the module Introduction to Scientific Work and of the modules in the specialisation as well as the aquisiton of at least further 30 credit poins.
Responsible Persons	The dean of studies at the Department of Mathematics

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Modules of Type Introductory modules

The *introductory modules* are predominantly introductory courses in various advanced areas of mathematics. Many of these areas can be further specialised within the offered specialisation modules. The courses underlying these modules can also be included in the modules of the third year of the Bachelor of Science in Mathematics again. If the courses have already been included in the framework of a module in the Bachelor of Science in Mathematics, the module cannot be included in the Master of Science in Mathematics. The module descriptions indicate which study specialisations the respective module is assigned to; the assignment may depend on taking additional modules.

The module descriptions on the following pages are sorted by the module numbers. For clarity, we first provide a list of modules sorted by their title in alphabetical order and then sorted by whether they are offered regularly or irregularly. Subsequently, there are listings of the modules based on their inclusion in the study specialisations.

Modules which are offered regularly (at least once every two years)

Modules which are not offered regularly

Modules in the Specialisation Algebra and Geometry

The following modules belong to the specialisation Algebra and Geometry. Possible restrictions can be found in the module description.

Modules in the Specialisation Analysis and Differential Geometry

The following modules belong to the specialisation Analysis and Differential Geometry. Possible restrictions can be found in the module description.

Modules in the Specialisation Mathematical Physics

The following modules belong to the specialisation Mathematical Physics. Possible restrictions can be found in the module description.

Modules in the Specialisation Numerial Mathematics and Optimisation

The following modules belong to the specialisation Numerial Mathematics and Optimisation. Possible restrictions can be found in the module description.

Modules in the Specialisation Stochastics

The following modules belong to the specialisation Stochastics. Possible restrictions can be found in the module description.

Modul descriptions (Introductory modules)

Module Number: **MAT-30-30**	**Module Title:** Seminar: Mathematical Subjects		**Type of Module:** Compulsory Module
ECTS-Points	3		
Workload **- Time in Class** **- Self-Study**	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	every Semester		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Seminar, exercise classes, talk, practical course, self-study, group work		
Content	Various topics from the advanced fields of mathematics.		
Objectives	The students independently explore a coherent topic in mathematics and prepare it in a didactically appealing form. They learn how to present their work to a group, respond to questions regarding the content, and conduct a professional discussion. Additionally, they acquire knowledge about a technically sophisticated writing programme and use it to create a digital medium, which can later be utilised for teaching or learning purposes.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Seminar	S	o	2	3	yes	Pr	60-90	g	100

In addition to a successful talk, the acquisition of credit points also requires a regular active participation in the course, for example in the form of questions and contributions to the discussion or by completing assignments. In addition, a written elaboration of one's own presentation or the preparation of a handout for the participants may be part of the work to be done. This additional work constitutes the coursework for the module.

Transfer	-
Prerequisites	There are no further prerequisites.
Responsible Persons	The dean of studies at the Department of Mathematics

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-01	**Module Title:** Introduction to Commutative Algebra and Algebraic Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Rings and ideals. • Gröbner bases. • Localization. • Noetherian rings and modules. • Integral ring extensions. • Krull's principal ideal theorem and dimension theory. • Hilbert's Nullstellensatz and Noether normalisation. • Affine varieties, Zariski topology, morphisms.		
Objectives	The students have become familiar with the central concepts, results, and methods of commutative algebra and affine algebraic geometry. They have experienced the profound interplay between algebra and geometry through the example of affine varieties. Furthermore, the students understand how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Commutative Algebra and Algebraic Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Commutative Algebra' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Responsible Persons	Daniele Agostini, Jürgen Hausen

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-02	**Module Title:** Commutative Algebra		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Rings and Ideals. • Localisation and local rings. • Noetherian and Artinian rings and modules. • Integral ring extensions and Cohen-Seidenberg theorems. • Krull's principal ideal theorem and dimension theory. • Primary decomposition. • Normality, regularity and discrete valuation rings. • Hilbert's Nullstellensatz and Noether normalisation.		
Objectives	The students are familiar with and understand the language and methods of commutative algebra, which are essential for studying the fields of algebra, geometry, and number theory. They recognise how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Commutative Algebra	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Introduction to Commutative Algebra and Algebraic Geometry' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Responsible Persons	Daniele Agostini, Thomas Markwig

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-45-20**	**Module Title:** Introduction to Berkovich Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Non-Archimedean fields, valuations, and absolute value functions. • Ultrametric triangle inequality and induced topology. • Affinoid domains. • Berkovich affine and projective line. • Analytification of algebraic varieties.		
Objectives	The students have become familiar with the most important examples of non-Archimedean fields and their induced topology. They have gained an understanding of the fundamental challenges in developing a theory of analytic geometry over these fields and have studied Berkovich's approach to addressing these issues. The students have examined the projective line in Berkovich's framework in detail, both set-theoretically and topologically. In doing so, they have encountered a type of geometric space fundamentally different from other examples encountered in their studies (such as vector spaces, varieties, or manifolds). They are also familiar with the connections to algebraic geometry through the analytification functor. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see table below		
Literature	**Possible References :** • Annette Werner: Nichtarchimedische Geometrie. Vorlesungsskript.		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Berkovich Geometry	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Responsible Persons	Hannah Markwig

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-45-21</td><td colspan="2">Module Title:
Algebraic Number Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Rings of integers.
Class numbers.
Dirichlet’s unit theorem.
Extension of Dedekind rings.
Valuation theory.
Local fields.
Adeles and ideles.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have learned the central terms, results and methods of algebraic number theory. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Algebraic Number Theory</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
Jürgen Neukirch: Algebraische Zahlentheorie. Springer 2007.
Alexander Schmidt: Einführung in die algebraische Zahlentheorie. Springer 2007.
Andre Weil: Basic number theory. Springer 1995.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Responsible Persons	Daniele Agostini, Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-22	**Module Title:** Number Theory and Cryptography		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• RSA cryptosystem, primality tests, AKS algorithm. • Factorisation methods, number field sieve. • Quadratic reciprocity in cryptography. • Evaluation of the discrete logarithm. • Dynamical systems and Pollard's rho algorithm. • Elliptic curve cryptography. • Lattices and post-quantum cryptography. • Zero-knowledge proofs, digital signatures and hash functions.		
Objectives	The students know the basic concepts of elementary number theory and their applications in cryptography. They have deepened and extended their knowledge about neighbouring disciplines: They encounter methods of the theory of dynamical systems and become acquainted with elliptic curves over finite fields. They understand how fundamental cryptographic protocolls are working. Through studying many open problems of crytography, whose solutions may suprisingly come from most distinct branches of mathematics, the students learn to think critically. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see below		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Number Theory and Cryptography	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Stefan Müller-Stach, Jens Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg+Teubner 2011. • Joseph H. Silverman, John T. Tate: Rational points on elliptic curves. Springer 1992. • Nigel Smart: Cryptography: An introduction. McGraw-Hill 2003. (online version: https://www.cs.bris.ac.uk/~nigel/Crypto_Book/). • Lawrence C. Washington: Elliptic curves: Number theory and cryptography. Chaman & Hall/CRC 2008.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Elliptic Curves and Cryptography' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Thomas Markwig

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Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-45-23</td><td colspan="2">Module Title:
Cryptography</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">4.5</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
150 h</td><td>Time in Class:
45 h</td><td>Self-Study:
105 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 1 SWS</td></tr>
<tr><td>Content</td><td colspan="3">
• Brief review of key concepts and results from algebra and number theory.
• Historical ciphers and their cryptanalysis (Caesar, Vigenere, substitution); encryption schemes.
• Diffie-Hellman protocol and fast exponentiation.
• Discrete logarithms: Shanks’ algorithm and Pollard’s rho method.
• RSA: correctness, security, and attacks.
• Signature schemes.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students are familiar with the fundamental concepts and results of elementary number theory and algebra, as well as their application in cryptography. They can implement the methods covered in Python or SageMath in an exemplary manner and know what to pay attention to. Using classical ciphers, they understand typical strengths and weaknesses; they master the Diffie-Hellman protocol and are familiar with the man-in-the-middle attack. They can compute discrete logarithms in cyclic groups, understand the RSA scheme, and are able to interpret the recommendations of the Federal Office for Information Security (BSI). In various attack scenarios, they can identify weaknesses of RSA when the requisite conditions are not met. By engaging with numerous open problems in cryptography – whose solution approaches can, perhaps surprisingly, stem from very different areas of mathematics – students practise critical thinking. The exercises are central and support students in working independently and in a practice-oriented way, especially with CAS systems such as SageMath. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Cryptography</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">or.</td><td rowspan="2">20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>1</td><td>1,5</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired.</td></tr>
</table>

Literature	**Possible References :** • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Christian Karpfinger, Hubert Kiechle: Kryptologie, Algebraische Methoden und Algorithmen, Vieweg 2010. • Dan Boneh, Victor Shoup: A Graduate Course in Applied Cryptography. 2023 (online Version: https://toc.cryptobook.us/). • Jonathan Katz, Yehuda Lindell: Introduction to Modern Cryptography. Chapman and Hall/CRC 2020.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Number Theory and Cryptography' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Thomas Markwig

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-25	**Module Title:** Elementary Number Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Divisibility in the integers. • Prime numbers. • Congruences. • Quadratic residues. • Arithmetic functions. • Multiplicative functions. • Classical theorems. • Applications.		
Objectives	Students deepen their basic knowledge of integers and experience applying this knowledge to mathematical problems of various kinds. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Elementary Number Theory	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Friedhelm Padberg: Elementare Zahlentheorie. Spektrum Akademischer Verlag 2001. • Stefan Mueller-Stach, J. Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg 2006.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Thomas Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-26	**Module Title:** Introduction to Analytic Number Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Arithmetic functions and Dirichlet series, • Prime number theorem and Dirichlet's prime number theorem, • Zeros of the Riemannian zeta function, Riemann hypothesis and the explicit formula.		
Objectives	The students understand the interplay between analysis and number theory. They can apply analytical methods to number theoretic problems. They understand the mechanism of analytical continuation through integral representation and have learned to independently transfer it to other cases, such as automorphic L-functions. They have gained an understanding of the Riemann hypothesis, which is considered the most difficult problem of all math, and understand its depth. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see table below		
Literature	**Possible References :** • Komaravolu Chandrasekharan: Introduction to Analytic Number Theory. Springer 1968.		
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.		

Requirements for obtaining Credits / Grading (Weighting if applicable):

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Analytic Number Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-27	**Module Title:** Elliptic Curves and Cryptography		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Basic concepts of cryptography. • Symmetric cryptosystems, public key systems, discrete logarithm, RSA. • Factorisation into primes, attacks on cryptosystems. • Basic concepts of plane projective geometry. • Elliptic curves as Abelian groups. • Curves over finite fields, Frobenius morphism, endomorphism ring. • Counting points, Hasse bound, Weil conjectures, Schoof's algorithm. • Cryptosystems on elliptic curves, algorithms and attacks.		
Objectives	Students are familiar with the basic tasks and concepts of cryptography. They understand the cryptographically motivated questions relating to elliptic curves and have an insight into the advanced algebraic and geometric techniques for answering them. They understand the challenges of algorithmic implementation and are familiar with standard algorithms. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Elliptic Curves and Cryptography	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Albrecht Beutelspacher, Jörg Schwenk, Klaus-Dieter Wolfenstetter: Moderne Verfahren in der Kryptographie. Springer 2015. • Joseph H. Silverman: The arithmetic of elliptic curves. Springer 2009. • Ian Blake, Gadiel Seroussi, Nigel Smart: Elliptic curves in cryptography. CUP 1999.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the modules 'Number Theory and Cryptography' due to the large overlap in content
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory, • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.
Responsible Persons	Thomas Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**	**Type of Module:**
MAT-45-30	Software Obfuscation: Mixed Boolean-Arithmetic Expressions	Compulsory Module with Choice

ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		

Content

- What is software obfuscation and what is it used for? What techniques are there, and why is deobfuscation so difficult? What does a reverse engineer do? Introductory examples.
- Mathematical foundations: ideals in $\mathbb{Z}$, greatest common divisor, Euclidean algorithm, time complexity estimates, rings and fields, computations in residue class rings, permutations, matrices and determinants, linear algebra over rings.
- Boolean algebras and Boolean functions.
- Fundamentals of mixed Boolean-arithmetic expressions (MBA).
- Obfuscation using MBAs and permutation polynomials.
- Deobfuscation using MBAs; analysis and comparison of state-of-the-art tools and their weaknesses.
- Deobfuscation using program synthesis.

Objectives

The students are familiar with initial techniques in the field of software obfuscation and deobfuscation and understand the mathematical results required for this. They are able to apply these confidently. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Software Obfuscation: Mixed Boolean-Arithmetic Expressions	L	f	2	3	yes	or.	20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired.

Literature	**Possible References :** • Yongxin Zhou, Andrew Main, Yuxin Gu, Harrison Johnson. Information Hiding in Software with Mixed Boolean-Arithmetic Transforms. In: International Workshop on Information Security Applications (WISA). Springer, 2007, pp. 6175. • Bernhard Reichenwallner, Peter Meerwald-Stadler. Ecient Deobfuscation of Linear Mixed Boolean-Arithmetic Expressions. In: Proceedings of the 15th ACM Workshop on Articial Intelligence and Security (AISec 2022). ACM, 2022, pp. 119130.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowlegde from Linear Algebra and the submodule Algebraic Structures are assumed.
Responsible Persons	Thomas Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-31	**Module Title:** Representation Theory of Finite Groups		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Groups and group actions. • Representations, irreducibility, Schursch's lemma. • Semisimplicity, Maschke's theorem. • Characters, orthogonality relations. • Isotypical decomposition, character tables. • Representations of the symmetric group. • Semi-simple Artinian algebras.		
Objectives	In the lecture, students learn the basic principles of of representation theory and develop an understanding for the interaction of geometric and algebraic methods. methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Representation Theory of Finite Groups	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • William Fulton, Joe Harris: Representation theory. Springer 1991. • Bertram Huppert: Character theory of finite groups. De Gruyter 1998. • Serge Lang: Algebra. Springer 2002. • Jean-Pierre Serre: Linear representations of finite groups. Springer 1977.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. It can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Group Representations in Physics' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Jürgen Hausen, Thomas Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-45-32**	**Module Title:** Problem Solving Strategies		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Basic Techniques and Problem Reduction. • Proofs and Logic. • Sets and Functions. • Elementary Combinatorics. • Counting Techniques and Cardinality. • The Pigeonhole Principle (Box Principle). • Bounds, Estimation, and Inequalities. • The Extremal Principle. • The Invariance Principle: Invariants and Monovariants. • Proofs by Colouring. • Graph-Theoretic Modelling.		
Objectives	Students acquire a versatile toolkit of mathematical and heuristic problem-solving strategies that they can draw upon both in their subsequent studies and, where appropriate, in competition-oriented mathematical problem solving. This includes fundamental results and proof techniques, as well as the ability to recognise characteristic structural features of a problem that indicate suitable solution approaches or allow unsuitable strategies to be ruled out at an early stage. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Problem Solving Strategies	L	f	2	3	yes	or.	20-30	g	100
		E	f	1	1,5					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired.									
Literature	**Possible References :** • George Pólya: How to Solve It: A New Aspect of Mathematical Method. Princeton University Press 1973. • Arthur Engel: Problem-Solving Strategies. Springer 1998.									
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.									
Prerequisites	Knowlegde from Linear Algebra and the submodule Algebraic Structures are assumed.									
Responsible Persons	Thomas Markwig									

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-02	**Module Title:** Convex Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Cones, polytopes, polyhedra, fans, polyedral complexes. • Normal fans of polygons. • Triangulations, subdivisions, secondary fans, discriminants.		
Objectives	In the lecture the students learn basic terms, results and methods of convex geometry. They develope a deepened understanding for the concept of duality of mathematical objects on the example of polytopes and fans. Furthermore they enhance their geometric view and their spatial sense. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Convex Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Günter M. Ziegler: Lectures on Polytopes. Springer 1998.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-04	**Module Title:** Tropical Enumerative Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Enumerative geometry of algebraic curves, in particular in the plane. • Tropical enumerative problems and multiplicities. • Combinatorial methods, floor diagrams and lattice paths. • Correspondence theorems for curves in the plane through given points. • Tropical and classic Gromov-Witten theory in genus 0. • Real counts, Welschinger invariants and polynomial invariants. • Hurwitz numbers. • Tropical correspondences for Hurwitz numbers.		
Objectives	The students know basic terms, results and methods of enumerative geometry in the context of tropical geometry methods. They develop a deeper understanding of the possibilities and limitations of the tropical access in connection with more complex issues. Furthermore, they deepen their knowledge in the field of algebraic geometry towards modular spaces and Gromov-Witten theory. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see table below		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Tropical Enumerative Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-05	**Module Title:** Tropical Enumerative Geometry - Part 1		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Enumerative geometry of algebraic curves, in particular in the plane. • Tropical enumerative problems and multiplicities. • Combinatorial methods, floor diagrams and lattice paths. • Correspondence theorems for curves in the plane through given points. • Real counts, Welschinger invariants and polynomial invariants.		
Objectives	The students know basic terms, results and methods of enumerative geometry in the context of tropical geometry methods. They develop a deeper understanding of the possibilities and limitations of the tropical access in connection with more complex issues. Furthermore, they deepen their knowledge in the field of algebraic geometry towards moduli spaces. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Tropical Enumerative Geometry - Part 1	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is conceptually part of the module Tropical Enumerative Geometry and cannot be counted alongside it.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-10	**Module Title:** Geometry of Manifolds 1		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Manifolds and submanifolds. • Vector fields and flows. • Metrics, foundations of Riemannian geometry. • Complex structures. • Theorem of Gauß-Bonnet on surfaces.		
Objectives	The students know and understand the fundamental concepts of real and complex differential geometry and the basic techniques for handling them. Especially they have deepened their understanding of differential and integral calculus and have exemplarily experienced how mathematical concepts are used in a natural way in geometry. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Geometry of Manifolds 1	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. Due to a significant overlap in contents the module cannot be taken together with either of the modules 'Geometry in Physics' and 'Introduction to Differential Geometry'.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Christoph Bohle, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-50-12</td><td colspan="2">Module Title:
Information Geometry</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">3</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
90 h</td><td>Time in Class:
30 h</td><td>Self-Study:
60 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Foundations of Information Geometry (e.g. Fisher information metrics and dual relationships for parametric statistical models, Kullback-Leibler divergence, natural gradient).
Application to neural data processing (in particular supervised learning in artificial neural networks).</td></tr>
<tr><td>Objectives</td><td colspan="3">Students have an elementary understanding of how to apply concepts of differential geometry to problems in information theory and statistics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3"><table><tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr><tr><td>Information Geometry</td><td>L</td><td>f</td><td>2</td><td>3</td><td>no</td><td>wr. o. or.</td><td>90-180
o. 20-30</td><td>g</td><td>100</td></tr></table>Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
Shun-Ichi Amari, Hiroshi Nagaoka: Methods of Information Geometry. AMS 2001.
Anthony C. C. Coolen, Reimer Kuehn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005.
Shun-Ichi Amari: Natural Gradient works Efficiently in Learning. Neural Computation 1998.
Yann Ollivier: Riemannian Metrics for Neural Networks I - Feedforward Networks. Information and Inference, IMA 2015.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisations Algebra and Geometry, Analysis and Differential Geometry and Stochastics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.</td></tr>
</table>

Responsible Persons	Christoph Bohle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-50-13</td><td colspan="2">Module Title:
Mathematical Aspects of 3d Computer Vision</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Foundations of projective geometry.
Visual odometry.
Classical Simultaneous Localisation and Mapping (SLAM) methods and deep learning variants.
Computer-based experiments using the Robot Operating System (ROS2).</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have acquired the mathematical foundations of 3D computer vision. They are familiar with modern architectures and the mathematical questions arising from them. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Mathematical Aspects of 3d Computer Vision</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o.
or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 5.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
Richard Hartley, Andrew Zisserman: Multiple View Geometry in Computer Vision. Cambridge University Press 2003.
Sebastian Thrun, Wolfram Burgard, Dieter Fox: Probabilistic Robotics. MIT Press 2005.
Luca Carlone, Ayoung Kim, Timothy D. Barfoot, Daniel Cremers, Frank Dellaert: SLAM Handbook: From Localization and Mapping to Spatial Intelligence. Cambridge University Press 2026.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry*, *Analysis and Differential Geometry* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics. In addition, prior knowledge of deep learning is required, such as that provided in module MAT-50-14.
Responsible Persons	Christoph Bohle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-50-14</td><td colspan="2">Module Title:
Mathematical Aspects of Deep Learning 1</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">3</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
90 h</td><td>Time in Class:
30 h</td><td>Self-Study:
60 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">
Artificial neural networks and their training using backpropagation/stochastic gradient descent. Probabilistic interpretation. Mathematical problems arising in neural network science.
Classical architectures for computer vision (CNNs) and sequential data (RNNs). (Self-)supervised learning methods.
Transformer models, a modern architecture based on the attention mechanism, and its applications in natural language processing (GPT) and computer vision.
Simple neuroscience models for neural networks. Biological plausibility of backpropagation and transformers.
</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have learned the fundamentals of information processing using artificial neural networks and biologically more plausible alternatives. They are familiar with contemporary architectures and mathematical questions arising in the framework. The students can name and prove the central results of the lecture and they can explain their intrinsic connections.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td>Mathematical Aspects of Deep Learning 1</td><td>L</td><td>f</td><td>2</td><td>3</td><td>no</td><td>wr. o. or.</td><td>90-180
o. 20-30</td><td>g</td><td>100</td></tr>
</table>
Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
Ian Goodfellow, Yoshua Bengio, Aaron Courville: Deep Learning. MIT 2016.
Simon J. D. Price: Understanding Deep Learning. MIT Press 2023.
Christopher M. Bishop, Hugh Bishop: Deep Learning - Foundations and Concepts. Springer 2024.
</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisations Algebra and Geometry, Analysis and Differential Geometry und Stochastik. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
</table>

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.
Responsible Persons	Christoph Bohle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-50-15</td><td colspan="2">Module Title:
Introduction to Riemann Surfaces</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">4.5</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
150 h</td><td>Time in Class:
45 h</td><td>Self-Study:
105 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 1 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Coverings and fundamental groups.
Topological classification of the surfaces.
Theorem of Riemann-Hurwitz.
Differential forms and integration.
Sheaves and cohomology.
Theorem of Riemann-Roch.
Serre duality.
Kobayashi metric.
Theorem of Picard.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. Using the sheaf concept, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Introduction to Riemann Surfaces</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>1</td><td>1,5</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 5.</td></tr>
</table>

Literature	**Possible References :** • Hershel M. Farkas, Irwin Kra: Riemann Surfaces. Springer 1992. • Otto Forster: Riemannsche Flächen. Springer 1977. • Klaus Lamotke: Riemannsche Flächen. Springer 2009. • Jürgen Jost: Compact Riemann surfaces. Springer 2006.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Riemann surfaces' due to the large overlap in content
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-20	Module Title: Topology		Type of Module: Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Review of metric spaces: closed sets, environment, continuity, complete metric spaces, compactness in metric spaces metric spaces. • Set-theoretic topology: topological spaces, continuity convergence, compactness, separation axioms. • Spaces of continuous functions: Urysohn's lemma and applications, Stone-Cech compactification, the theorem of Stone-Weierstraß, notions of convergence in functions, compactness in spaces of functions. • Baire's spaces and application of Baire's theory: Baire's function classes, existence theorems. • Outlook on algebraic topology.		
Objectives	Students have familiarised themselves with the central concepts, results and methods of set-theoretical topology and have understood that this theory can be used to describe many phenomena in different areas of mathematics. In this way, they link their knowledge of very different areas of mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Topology	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Felix Hausdorff: Grundzüge der Mengenlehre. Von Veit & Comp. 1914. • Boto von Querenburg: Mengentheoretische Topologie. Springer 2001. • Volker Runde: A Taste of Topology. Springer 2005.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Ulrich Groh, Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-21	**Module Title:** Algebraic Topology 1		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Set theoretical topology. • Basic concepts of category theory. • The fundamental group of a punctured topological space. • Theory of covering spaces. • Basic concepts of singular homology theory. • Applications.		
Objectives	The students learn how to realise ideas in topology, e.g. the detection of holes in topological spaces, into a precise theory, even with a sophisticated technique. In particular, they recognise how abstract concepts, e.g. from category theory and homological algebra, provide effective ways of speaking that enable the formation of ideas to be adequately implemented. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Topology	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Anton Deitmar, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-29	**Module Title:** Algebraic Curves and Riemann Surfaces		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Compact Riemann surfaces. • Normalisation of plane curves. • Topological genus. • Coverings. • Forms and integration. • Sheaves and cohomology. • Hodge theory. • Arithmetic and geometric genus. • Abel's theorem. • Riemann-Roch theorem. • Serre duality. • Jacobian and Abelian varieties. • Riemann bilinear relations. • Jacobi inverse problem. • Elliptic curves and functions. • j-Invariant. • Uniformisation. • Topology of non-compact Riemann surfaces.		
Objectives	Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. By example of the concept of sheaves, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Curves and Riemann Surfaces	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 6 credit points will be awarded for the module instead of 9.

Literature

Possible References :

- Frederice Mangolte: Real Algebraic Varieties. Springer 2020.
- Robert Silhol: Real Algebraic Surfaces. Springer 1989.
- Riccardo Benedetti, Jean-Jacques Risler: Real Algebraic and Semi-algebraic Sets. Editions Herrmann 1990.
- Alex Degtyarev, Viatcheslav Kharlamov: Topological properties of real algebraic varieties: du côté de chez Rokhlin. arXiv:math/0004134.

Transfer

The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
The module cannot be taken together with the module *Introduction to Riemann Surfaces* due to the large overlap in content.

Prerequisites

In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required:

- MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.

Responsible Persons

Daniele Agostini, Ivo Radloff

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-50-30</td><td colspan="2">Module Title:
Geometric Group Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Group actions on graphs, free groups.
Quasi isometries.
Growth types.
Hyperbolic groups.
Ends.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students learn to explore properties of finitely generated groups using geometric tools, starting from the Cayley graph of the group. They are able to investigate the geometric properties of the Cayley graphs with the help of analytical methods and to work out their connections to the underlying group. Students understand how algebra and analysis can work together to develop a new theory at the interface of algebra and geometry that leads to interesting statements about groups. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Geometric Group Theory</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
Clara Löh: Geometric Group Theory - an Introduction. Springer 2017.
Thorsten Camps, Volkmar Große Rebel, Gerhard Rosenberger: Einführung in die kombinatorische und die geometrische Gruppentheorie. Heldermann Verlag 2008.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-50	**Module Title:** Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	Starting from a system of axioms for plane absolute geometry with the basic concepts of incidence and congruence, the associated Bachmann reflection geometry is developed. After the introduction of the hyperbolic axiom, this is continued with reflection-geometric end theory. A Euclidean field is created from the rotations around an end and the translations along a straight line, with the help of which the hyperbolic plane under consideration is described algebraically.		
Objectives	The students have learnt to look at one and the same mathematical object (in this case absolute and hyperbolic planes) from completely different perspectives and to link them together. In particular, they have learnt about Bachmann's group-theoretically oriented reflection geometry, which rarely appears in the curriculum, and thus deepen their knowledge of groups. They also deepened their knowledge about the interweaving of geometry and algebra. The students are able to name and prove the main statements of the lecture and to categorise and explain the relationships presented. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Friedrich Bachmann: Aufbau der Geometrie aus dem Spiegelungsbegriff. Springer 1959.
- Robin Hartshorne: Geometry: Euclid and beyond. Springer 2000.
- Helmut Karzel, Kay Sörensen, Dirk Windelberg: Einführung in die Geometrie. Vandenhoeck und Ruprecht 1973.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures. Knowledge from the following modules is helpful but not strictly required: • MAT-20-06 Introduction to Geometry.
Responsible Persons	Hermann Hähl, Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-01	**Module Title:** Functional Analysis		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Normed spaces, Banach spaces, dual spaces. • Hahn-Banach theorem, uniform boundedness principle. • Closed graph theorem, open mapping theorem, Banach-Alaoglu theorem. • Compact operators, normal operators, spectral theorems.		
Objectives	The students are aquainted with the basic principles and techniques of the theory of infinte dimensional spaces and can apply them to problems in analysis and geometry. They understand the complexity of problems of spectral theory and can use its results for the solution of analytical problems. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Functional Analysis	L	f	4	6	yes	K o. mP o. H	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Nicolas Bourbaki: Topological vector spaces. Springer 1987. • Adam Bowers, Nigel Dalton: An introductory course in functional analysis. Springer 2014. • Harro Heuser: Funktionalanalysis. Teubner 2006. • Markus Haase: Functional analysis. American Mathematical Society 2014. • Peter D. Lax: Functional analysis. Wiley 2002. • Gert Kjaergaard Pedersen: Analysis now. Springer 1995. • Walter Rudin: Functional analysis. McGraw-Hill 1991. • Dirk Werner: Funktionalanalysis. Springer 2011. • Kosaku Yosida: Functional analysis. Springer 1995. • Hans Wilhelm Alt: Lineare Funktionalanalysis. Springer 2012.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Non-Stationary Differential Equations, it can be included in the study focus *Numerical Mathematics and Optimisation*.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-07	**Module Title:** Linear Control Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	Mathematical methods are indispensable for the management and control of complex systems and processes. The underlying theory is not only fascinating due to its diverse applications, but also, in its abstract form, due to the clarity and elegance of its methods and results. In this lecture, finite-dimensional systems are dealt with first, for which a good knowledge of analysis and linear algebra is sufficient. The aims are Kalman's controllability criterion and the resulting criteria for stabilisability. If there is enough time, we will extend the theory to infinite-dimensional systems. In the exercise classes we will apply the theory to concrete examples.		
Objectives	Students have learnt basic methods of linear control theory. At the same time, they have experienced and understood the interaction of various theoretical concepts from linear algebra and analysis and their benefits for specific applications. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Linear Control Theory	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Hans Wilhelm Knobloch, Huibert Kwakernaak: Lineare Kontrolltheorie. Springer 1985.
- Jerzy Zabczyk: Mathematical Control Theory. Birkhäuser 1992.
- Ruth F. Curtain, Hans Zwart: An Introduction to Infinite-Dimensional Systems Theory. Springer 1995.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Rainer Nagel

Abbreviations:

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Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-21	**Module Title:** Introduction to Partial Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces. • L^2 theory. • Important examples (Laplace equation, wave equation, heat equation). • Fundamental solutions (elliptic situation). • Weak solutions of elliptic equations.		
Objectives	The students got to know a central branch of analysis, whose terms and methods are fundamental for many fields, like numerics or stochastics. Also evolutionary equations, who have strong connections to geometry, are issue of the lecture. The students are acquainted with central terms, results and methods of linear partial differential equations and are able to use these methods in advanced courses. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Partial Differential Equations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Non-Stationary Differential Equations, it can be included in the study focus *Numerical Mathematics and Optimisation*.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-25</td><td colspan="2">Module Title:
Introduction to Partial Differential Equations – Part 1</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">4.5</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
150 h</td><td>Time in Class:
45 h</td><td>Self-Study:
105 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 1 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Harmonic functions.
• Maximum principles.
• Sobolev spaces.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have familiarised themselves with the first basic features of a central area of analysis, the concepts and methods of which are fundamental for many other areas, such as numerics and stochastics. Students are familiar with the central concepts, results and methods of linear partial differential equations and can successfully apply these methods in the more advanced courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Introduction to Partial Differential Equations – Part 1</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>1</td><td>1,5</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010.
• David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001.
• Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. In combination with one of the modules 'Numerics of Stationary Differential Equations' or 'Numerics of Nonstationary Differential Equations', it can be included in the Specialisation *Numerical Mathematics and Optimisation*. The module is part of the module Introduction to Partial Differential Equations and cannot be taken together with this module.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-34	Module Title: Introduction to Dynamical Systems		Type of Module: Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Kepler's laws. • Equilibrium positions. • Stability. • Predator-prey model. • Poincaré-Bendixson theorem. • Limit sets. • Periodic orbits. • Celestial mechanics.		
Objectives	The students can ask and examine qualitative questions about the solutions of ordinary differential equations, like e.g.: How long do mathematical solutions exist? Are there equilibrium states or periodic orbits? When are trajectories stable? They are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introducation to Dynamical Systems	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Literature

Possible References :

- Morris W. Hirsch, Stephen Smale: Differential equations, dynamical systems, and linear algebra. Academic Press 1974.
- Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 2010.
- Carl Ludwig Siegel, Jürgen Moser: Lectures on celestial mechanics. Springer 1995.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Dynamical Systems' due to the large overlap in content.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations.
Responsible Persons	Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-41</td><td colspan="2">Module Title:
Introduction to Geometric Measure Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Measures, covering theorems, differentiation of measures, Hausdorff measures and densities.
Isodiametric inequality.
Rademacher’s theorem and Whitney’s embedding theorem.
Surface- and cosurface formula.
Countable rectifiable sets, rectifiable varifolds.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Introduction to Geometric Measure Theory</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o.
or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**	**Type of Module:**
MAT-55-44	Introduction to Geometric Measure Theory – Measure Theoretic Methods	Compulsory Module with Choice
ECTS-Points	4.5	
Workload - Time in Class - Self-Study	Workload: 150 h; Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester	
Frequency	not regularly	
Term	1-3	
Language of Instruction	German or English	
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS	
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem.	
Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.	

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Geometric Measure Theory – Measure Theoretic Methods	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992.
- Herbert Federer: Geometric measure theory. Springer 1969.
- Leon Simon: Lectures on geometric measure theory. Australian National University 1984.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is part of the module 'Introduction to Geometric Measurement Theory' and cannot be taken together with this module.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-45**	**Module Title:** Introduction to Geometric Measure Theory – Varifolds		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Surface- and cosurface formula. • Countable rectifiable sets, rectifiable varifolds.		
Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Geometric Measure Theory – Varifolds	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992.
- Herbert Federer: Geometric measure theory. Springer 1969.
- Leon Simon: Lectures on geometric measure theory. Australian National University 1984.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is part of the module 'Introduction to Geometric Measurement Theory' and cannot be taken together with this module.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-49</td><td colspan="2">Module Title:
Calculus of Variations</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">4.5</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
150 h</td><td>Time in Class:
45 h</td><td>Self-Study:
105 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 1 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Direct method of calculus of variations.
Euler-Lagrange equations.
Palais-Smale condition.
Mountain-Pass Lemma according to Ambrosetti-Rabinowitz.</td></tr>
<tr><td>Objectives</td><td colspan="3">In the first part of the course, students have learnt the direct method of calculus of variations, which is primarily used to prove the existence of weak solutions of partial differential equations, but also has applications in e.g. differential geometry. They have also acquired the necessary basics from functional analysis and partial differential equations and can also use these in a different context, e.g. geometric analysis. In the second part of the course, students learnt about a so-called mountain-pass lemma. With its help, they can analyse non-uniqueness in the existence of solutions of partial differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Calculus of Variations</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o.
or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>1</td><td>1,5</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
Michael Struwe: Variational Methods, Springer 2008.
David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order, Springer 1998.
Walter Rudin: Functional Analysis, Mc Graw Hill Education 1991.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-51	**Module Title:** Lie Groups		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Manifolds and Lie groups, • Lie algebras and exponential map, • Covering spaces and classification of Lie groups by their Lie algebras, • Classical Lie groups, • Operations of Lie groups and homogeneous spaces.		
Objectives	Lie groups lie at the interface between geometry, algebra and analysis. They are suitable for describing the symmetries of geometric objects, but also algebraic equations or solutions of differential equations, in particular if these symmetries form a continuous set. The students learn from a prominent example how different disciplines of mathematics can work together very successfully and how a convincing formalism is developed that can precisely describe a variety of symmetry phenomena. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Lie Groups	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Joachim Hilgert, Karl-Hermann Neeb: Liegruppen und Lie-Algebren. Vieweg 1991.
- Gerhard P. Hochschild: The structure of Lie groups. Holden-Day 1965.
- Frank W. Warner: Foundations of differentiable manifolds and Lie groups. Springer 1983.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-01 Measure and Integration Theory, • MAT-20-04 Integration on Submanifolds of $\mathbb{R}^n$.
Responsible Persons	Anton Deitmar, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-53	**Module Title:** Automorphic Forms		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Modular forms for the module group and its congruence subgroups. • Examples: Eisenstein series, Ramanujan delta function, theta series. • Modular curves. • Arithmetic applications and conjectures. • Hecke operators. • The L-function of a modular form and its connections with elliptic curves.		
Objectives	Students have familiarised themselves with the central concepts and methods of the theory of automorphic forms in examples and are able to use them. They have understood the connection between modular, real representation theory and adelic L-functions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Automorphic Forms	L	f	2	3	yes	K o. mP o. H	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Deitmar, Anton: Automorphic Forms. Springer 2012.
- Goldfeld, Dorian: Automorphic forms and L-functions for the group GL(n,R). Cambridge University Press 2015.
- Serre, Jean-Pierre: A course in arithmetic. Springer 1973.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is part of the module Introduction to Geometric Measurement Theory and cannot be taken together with this module.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-60	**Module Title:** Introduction to Mathematical Logic		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Propositional logic. • Languages of the first order: – Completeness and compactness. • Theory of computations: – Register machines; – Gödelisation. • Incompleteness of arithmetic: – First and second incompleteness theorem. • Set theory: – Ordinal- and cardinal numbers; – Incompleteness of set theory.		
Objectives	Students are able to understand mathematical theorems and theories in the context of mathematical logic. They understand the limits of possible mathematical knowledge, recognise the difference between truth and provability and can apply basic theoretical model thinking to mathematical content. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to the Mathematical Logic	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classess; in this case, only 3 credit points will be awarded for the module instead of 4.5.

Literature	**Possible References :** • Rautenberg, Wolfgang: Einführung in die Mathematische Logik. Vieweg+Teubner 2008. • Ziegler, Martin: Mathematische Logik. Birkhäuser 2016.
Transfer	The module is not assigned to a specialisation. It can be included in the Sections *Advanced Knowledge in Mathematics* or *Elective Specialisation* in accordance with the restrictive requirements of the respective section.
Prerequisites	There are no prerequisites.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-62	Module Title: Consistency Proofs		Type of Module: Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Historical examples of the question of consistency (limits; parallel axiom; set-theoretic paradoxes). • Philosophical foundational programs (logicism; formalism; intuitionism). • The Hilbert program and Gödel's theorems. • Gentzen's transfinite consistency proof for number theory. • Alternative approaches to consistency (including Gödel's T). • Current situation of consistency proofs.		
Objectives	Students learn about the historical context from which the question of the non-contradiction of formal mathematical theories arose, as well as the relevant modern techniques for investigating this question mathematically. They are able to categorise the problem of non-contradiction in mathematics both historically and philosophically. In addition, they have acquired the mathematical tools to be able to comprehend the corresponding proofs of non-contradiction and, to a certain extent, to carry them out themselves. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Consistency Proofs	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Kurt Gödel: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatsh. f. Mathematik und Physik 38, 173-198 (1931). • Gerhard Gentzen: Die Widerspruchsfreiheit der reinen Zahlentheorie. Math. Ann. 112, 493-565 (1936). • Reinhard Kahle, Michael Rathjen (Hrsg.): Gentzen's Centenary: The quest for consistency. Springer 2015. • Reinhard Kahle, Michael Rathjen (Hrsg.): The Legacy of Kurt Schütte. Springer 2020.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Reinhard Kahle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-64	**Module Title:** Theory of Mathematical Proofs		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Axiomatic theories, incompleteness. • Gentzen's proof of consistency for arithmetic. • Ordinal number analysis. • Provable recursive functions. • Predicative analysis. • Theories of inductive definitions.		
Objectives	Students are familiar with the methods and results of modern proof theory, in particular with calculations for mathematical theories and their metamathematical properties. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Theory of Mathematical Proofs	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Wolfram Pohlers. Proof Theory. Springer 2009.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Reinhard Kahle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-65	**Module Title:** Explicit Mathematics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Applicative theories. • Explicit mathematics. • Universes in explicit mathematics. • Applications in proof theory.		
Objectives	Students are familiar with an alternative logical theoretical framework for arithmetic and subsystems of analysis and are familiar with their function in proof-theoretical investigations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Explicit Mathematics	L	f	2	3	yes	or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 6.

Literature	**Possible References :** • Solomon Feferman: A language and axioms for explicit mathematics, in Algebra and Logic. Lecture Notes in Mathematics, 450, pp. 87-139, Springer-Verlag, Berlin, 1975. • Solomon Feferman: Constructive theories of functions and classes. In Logic Colloquium ?78, (Proc. Mons Colloq.), pp. 159-224, North-Holland, Amsterdam, 1979. • Gerhard Jäger, Reinhard Kahle, Thomas Strahm: On applicative theories. In Andrea Cantini, Ettore Casari, and Pierluigi Minari, editors, Logic and Foundations of Mathematics, pages 83–92, Kluwer, 1999. • Reinhard Kahle: The applicative realm. Textos de Matematica, 40, Departamento de Matemática, Universidade de Coimbra, 2007. • Gerhard Jäger, Reinhard Kahle, Thomas Studer: Universes in explicit mathematics. Annals of Pure and Applied Logic, 109(3),141-162, 2001.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Reinhard Kahle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-65-05</td><td colspan="2">Module Title:
Groups and Representations</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">

Groups: subgroups, homomorphisms, isomorphisms, group actions, orbits, stabilisers, equivalence classes, normal subgroups, cosets, factor groups.
Representations: faithful, unitary and irreducible representations, reducibility, characters, Schur’s lemma(s), orthogonality of irreducible representations.
Applications: symmetries and degeneracies in quantum mechanics, selection rules.
Representations of finite groups: group algebra, regular representation, ideals, idempotents.
Symmetric groups: Young tableaux, Young operators, dimensions and characters.
Applications: identical particles in quantum theories.
Lie groups: Haar measure, representations, Lie algebras.
Tensor representations of classical groups: symmetry classes, Young tableaux.
Applications: SU(2) and SU(3) in particle physics (spin, isospin, flavour)
Moreover a selection of the following:

Irreducible representations of the Lorentz and Poincaré groups.
Applications: notion of particles in quantum theories.
Roots and weights; Killing-Cartan classification of semi-simple Lie algebras

</td></tr>
<tr><td>Objectives</td><td colspan="3">The studens know the basic concepts of group and representation theory. They are able to apply these abstract algebraic concepts in the context of theoretical physics and have, thus, developed a deepend understanding for the connections between mathematics and physics. The students are familiar with a number of complex examples of applications of the representation theory of groups in physics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
</table>

<table>
<tr><td rowspan="4">Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Groups and Representations</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
<tr><td colspan="10">In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="10">Possible References :
• Irene Verona Schensted: A course on the Application of Group Theory to Quantum Mechanics. NEO Press 1976.
• Barry Simon: Representations of Finite and Compact Groups. AMS 1996.
• Wu-Ki Tung: Group Theory and Physics. World Scientific 1985.</td></tr>
<tr><td>Transfer</td><td colspan="10">The module belongs to the Study Specialisations Algebra and Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.
The module cannot be taken together with the module 'Representation Theory of Finite Groups' due to the large overlap in content.</td></tr>
<tr><td>Prerequisites</td><td colspan="10">In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required:
• MAT-10-12 / MAT-20-06 Algebraic Structures.</td></tr>
<tr><td>Responsible Persons</td><td colspan="10">Stefan Keppeler</td></tr>
</table>

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-65-11</td><td colspan="2">Module Title:
Geometry in Physics</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">regularly in Winter Semester</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">The module provides an introduction to fundamental methods of differential geometry and their relevance for physics. Particular topics are manifolds, differential forms, Riemannian metrics and associated notions of curvature, Riemannian geometry of submanifolds, real vector bundles, and connections. Applications of these concepts in Physics are discussed.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students obtain knowledge, understanding, and acquaintance with the use of the listed notions of differential geometry. They develop, in particular, a deeper understanding of differential and integral calculus and experience through examples how the mathematical notions are naturally applied within physical theories. Students obtain knowledge, understanding, and acquaintance with the use of the listed notions of differential geometry. In particular, they develop a deeper understanding of differential and integral calculus and experience through examples how the mathematical notions are naturally applied within physical theories. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework.
Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Geometry in Physics</td><td>L</td><td>o</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>o</td><td>2</td><td>3</td></tr>
</table>
In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• John Lee: Introduction to smooth manifolds. Springer 2012.
• John Lee: Riemannian manifolds: An introduction. Springer 1997.
• Chris Isham: Modern differential geometry for physicists. World Scientific 1999.
• Mikio Nakahara: Geometry, Topology and Physics. IOP Publishing 2003.</td></tr>
</table>

Transfer	Participation in the module is a prerequisite for participation in the module Mathematical Relativity. Successful completion of the module may be a prerequisite for participation in the module Seminar Knowledge Extension and is so for the participation in the module Scientific Project.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Christoph Bohle, Carla Cederbaum, Stefan Teufel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-12	**Module Title:** Mathematical Quantum Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Winter Semester		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	The module provides an introduction to mathematical quantum theory. Particular topics are the stationary and time-dependent Schrödinger equation, fundamental approximation methods as Rayleigh-Schrödinger perturbation theory and Hartree- resp. Hartree-Fock theory, the Fock space formalism, and elements of scattering theory. Optionally, other topics such as adiabatic theory and semiclassical approximations can be discussed.		
Objectives	Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from quantum theory. They are able to interrelate physical problems in atom, solid state and particle pysics and their mathematical models via spectral and interference theoretical methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Students experience through examples how the mathematical notions are naturally applied within physical theories. Thereby, they enhance their knowledge on methods and subjects. Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from quantum theory. They are able to interrelate physical problems in atom, solid state and particle pysics and their mathematical models via spectral and interference theoretical methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Students experience through examples how the mathematical notions are naturally applied within physical theories. Thereby, they enhance their knowledge on methods and subjects. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Quantum Theory	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.

Transfer	Successful completion of module Mathematical Quantum Theory is a prerequisite for the participation in the module Advanced Topics in Mathematical Quantum Theory. Successful completion of one of the modules Mathematical Quantum Theory and Mathematical Relativity is a prerequisite for the participation in the module Scientific Project.

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Stefan Teufel, Marius Lemm

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-14	**Module Title:** Mathematical Statistical Physics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly, in Summer Semester		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	The module provides an introduction to mathematical statistical physics. Particular topics are concepts of probability theory, classical statistical mechanics of gases (equivalence of ensembles, thermal equilibrium, Boltzmann equation, entropy), Brownian motion (stochastic processes, Wiener process), lattice models (Ising model, Gibbs measure, thermodynamic limit, phase transitions), statistical quantum mechanics (quantum mechanical ensembles, transition to thermal equilibrium, Bose-Einstein condensate). Optionally, other topics such as open quantum systems, transport phenomena, renormalization group theory and the fluctuation-dissipation theorem can be discussed.		
Objectives	Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from statistical physics. They are able to interrelate fundamental physical concepts, such as equilibrium, irreversibility and entropy, with their mathematical models via probabilistic methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Thereby, they enhance their knowledge on methods and subjects gained throughout the first semester, in particular on probability theory. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Statistical Physics	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.

Transfer	Successful completion of module is a prerequisite for the participation in the module Advanced Topics in Mathematical Statistical Physics.

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Responsible Persons	Roderich Tumulka

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-65-15</td><td colspan="2">Module Title:
Foundations of Quantum Mechanics</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">regularly every two years</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">The module provides an introduction to fundamental questions of quantum mechanics, including its mathematical and philosophical aspects. Various interpretations such as Copenhagen, Bohmian mechanics, many worlds and spontaneous collapse of the wave function are presented and analysed mathematically and physically. Other topics include Born's rule, Heisenberg's uncertainty principle, the quantum measurement problem, Bell's non-locality theorem, identical particles and no-hidden-variable theorems.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students know and can apply the rules of quantum mechanics in different environments and understand several important theories of how the quantum world works. They acquire mathematical knowledge relevant to the application of these rules and theories and can connect the mathematical treatment with the physical meaning. They will familiarise themselves with the surprising phenomena and paradoxes of quantum mechanics. They will appreciate what is controversial about the orthodox interpretation and why, and will be able to follow the current debate on fundamental issues. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Foundations of Quantum Mechanics</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
The type of examination is set by the instructor.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisation Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required:
• MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.</td></tr>
<tr><td>Responsible Persons</td><td colspan="3">Roderich Tumulka</td></tr>
</table>

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-39	**Module Title:** Propagation of Chaos	**Type of Module:** Compulsory Module with Choice
ECTS-Points	9	
Workload - Time in Class - Self-Study	Workload: 270 h; Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester	
Frequency	not regularly	
Term	1-3	
Language of Instruction	German	
Forms of Teaching and Learning	Lecture 4 SWS	
Content	• Interacting many body systems (quantum and classical), importance of correlations. • Mean-field situations (e.g., Vlasov) and collisions (Boltzmann). • Explicit treatment of correlations. • Large deviations from the expected value.	
Objectives	Students learn how different kinds of many-body systems can be described by effective, non-linear equations. They are able to distinguish and compare different types of convergence of microscopic many-body systems against the effective theory, both in classical and quantum mechanical situations. Based on an argument similar to the law of large numbers, they understand how the independence of particles leads to the effective equation. They learn to prove that independence is indeed preserved - at least approximately - under the evolution of time (propagation of chaos). Building on this, they understand various proof strategies adapted to the respective situation. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.	

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Propagation of Chaos	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	?	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Louis-Pierre Chaintron, Antoine Diez: Propagation of chaos: a review of models, methods and applications. arXiv:2203.00446.
- Francois Golse: Mean-Field Limits in Statistical Dynamics. arXiv:2201.02005.

Transfer

The module belongs to the *Study Specialisations Mathematical Physics* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Responsible Persons	Peter Pickl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-65-40</td><td colspan="2">Module Title:
Interacting Many-Body Quantum Systems</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Bosons and fermions.
• Bose-Einstein condensation.
• Mean-field theory.
• Nonlinear Schrödinger equation.
• Hartree- and Hartree-Fock theory.
• Bogoliubov approximations.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students have learned about the difference between Fermions and Boson and its impact on many body systems. They understand that, at cold temperatures, the fluctuations of the particle number in a certain area is significantly suppressed for Fermions. They understand how the Bosonic nature leads to the famous effect of Bose Einstein condensation and that, in certain scaling regimes, interacting many body systems can in good approximation be described by nonlinear evolution equations. Finally they are able to give higher order corrections and to calculate the leading order correlations between the particles. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Propagation of Chaos</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Elliott H. Lieb, Robert Seiringer, Jan Philip Solovej, Jakob Yngvason: The Mathematics of the Bose Gas and its Condensation. Birkhäuser 2000.
• Niels Benedikter , Marcello Porta , Benjamin Schlein: Effective Evolution Equations from Quantum Dynamics. Springer 2016.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Peter Pickl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-01	**Module Title:** Algorithms of Numerical Mathematics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		

Content

Advanced, important algorithms of numerics (without differential equations) such as:

- Fast Fourier transformation;
- QR algorithms for the calulation of eigenvalues;
- Method of conjugated gradients and more general Krylov space methods as iterative methods in numeric linear algebra and in non-linear optimisation;
- Simplex method and interior point methods in linear optimisation.

Objectives

The students have learned the key concepts, results, and methods of algorithmic numerical mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algorithms of Numerical Mathematics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Peter Deuflhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008.
- Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg 2009.

Transfer

The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Responsible Persons	Christian Lubich, Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-70-02</td><td colspan="2">Module Title:
Numerics of Stationary Differential Equations</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Numerical covering of boundary value problems of stationary (i.e. time independent) ordinary and elliptic partial differential equations, with emphasis to the methods of finite elements.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have learned the central terms, results and methods of the numerical treatment of boundary value problems of stationary differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Numerics of Stationary Differential Equations</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Dietrich Braess: Finite Elemente. Springer Spektrum 2013.
• Wolfgang Hackbusch: Theorie und Numerik elliptischer Differentialgleichungen. Teubner 1986.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisation Numerical Mathematics and Optimisation. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
</table>

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations, • MAT-20-11 / MAT-20-13 Numerical Mathematics. Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.
Responsible Persons	Christian Lubich

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-04	**Module Title:** Ordinary Differential Equations - Analysis and Numerics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Non-linear ordinary differential equations: Theorems of Hartman-Grobman and Poincare-Bendixson, bifurcation theory. • Numerical approximation: linear multi-step processes, adaptive processes, geometric integration.		
Objectives	Students are familiar with the basic methods for studying qualitative behavior and for simulating solutions of non-linear ordinary differential equations. They have learned constructive methods for solving them and are in principle able to implement these with the help of the computer. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Ordinary Differential Equations - Analysis and Numerics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Lawrence Perko: Differential equations and dynamical systems. Springer 1993. • David Griffiths, Desmond J. Higham: Numerical methods for ordinary differential equations. Springer 2010.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics. Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-70-05</td><td colspan="2">Module Title:
Optimal Control Theory with Ordinary Differential Equations</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">4.5</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 1 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Brief overview of existence and uniqueness theory for ODEs.
• Numerical solutions to ODEs.
• Introduction to optimal control problems with ODEs.
• Existence and uniqueness theory for linear quadratic optimal control problems (LQ problems).
• Pontryagin's maximum principle.
• Numerical approximation of LQ problems.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students are familiar with the problems of optimal control using ordinary differential equations and various approaches to solving the problem. They are also familiar with qualitative statements on unambiguous solvability. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Optimal Control Theory with Ordinary Differential Equations</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>1</td><td>1,5</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Matthias Gerdts: Optimal Control of ODEs and DAEs. De Gruyter 2012.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-11	**Module Title:** Stochastic Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		

Content

- Stochastic processes, filtrations, martingales.
- Wiener process, random walk, Donsker's theorem.
- Diffusion semigroup, Ito's integral.
- Solution of a stochastic differential equation.
- Markov property, Malliavin calculus, rough path theory.

Objectives

Students master the basic principles and techniques for constructing solutions of stochastic differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Stochastic Differential Equations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Bernt Oksendal: Stochastic differtial equations. Springer 2000.

Transfer

The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-70-12</td><td colspan="2">Module Title:
Introduction to Stochastic Differential Equations - Part 1</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">4.5</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 1 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Introduction to Brownian motion and stochastic integration.
Solution concepts for stochastic differential equations.
Stability of stochastic differential equations.
Numerical approximation of stochastic differential equations.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students master the basic principles and techniques for constructing solutions of stochastic differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Introduction to Stochastic Differential Equations - Part 1</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>1</td><td>1,5</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
Bernt Oksendal: Stochastic differential equations. Springer 2000.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisations Numerical Mathematics and Optimisation and Stochastics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
</table>

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-15	**Module Title:** Numerics of Stochastic Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Random number generator, Ito-Taylor expansion. • Strong and weak approximation, consistency. • Euler-Maruyama method, Milstein method, stochastic Runge-Kutta method. • Approximation of stopped diffusion processes.		
Objectives	Students master the basic principles and techniques for the numerical approximation of solutions of stochastic differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Numerics of Stochastic Differential Equations	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Peter E. Kloeden, Eckhard Platen: Numerical solution of stochastic differential equations. Springer 1999.
Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-20	**Module Title:** Introduction to Optimisation		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 3 SWS + Ex.cl. 1 SWS		
Content	• Optimality theory for smooth, convex and linear optimisation problems optimisation problems with constraints. • Foundations of the theory of convex sets and functions. • Duality theory for convex and linear optimisation problems. • Solution methods for linear optimisation problems.		
Objectives	Students know and understand methods and algorithms for solving convex and linear optimisation problems. They have learnt to apply the methods to simple problems related to economics, technology or physics. They will be able to critically assess the possibilities and limitations of using the methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Optimisation	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Florian Jarre, Joseph Stoer: Optimierung: Einführung in mathematische Theorie und Methoden. Springer 2019.
- Jorge Nocedal, Stephen J. Wright: Numerical optimization. Springer 2006.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Christian Lubich

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-70-22</td><td colspan="2">Module Title:
Optimisation with Differential Equations</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">
• Direct method in the calculus of variations, Euler-Lagrange equation.
• Brouwer-Minty theorem, non-linear evolution equations.
• Gateaux and Frechet differentiability.
• Proof of existence of optimal controls, necessary optimality conditions.
• Adjoint, convergent optimisation methods in Banach spaces.
• Variational discretisation concepts.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students master the basic principles and techniques for deriving optimality conditions for prototypical control problems with constraints in the form of partial differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Optimisation with Differential Equations</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Michael Hinze, Rene Pinnau, Michael Ulbrich, Stefan Ullrich: Optimization with PDE constraints. Springer 2009.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-70-30</td><td colspan="2">Module Title:
Theoretical Aspects of Machine Learning</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">The lecture covers some recent aspects of theoretical machine learning such as:
The theory of Reproducing Kernel Hilbert Spaces (RKHS).
Applications of RKHS theory such as SVMs, kernel regression, kernel PCA, and kernel mean embeddings.
Approximation capabilities of neural networks.
Dynamics of neural networks and the neural tangent kernel.
Recent advances in high dimensional statistics, in particular overparametrisation and generalisation.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students learn the mathematical foundations of supervised learning theory, neural networks, support vector machines and kernel methods. They are familiar with fundamental modern topics in machine learning and with their theoretical basis, mathematical approach and conceptual tools as needed for the discussion and justification of algorithms. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area.
Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Theoretical Aspects of Machine Learning</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may, in exceptional cases, be offered without exercises at the discretion of the lecturer; in this case, only 3 credit points are awarded instead of 6.</td></tr>
</table>

Literature	**Possible References :** • Mehryar Mohri, Afshin Rostamizadeh, Ameet Talwalkar: Foundations of Machine Learning. MIT Press 2012. • Shai Shalev-Shwartz, Shai Ben-David: Understanding Machine Learning: From Theory to Algorithms. CUP 2014. • Peter L. Bartlett, Andrea Montanari, Alexander Rakhlin: Deep learning: a statistical viewpoint. Acta Numerica 2021. • Daniel A. Roberts, Sho Yaida, Boris Hanin: The Principles of Deep Learning Theory: An Effective Theory Approach to Understanding Neural Networks. Cambridge University Press 2022.
Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-31	**Module Title:** Statistical Learning 1	**Type of Module:** Compulsory Module with Choice
ECTS-Points	9	
Workload - Time in Class - Self-Study	Workload: 270 h; Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester	
Frequency	regularly	
Term	1-3	
Language of Instruction	German	
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS	
Content	• Non-parametric regression, regression estimator. • (Universal) consistency. • Rate convergence. • Stone's theorem. • Kernel estimator, k-NN estimator. • Slow rate convergence, minimax convergence rates.	
Objectives	Students are familiar with basic non-parametric regression estimators, in particular with their universal consistency and rate convergence. They are familiar with the basic principles and methods of stochastic learning as required for machine learning applications. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.	

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Statistical Learning 1	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Laslo Györfi, Michael Kohler, Adam Krzyzak, Harro Walk: A distribution-free theory of nonparametric regression. Springer 2002.

Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-33	**Module Title:** Theory and Numerics for Constrained Optimisation Problems		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	We start with the unconstrained convex minimisation problem (on spaces), and the gradient method with step size control according to Armeijo for the approximate calculation of a minimum, as well as its variants. The simplex method solves linear programmes on polyhedra. Central to this is the convex (non-linear) minimisation task on sets, and the characterisation of a minimum with (necessary) optimality conditions (tangent cone, linearised tangent cone, Abadie condition, Karush-Kuhn-Tucker conditions). In addition, numerical solution methods based on these theoretical concepts (interior points method, penalty methods, SQP method) are presented and analysed.		
Objectives	The participants have become familiar with relevant current algorithms for the optimisation of constrained optimisation problems: these include gradient methods with step size control, the simplex method, interior point methods, penalisation methods and the SQP method. You will be able to analyse the algorithms and compare their complexity. he students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Theory and Numerics for Constrained Optimisation Problems	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 6 credit points will be awarded for the module instead of 9.

Literature

Possible References :

- Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.

Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-34	**Module Title:** Mathematical Introduction to Data Science		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	The lecture provides a mathematically rigorous survey of core data-science techniques from regression and k-nearest neighbours to high-dimensional phenomena, dimensionality reduction, and modern classifiers (Perceptron, SVMs, Kernel Methods, Neural Networks). Emphasis is put on both theoretical foundations and hands-on Python implementation with real datasets.		
Objectives	The students can formulate and solve linear, polynomial and logistic regression models. They are able to implement k-nearest neighbours classification and build and evaluate classifiers (Perceptron, SVM (kernel methods), Neural Networks). Moreover, they know how to optimise convex objectives using gradient-descent techniques. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Introduction to Data Science	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may, in exceptional cases, be offered without exercises at the discretion of the lecturer; in this case, only 3 credit points are awarded instead of 6.

Literature	**Possible References :** • Sven A. Wegner: Mathematical Introduction to Data Science. Springer 2024.
Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-70-35</td><td colspan="2">Module Title:
Optimal Transport</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">The course introduces the theory of optimal transport at the interface of analysis, probability, and optimisation. Starting from the classical problems of Monge and Kantorovich, the fundamental measure-theoretic and analytical tools are developed.
Building on this, central results are presented, including the existence and duality of optimal transport plans, c-cyclical monotonicity, Brenier’s theorem, and the geometric structure of Wasserstein spaces.
In the second part of the course, the algorithmic perspective of optimal transport is explored. Topics include entropic regularisation, the Sinkhorn algorithm, and selected applications, for example in image processing, statistics, and machine learning.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students are familiar with the fundamental concepts, methods, and key results of the theory of optimal transport and are able to situate these within a broader mathematical context spanning analysis, probability, and optimisation. They can understand the classical formulations due to Monge and Kantorovich, as well as essential structural concepts such as duality, c-cyclical monotonicity, and the geometry of Wasserstein spaces, and apply them in simple settings.
Furthermore, they are acquainted with basic algorithmic approaches to optimal transport and can assess their relevance for selected applications. In the exercise classes, they deepen their understanding of the lecture material, further develop their ability to construct precise mathematical arguments, and practise the written and oral presentation of their own solution methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Optimal Transport</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Alessio Figalli, Federico Glaudo: An Invitation to Optimal Transport, Wasserstein Distances, and Gradient Flows. EMS 2023. • Gabriel Peyré, Marco Cuturi: Computational Optimal Transport. Now Publishers 2019.
Transfer	The module belongs to the *Study Specialisation Analysis*, *Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 Submodules Measure and Integration Theory, and Integration on Submanifolds of $\mathbb{R}^n$. • MAT-20-12 Stochastics.
Responsible Persons	Nicolai Jork, Andreas Prohl

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-70-40**	**Module Title:** Game Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The focus is on Nash and generalised Nash equilibrium problems and their numerical solution.		
Objectives	Students are familiar with the fundamental issues of game theory. They are familiar with analytical and numerical approaches to analysing them. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Game Theory	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Christian Kanzow, Alexandra Schwartz: Spieltheorie. Birkhaeuser 2018.
Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-70-51</td><td colspan="2">Module Title:
Financial Mathematics and Numerics</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">The course is designed to provide a comprehensive introduction to the mathematical models and numerical techniques that are essential for understanding and solving problems in modern finance. The course will delve into stochastic processes, modeling random phenomena in financial markets, and the numerical methods necessary for approximating solutions to complex financial equations.
Key topics include the mathematics behind derivative pricing models, such as the Black-Scholes framework, and the use of stochastic calculus (SDE theory) in risk management and portfolio optimisation. By integrating theory with numerical practice, this course bridges the gap between financial mathematics and real-world applications, equipping students with the quantitative skills required for careers in quantitative finance, risk analysis, and financial engineering.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students know important mathematical models for the description of problems in financial mathematics and can apply numerical approaches to their solutions in a targeted manner. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area.
Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Financial Mathematics and Numerics</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may, in exceptional cases, be offered without exercises at the discretion of the lecturer; in this case, only 3 credit points are awarded instead of 6.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Steven Shreve: Stochastic Calculus for Finance. Springer 2005.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-11 / MAT-20-13 Numerical Mathematics, • MAT-20-12 Stochastics.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-75-01</td><td colspan="2">Module Title:
Probability Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">regularly in Winter Semester</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Characteristic functions and additions to the central limit theorem.
Conditional expectations and further measure-theoretic foundations.
Markov chains and martingales in discrete time, classification, asymptotic behaviour, stopping times, stationarity, ergodicity.
Introduction to processes in contiuous time like Poisson processes and Brownian motion.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students got to know the central terms results and methods of probability theory. They can model, analyse and interprete stochastic dependency structures of random quantities in a measure theoretically founded manner. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Probability Theory</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Richard Durrett: Probability, Theory and Examples. Cambridge University Press 2019. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Jean Jacod, Philip E. Protter: Probability essentials. Springer 2004. • Olav Kallenberg. Foundations of Modern Probability. Springer 2021. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2020. • David Meintrup, Stefan Schäffler: Stochastik. Springer 2005. • Albert N. Shiryaev: Probability-1. Springer 2016.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Responsible Persons	Martin Möhle, Elmar Teufl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-03	**Module Title:** Mathematical Statistics	**Type of Module:** Compulsory Module with Choice
ECTS-Points	9	
Workload - Time in Class - Self-Study	Workload: 270 h; Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester	
Frequency	regularly	
Term	1-3	
Language of Instruction	German	
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS	
Content	• Statistical models, exponential families, sufficient statistics. • Rao-Blackwell theorem, Lehmann-Scheffe theorem, Cramer-Rao theorem. • Estimation methods, UMVU estimator, quality criteria, asymptotic behaviour of estimators. • Hypothesis testing, confidence interval, Neyman-Pearson lemma. • Testing methods, UMPU tests, 1- and 2-sample tests. • Models with growing density quotients, non parametric models. • Introduction in regression and variance analysis.	
Objectives	Students can model statistical relationships mathematically. They can mathematically construct, analyse, compare and apply statistical estimation and test methods and interpret their results. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.	

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Statistics	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Peter J. Bickel, Kjell A. Doksum: Mathematical Statistics: Basic Ideas and Selected Topics. Chapman & Hall 2016. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Erich L. Lehmann, Joseph P. Romano: Testing statistical hypotheses. Springer 2005. • Erich L. Lehmann, George Casella: Theory of point estimation. Springer 1998. • Wiebe R. Pestman: Mathematical Statistics. De Gruyter 2009 • Helmut Pruscha: Vorlesungen über Mathematische Statistik. Springer Vieweg 2000. • Mark J. Schervish: Theory of Statistics. Springer 1995.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics. Knowledge from the following modules is helpful but not strictly required: • MAT-75-01 Probability Theory.
Responsible Persons	Martin Möhle, Elmar Teufl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-10	**Module Title:** Graph Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Basic concepts in graph theory, • Basic graph theory algorithms, • Flows, cuts, connectedness, matchings, • Cycle and cut space (cohomology theory), • Spectral graph theory, matrix tree theorem, • Planar graphs, theorem of Kuratowski and Wagner, • Planar embeddings, • Graph colorings, • Theory of minors.		
Objectives	Students know the basic concepts of graph theory, can analyse the structure of graphs and use graph theory methods in practice. They will also recognise connections to geometry and algebra and be able to benefit from them. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Graph Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Bela Bollobas: Modern graph theory, Springer, 1998. • John Adrian Bondy, Uppaluri Siva Ramachandra Murty: Graph theory, Springer, 2008. • Reinhard Diestel: Graph theory, Springer, 2018. • Jonathan L. Gross, Jay Yellen, Mark Anderson: Graph theory and its applications, CRC Press, 2019.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Responsible Persons	Elmar Teufl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-21	**Module Title:** Bayesian Networks and Causality		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	Uncertainty is a fact of life, and robust artificial intelligence for real life application has to be able to deal with it. Therefore, the development of mathematical representations that effectively incorporate probabilities was a key step in the development of artificial intelligence. However, human understanding goes further than that: We go beyond observing events, such as *Whenever the sprinkler is on in my greenhouse, the plants are wet* or *Whenever the plants are wet, the sprinkler is on*, to postulating a relationship of cause and effect: *If I turn on the sprinkler, the plants will be wet, but if I water the plants, that will certainly not activate the sprinkler*. In this course, Bayesian networks are studied, which are a widely used representation for probability distributions. In particular, commonly used inference and learning algorithms are discussed. Moreover, it will be shown that Bayesian networks do not only represent probability distributions, but are also able to express causal relationships. Finally the causal expressivity of Bayesian networks will be looked at, aiming to learn causal structure from observational data. • Part I: Bayesian Networks as an Efficient Representation of Probability Distributions: – Computing probabilities using Bayesian networks. – d-Separation: A graphical criterion for probabilistic independence. – Parameter and structure learning in Bayesian networks. • Part II: Bayesian Networks as a Representation for Causal Knowledge: – Functional causal models: A representation of causal knowledge. – Pearl's causal ladder: Predicting the effects of external interventions and reasoning with counterfactuals. – Causal Bayesian networks. – Causal structure discovery: Learning causal relationships from data. – Counterfactual identifiability: Answering counterfactual. questions using causal Bayesian networks.		
Objectives	In the first part of the course, students have learned how Bayesian networks represent probability distributions and how this representation can be used to efficiently compute probabilities or determine whether two random variables are independent. In the second part, students have learned to distinguish inference tasks in artificial intelligence according to Pearl's causal ladder: probabilistic, interventional, and counterfactual reasoning, which generally require increasingly detailed knowledge. They are familiar with identifiability results, which provide assumptions under which certain queries on the causal ladder can be answered using only knowledge from lower levels. For instance, they know how to answer queries about the effects of external interventions using only knowledge of the correct probability distribution, which can often be estimated from observational data. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

<table>
<tr><td rowspan="4">Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Bayesian Networks and Causality</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>1</td><td>1,5</td></tr>
<tr><td colspan="10">Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="10">Possible References :
• Judea Pearl: Causality: Models, Reasoning and Inference. Cambridge University Press 2009.</td></tr>
<tr><td>Transfer</td><td colspan="10">The module belongs to the Study Specialisations Numerical Mathematics and Optimisation and Stochastics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
<tr><td>Prerequisites</td><td colspan="10">In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor’s programme is required:
• MAT-20-01 Measure and Integration Theory,
• MAT-20-12 Stochastics.</td></tr>
<tr><td>Responsible Persons</td><td colspan="10">Stephan Eckstein</td></tr>
</table>

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master’s thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-75-22</td><td colspan="2">Module Title:
Information Theory, Pattern Recognition, and Neural Networks</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">3</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
90 h</td><td>Time in Class:
30 h</td><td>Self-Study:
60 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">
Introduction to information theory:
The possibility of reliable communication over unreliable channels. The Hamming code and repetition codes.
Entropy and data compression:
Entropy, conditional entropy, mutual information, Shannon information content. The idea of typicality and the use of typical sets for source coding. Shannon's source coding theorem. Codes for data compression. Uniquely decodeable codes and the Kraft-MacMillan inequality. Completeness of a symbol code. Prefix codes. Huffman codes. Arithmetic coding.
Communication over noisy channels:
Definition of channel capacity. Capacity of binary symmetric channel; of binary erasure channel; of Z channel. Joint typicality, random codes, and Shannon's noisy channel coding theorem. Real channels and practical error-correcting codes. Hash codes.
Statistical inference, data modelling and pattern recognition:
The likelihood function and Bayes' theorem. Clustering as an example.
Approximation of probability distributions:
Laplace's method. (Approximation of probability distributions by Gaussian distributions.) Monte Carlo methods: Importance sampling, rejection sampling, Gibbs sampling, Metropolis method. (Slice sampling, Hybrid Monte Carlo, Overrelaxation, exact sampling.) Variational methods and mean field theory. Ising models.
Neural networks and content-addressable memories:
The Hopfield network.
</td></tr>
<tr><td>Objectives</td><td colspan="3">Students are familiar with fundamental concepts of information theory – such as entropy and noisy channel coding – and how these are applied in computing and approximating probabilities. They are able to name and prove the main statements of the lecture as well as categorise and explain the relationships presented. Students will be able to reproduce and critically scrutinise the current state of research in the specialist area addressed.</td></tr>
</table>

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Information Theory, Pattern Recognition, and Neural Networks	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Literature	**Possible References :** • David J.C. MacKay: Information Theory, Inference, and Learning Algorithms. CUP 2003.									
Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.									
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.									
Responsible Persons	Stephan Eckstein									

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-23	**Module Title:** Nonparametric Estimation of Distributions		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload **- Time in Class** **- Self-Study**	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	This course explores a fundamental question in statistics: Given n data points from an unknown random vector, how well can we infer the true characteristics of the random vector, i.e., its distribution? In particular, we will focus on the role of the dimension of the vector, leading to a treatment of the curse of dimensionality. Further, we will explore how prior knowledge (e.g., smoothness or structural understanding of the random vector) can help overcome the curse of dimensionality. In contrast to parametric statistics, where the distribution is assumed to belong to a finite-dimensional family (e.g., estimating mean and covariance matrix of a normal distribution), the focus is purely on nonparametric settings, where no such parametric forms are given a priori. These settings require suitably chosen evaluation criteria based on distances between probability distributions, and we will treat both density estimation with L^p-type losses and estimation in Wasserstein distance.		
Objectives	The statistical learning problem for probability distributions can be formulated precisely. The importance of the chosen metric and of the dimension of the distribution is well understood. Students are familiar with the fundamental proof techniques for establishing both upper bounds, using explicit estimators, and lower bounds, using information-theoretic methods. They develop a structural understanding of why$(G!7(Bandwhich$(G!7(Bassumptions are necessary to achieve convergence rates that are better than those dictated by the curse of dimensionality. Through the exercise classes, students demonstrate their ability to work independently on topics covered in the lectures. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Nonparametric Estimation of Distributions	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Alexandre B. Tsybakov: Introduction to Nonparametric Estimation. Springer 2009. • Jonathan Niles-Weed, Quentin Berthet: Minimax estimation of smooth densities in Wasserstein distance. The Annals of Statistics 50.3 (2022): 1519-1540. • Nicolas Fournier, Arnaud Guillin: On the rate of convergence in Wasserstein distance of the empirical measure. Probability Theory and Related Fields 162.3 (2015): 707-738. • Daniel Bartl, Stephan Eckstein: Fast Wasserstein rates for estimating probability distributions of probabilistic graphical models. arXiv:2510.09270 (2025).
Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Responsible Persons	Stephan Eckstein

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Modules of Type Specialisation modules

The *specialisation modules* listed here are modules that generally build on one or more of the introductory modules and deepen the knowledge and competencies acquired there. If there are content dependencies on such modules or between each other, these are indicated in the respective module descriptions. There is no requirement to complete specific modules successfully to prevent any delays in the study programme due to missing individual examination components. The courses underlying the modules may have been included, in justified cases, in the modules of the third year of the Bachelor of Science in Mathematics. If the courses have already been included in the framework of a module in the Bachelor of Science in Mathematics, the module cannot be included in the Master of Science in Mathematics again. The module descriptions indicate which study specialisations the respective module is assigned to; the assignment may depend on taking additional modules.

The module descriptions on the following pages are sorted by the module numbers. For clarity, we first provide a list of modules sorted by their title in alphabetical order and then sorted by whether they are offered regularly or irregularly. Subsequently, there are listings of the modules based on their inclusion in the study specialisations.

Modules which are offered regularly (at least once every two years)

Modules which are not offered regularly

Modules in the Specialisation Algebra and Geometry

The following modules belong to the specialisation Algebra and Geometry. Possible restrictions can be found in the module description.

Modules in the Specialisation Analysis and Differential Geometry

The following modules belong to the specialisation Analysis and Differential Geometry. Possible restrictions can be found in the module description.

Modules in the Specialisation Mathematical Physics

The following modules belong to the specialisation Mathematical Physics. Possible restrictions can be found in the module description.

Modules in the Specialisation Numerial Mathematics and Optimisation

The following modules belong to the specialisation Numerial Mathematics and Optimisation. Possible restrictions can be found in the module description.

Modules in the Specialisation Stochastics

The following modules belong to the specialisation Stochastics. Possible restrictions can be found in the module description.

Modul descriptions (Specialisation modules)

Module Number: MAT-40-11	**Module Title:** Seminar: Advanced Knowledge in Mathematics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	every Semester		
Term	2-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Seminar, talk, presentation, e-learning, blended learning		
Content	Various topics from the Study Specialisations of the study programme.		
Objectives	Students work independently on in-depth questions from the area of their Study Specialisation and prepare them in a didactically appealing and scientifically sound form. They train their presentation techniques and sharpen their professional discussion style.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Seminar	S	o	2	3	yes	Pr	60-90	g	100

In addition to a successful talk, the acquisition of credit points also requires a regular active participation in the course, for example in the form of questions and contributions to the discussion or by completing assignments. In addition, a written elaboration of one's own presentation or the preparation of a handout for the participants may be part of the work to be done. This additional work constitutes the coursework for the module.

Transfer	-
Prerequisites	There are no further prerequisites.
Responsible Persons	The dean of studies at the Department of Mathematics

Abbreviations:

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Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-03	**Module Title:** Computer Algebra		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Canonical forms and standard bases for ideals and modules. • Computation of important operations for ideals and modules. • Syzygies, free resolutions and the proof of the Buchberger criterion. • Calculation of the primary decomposition of ideals. • Hilbert functions.		
Objectives	Students are familiar with important problems in the interplay of commutative algebra and algebraic geometry as well as algorithmic approaches to solving them. In particular, they are familiar with the theory of standard bases and their diverse applications. They are also familiar with important software packages in the field of symbolic computing and have implemented algorithms in these. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse. They also learnt about important software packages in the field of symbolic computing and have implemented algorithms in them themselves.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Computer Algebra	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Gert-Martin Greuel, Gerhard Pfister: A SINGULAR Introduction to Commutative Algebra. Springer 2008. • Wolfram Decker, Christoph Lossen: Computing in algebraic geometry. A quick start using SINGULAR. Springer 2006. • Wolfram Decker, Gerhard Pfister: A first Course in computational algebraic geometry. Cambridge University Press 2013. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-02 Commutative Algebra. Knowledge from the following modules is helpful but not strictly required: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, • MAT-45-11 Algebraic Geometry.
Responsible Persons	Daniele Agostini, Hannah Markwig, Thomas Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-11	**Module Title:** Algebraic Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Summer Semester		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Prevarieties and varieties. • Projektive varieties and homogeneous spectrum. • Finite and proper morphisms. • Blow-up and Grassmannians. • Rational maps. • Divisors and line bundles, class group and Picard group.		
Objectives	The students learn central terms, results and methods of modern Algebraic Geometry and they develop a deeper understanding of the interconnections between Geometry and Algebra. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Robin Hartshorne: Algebraic geometry. Springer 2006. • Klaus Hulek: Elementare algebraische Geometrie. Vieweg 2012. • Ernst Kunz: Einführung in die algebraische Geometrie. Vieweg 1997. • David Mumford: The red book of varieties and schemes. Springer 1999. • Miles Reid: Undergraduate algebraic geometry. Cambridge University Press 1988. • Igor R. Shafarevich: Basic algebraic geometry. Springer 1994.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Geometry and Toric Varieties' due to the large overlap in content.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-02 Commutative Algebra.
Responsible Persons	Daniele Agostini, Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-45-12</td><td colspan="2">Module Title:
Algebraic Geometry and Toric Varieties</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">regularly in Summer Semester</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Projective space.
• Prevarieties, morphisms, tangent space and singularities.
• Products and separation.
• Projective varieties and Grassmannians.
• Divisors and line bundles, class group and Picard group.
• Toric varieties.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students learn the central concepts, results and methods of modern Algebraic Geometry and they develop an advanced understanding of the relationships between Geometry and Algebra. Using the example of the class of toric varieties, they also learn how methods of Convex Geometry can facilitate the investigation of an important example class of Algebraic varieties, and expand the interplay of Algebra and Geometry by a further component. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Algebraic Geometry and Toric Varieties</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • David A. Cox, John B. Little, Henry K. Schenck: Toric varieties. American Mathematical Society 2011: • Robin Hartshorne: Algebraic geometry. Springer 2006. • Klaus Hulek: Elementare algebraische Geometrie. Vieweg 2012. • Ernst Kunz: Einführung in die algebraische Geometrie. Vieweg 1997. • David Mumford: The red book of varieties and schemes. Springer 1999. • Miles Reid: Undergraduate algebraic geometry. Cambridge University Press 1988. • Igor R. Shafarevich: Basic algebraic geometry. Springer 1994.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Geometry' due to the large overlap in content.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry.
Responsible Persons	Jürgen Hausen, Daniele Agostini

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-45-13</td><td colspan="2">Module Title:
Algebraic Transformation Groups</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Operations of algebraic groups on algebraic varieties, homogeneous spaces.
• Elements of the structure theory of affine-algebraic groups and their Lie algebras.
• Elements of the representation theory of affine-algebraic groups and their Lie algebras.
• Quotients in algebraic geometry.
• Classical invariant theory: Hilbert’s finiteness theorem, calculation of invariants.
• Geometrical invariant theory: Mumford’s construction of quotients, variation of quotients.
• Additionally certain aspects of topics from the following list are covered:
– Toric varieties;
– Spheric varieties.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students learn basic methods for mathematical work with symmetries on geometric structures. At the same time, they experience the interaction of different algebraic concepts, for example from group and ring theory, in algebraic geometry. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Algebraic Transformation Groups</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Armand Borel: Linear algebraic groups. Springer 1991. • Jean A. Dieudonne, James B. Carrell: Invariant theory. Academic Press 1971. • David Mumford: Geometric invariant theory. Springer 1965.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'Algebraic Groups' due to the large overlap in content.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, • MAT-45-12 Algebraic Geometry and Toric Varieties.
Responsible Persons	Daniele Agostini, Jürgen Hausen

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-45-14</td><td colspan="2">Module Title:
Algebraic Curves</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Projective curves, divisors, Theorem of Riemann-Roch.
• Ramified coverings, Theorem of Hurwitz.
• Linear systems, embeddings, Castelnuovo inequality.
• Singularities of plane curves, Puiseux expansions.
• Classification and moduli spaces, Jacobi variety.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students have familiarised themselves with the central concepts, results and methods in a selected sub-area of algebraic geometry. They have developed an in-depth understanding of algebraic curves and their classification. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Algebraic Curves</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o.
or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Robin Hartshorne: Algebraic geometry. Springer 2006.
• Gerd Fischer: Ebene algebraische Kurven. Vieweg 1994.
• Rick Miranda: Algebraic Curves and Riemann Surfaces. AMS 1995.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following (sub)modules of the Bachelor's and Master's programmes is required (for the latter, please also note the prerequisites specified there): • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.
Responsible Persons	Daniele Agostini, Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-15	**Module Title:** Toric Varieties and Mori Dream Spaces		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	In the lecture Mori Dream Spaces are considered as generalisations of toric varieties: • Geometry and combinatorial theory for toric varieties and Mori Dream Spaces. • Divisors on toric varieties and Mori Dream Spaces. • Quotient representation and Cox ring for toric varieties and Mori Dream Spaces. • Sheaves of divisorial algebras. • Cox sheaves and characteristic space. • Quotients of H-factorial affine varieties. • Shaded rings. • Varieties with torus operations.		
Objectives	Students have deepened their knowledge and understanding of the central concepts, results and methods of modern algebraic geometry in its interplay between geometry and algebra. With the class of Mori dream spaces, they have become familiar with a generalisation of toric varieties and their investigation using methods of convex geometry. In doing so, they have added another important methodological component to the interplay between algebra and geometry. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Toric Varieties and Mori Dream Spaces	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, and Antonio Laface. Cox rings. Cambridge University Press 2014. • Yi Hu, Sean Keel. Mori dream spaces and GIT. Michigan Math. J. 48: 331-348, 2000.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, • MAT-45-12 Algebraic Geometry and Toric Varieties.
Responsible Persons	Jürgen Hausen

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-16	**Module Title:** Introduction to Schemes	**Type of Module:** Compulsory Module with Choice
ECTS-Points	9	
Workload - Time in Class - Self-Study	Workload: 270 h; Time in Class: 90 h; Self-Study: 180 h	
Duration	1 Semester	
Frequency	not regularly	
Term	1-3	
Language of Instruction	German or English	
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS	
Content	• Affine schemes. • General schemes: local and global properties. • Morphisms. • Projective schemes. • Coherent sheaves. • Cohomology.	
Objectives	The students understand the scheme-theoretic foundations of algebraic geometry. They can translate classical varieties into the language of schemes, work with sheaves and locally ringed spaces, and analyse morphisms of schemes and their key properties. They also gain a basic understanding of sheaf cohomology and its role in studying the geometric properties of schemes. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.	

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Introduction to Schemes	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Andreas Gathmann: Algebraic Geometry. Lecture notes, RPTU Kaiserslautern. • Robin Hartshorne: Algebraic geometry. Springer 2006. • Ravi Vakil: The Rising Sea: Foundations of Algebraic Geometry. Princeton University Press 2025.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry.
Responsible Persons	Daniele Agostini, Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-17	**Module Title:** Toric Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Lattice and algebraic tori. • Monoids, monoidal algebras. • Affine toric varieties, normality. • Fundamentals of convex geometry cones, polytopes, fans. • Affine, complete and projective toric varieties. • Orbital layout of a toric variety. • Smoothness and singular points. • Singularity resolution. • Divisor class group and Picard group. • Intersection sheaves and homogeneous coordinates.		
Objectives	Students have deepened their knowledge and understanding of the central concepts, results and methods of modern algebraic geometry in its interplay between geometry and algebra and combinatorics. With the class of toric varieties and their investigation, they have learnt a way of combining algebraic geometry with methods of combinatorics. In doing so, they have added another important methodological component to the interplay between algebra and geometry. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Toric Geometry	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • David Cox, John Little, Hal Schenk: Toric varieties. AMS 2011. • William Fulton: Introduction to toric varieties. PUP 1993.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry.
Responsible Persons	Jürgen Hausen, Hannah Markwig, Daniele Agostini

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-18	Module Title: Cox Rings		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Divisorial algebras. • Cox rings. • Charakteristic spaces. • Good quotients. • Geometric invariant theory. • Gale-duality. • Connections to toric geometry. • Defining data for varieties with finitely generated Cox ring. • Singularities. • Picard group. • Basis locus. • Ampleness. • Kanonical class. • Intrinsic quadrics. • k*-surfaces. • Varieties with torus action.		
Objectives	Students have deepened their knowledge and understanding of the central concepts, results and methods of modern algebraic geometry in its interplay between geometry and algebra and combinatorics. They have familiarised themselves with the Cox ring as an algebraic object for investigating special classes of geometric spaces. In doing so, they expand the interplay between algebra and geometry with another important methodological component. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Cox Rings	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, Antonio Laface: Cox Rings. CUP 2014.

Transfer

The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites

Knowledge from the following modules is required; please also note the prerequisites specified for these:

- MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry,
- MAT-45-12 Algebraic Geometry and Toric Varieties.

Responsible Persons

Jürgen Hausen

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-45-24**	**Module Title:** Elliptic Functions and Elliptic Curves		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload **- Time in Class** **- Self-Study**	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Elliptic functions, Weierstrass-P-function, Riemann surfaces, complex tori. • Plane projective curves, Theorem of Bezout, elliptic curves. • Curves over finite fields, rational points. • Applications in cryptography. • Additionally a selection of the following: – Modular forms; – Classification of elliptic curves; – Moduli spaces.		
Objectives	The students have expanded their competence in mathematical-interdisciplinary studies. They have learnt about elliptic curves as a class of mathematical objects, which has comprehensive relevance in a wide spectrum of mathematical areas. The students have studied the notions, methods and results from the disciplines Complex Calculus, Algebraic Geometry, Number Theory, Topology and Cryptography, which are relevant in the given context, and they understand their mutual interrelations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Elliptic Functions and Elliptic Curves	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Wolfgang Fischer, Ingo Lieb: Funktionentheorie. Vieweg 2005. • Gerd Fischer: Ebene algebraische Kurven. Vieweg 1994. • Joseph H. Silverman: The arithmetic of elliptic curves. Springer 2009. • Ian Blake, Gadiel Seroussi, Nigel Smart: Elliptic curves in cryptography. CUP 1999.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is helpful but not strictly required: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry.
Responsible Persons	Daniele Agostini

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-45-28	Module Title: Elliptic Curves and Taniyama-Shimura		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	A selection of the following topics will be covered: • Group-law, arithmetic of elliptic curves. • Modular curves and forms. • Riemann surfaces, abelian differentials, Jacobian. • Geometric version of Taniyama-Shimura-conjecture (i.e. Wiles' modularity theorem) explained. • Connection to Fermat's last theorem. • L-series.		
Objectives	The students have learnt and understood the interplay of methods of algebra, number theory and geometry to answer profound mathematical questions using the example of the conjecture of Taniyama-Shimura and its application to the proof of Fermat's theorem. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see table below In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.		
Literature	**Possible References :** • Joseph H. Silverman: The Arithmetic of Elliptic Curves. Springer 2009.		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Elliptic Curves and Taniyama-Shimura	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following (sub)modules of the Bachelor's and Master's programmes is required (for the latter, please also note the prerequisites specified there): • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry, • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Responsible Persons	Ivo Radloff

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-50-06**	**Module Title:** Tropical Enumerative Geometry - Part 2		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Plane curves of higher genus. • Multiplicities. • Welschinger invariants. • Lattice paths. • Floor diagrams. • Hurwitz numbers. • Tropical moduli spaces.		
Objectives	The students deepen their knowledge on the combinatorics of plane tropical curves. They get acquainted with various methods to enumerate tropical curves, as well as with various enumerative problems which can be solved with the aid of tropical geometry. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Tropical Enumerative Geometry - Part 2	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Diane Maclagan, Bernd Sturmfels: Introduction to tropical geometry. AMS 2015.
- Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.

Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is conceptually part of the module Tropical Enumerative Geometry and cannot be included alongside it.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-05 Tropical Enumerative Geometry - Part 1.
Responsible Persons	Hannah Markwig

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-11	**Module Title:** Geometry of Manifolds 2		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Global aspects of Riemannian geometry • Cohomology of manifolds. • Analysis of differential operators on manifolds. • Applications on Riemann surfaces (and complex manifolds).		
Objectives	The students are familiar with the fundamental concepts of global real and complex differential geometry. They have deepened their understanding of methods in differential geometry, and they have exemplarily experienced how local and global aspects in geometry interact. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Geometry of Manifolds 2	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011. • John Milnor: Morse Theory. PUP 1963. • Donu Arapura: Algebraic Geometry over the Complex Numbers. Springer 2012. • Sundararaman Ramanan: Global Calculus. AMS 2005.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-10 Geometry of Manifolds 1.
Responsible Persons	Christoph Bohle, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-50-16**	**Module Title:** Riemannian Geometry		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Riemannian manifolds. • Geodesics. • Curvature. • Geometry of submanifolds.		
Objectives	The students have learned and understood definitions and main examples of Riemannian manifolds from a classical point of view. In addition, topics related to geodesics were discussed. The students were exposed to important geometric results involving geodesics which are sufficient to study their role in different areas of differential geometry. Intuition for various notions of curvature was developed by the students and familiarity with computations in differential geometry was achieved in the exercise sessions. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Riemannian Geometry	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- John M. Lee: Riemannian manifolds: An introduction to curvature. Springer 1997.
- Barret O'Neill: Semi-Riemannian geometry. With applications to relativity. Academic Press 1983.

Transfer	The module belongs to the *Study Specialisations Algebra and Geometry*, *Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-11 Geometry in Physics.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-17	**Module Title:** Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory)		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	Integrable systems are differential or difference equations with extraordinarily large symmetry group. The course will focus on equations related to the Korteweg de Vries (KdV) equation and discrete counterparts. Originally a mathematical model for the soliton phenomenon discovered during a famous horse ride along a canal, equations of KdV type have now many applications and the underlying theory involves various mathematical disciplines. A fundamental idea for understanding and solving KdV type equations is their interpretation as spectrum preserving deformations of underlying auxiliary linear operators - in the simplest case symmetric matrices. We study an important class of explicit solutions that includes solitons and finite gap (or algebro-geometric) solutions. This class of solutions can be described using a combination of Riemann surface theory and classical mechanics. The relevant parts of classical mechanics, Riemann surface theory, and spectral theory will be explained in the lecture. We will also briefly touch upon an integrable systems interpretation of the QR-algorithm of numerical linear algebra. The KdV equation is related to geometry in several different ways: for example, it can be interpreted as a dynamical system on the space of parametrized curves in the plane; it is deeply related to the geometry of Lie algebras and Lie groups; the special solutions discussed in the lecture are related to the geometry of Riemann surfaces... In a sequel to the lecture, it is planed to explain how infinite dimensional projective geometry allows to interpret generalizations of the KdV equation as quadratic equations and to finally linearize their dynamics.		
Objectives	The students have seen and understood relations between classical topics like Riemann surfaces, mechanics, and spectral theory – as well as other branches of mathematics – that were discovered mainly in the second half of the twentieth century during the emergence of a branch of mathematics sometimes called *soliton theory* or *integrable mathematics* The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	<table><tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr><tr><td rowspan="2">Introduction to Integrable Systems</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr><tr><td>E</td><td>f</td><td>2</td><td>3</td></tr></table> In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.
Literature	**Possible References :** • Olivier Babelon, Denis Bernard, Michel Talon: Introduction to classical integrable systems. CUP 2004. • Leonid A. Dickey: Soliton equations and Hamiltonian systems. World Scientific 2003. • Alan C. Newell: Solitons in mathematics and physics. SIAM 1985. • Sergei P. Novikov, Sergei V. Manakov, Lev P. Pitaevskii, Vladimir E. Zakharov: Theory of Solitons - The Inverse Scattering Method. Consultants Bureau 1984).
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry*, *Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is helpful but not strictly required: • MAT-50-10 Geometry of Manifolds 1.
Responsible Persons	Christoph Bohle, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-50-18</td><td colspan="2">Module Title:
Integrable Systems (and Infinite Dimensional Lie Algebras)</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Integrable systems are differential or difference equations with extraordinarily large symmetry group. The course will focus on equations related to the Korteweg de Vries (KdV) equation and discrete counterparts. Originally a mathematical model for the soliton phenomenon discovered during a famous horse ride along a canal, equations of KdV type have now many applications and the underlying theory involves various mathematical disciplines.
A fundamental idea for understanding and solving KdV type equations is their interpretation as spectrum preserving deformations of underlying auxiliary linear operators - in the simplest case symmetric matrices.
This lecture is the continuation of the lecture called Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory). This continuation will investigate integrable equations using sl(2,C)–loop algebras. In particular, we will study explicit solutions that can be described using the theory of hyperelliptic Riemann surfaces.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have aquired a uniform point of view on integrable equations related to the loop algebra of sl(2,C). The students can name and prove the central results of the lecture and they can explain their intrinsic connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Integrable Systems (and Infinite Dimensional Lie Algebras)</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Olivier Babelon, Denis Bernard, Michel Talon: Introduction to classical integrable systems. CUP 2004. • Leonid A. Dickey: Soliton equations and Hamiltonian systems. World Scientific 2003. • Alan C. Newell: Solitons in mathematics and physics. SIAM 1985. • Sergei P. Novikov, Sergei V. Manakov, Lev P. Pitaevskii, Vladimir E. Zakharov: Theory of Solitons - The Inverse Scattering Method. Consultants Bureau 1984).
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry*, *Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-17 Introduction to Integrable Systems (Classical Mechanics, Riemann Surfaces, and Spectral Theory).
Responsible Persons	Christoph Bohle, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-50-19**	**Module Title:** Mathematical Aspects of Deep Learning 2		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Mechanisms behind (mathematical) reasoning in large language models (like GPT o1). Search and reinforcement learning. • Towards multimodal large language models: how to make large language models *see* and *draw* and allow comprehension and creation of images. • Uses of neural networks in applied mathematics (engineering) e.g. modeling with PDEs, simulation, inverse problems. For example physics informed neural networks, neural operators, 3d reconstruction from 2d images.		
Objectives	Students are familiar with some current problems in Deep Learning and with applications within or with relevance to Mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see table below. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.		
Literature	**Possible References :** • Ian Goodfellow, Yoshua Bengio, Aaron Courville: Deep Learning. MIT 2016. • Simon J. D. Price: Understanding Deep Learning. MIT Press 2023. • Christopher M. Bishop, Hugh Bishop: Deep Learning - Foundations and Concepts. Springer 2024.		
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry*, *Analysis and Differential Geometry* und *Stochastik*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.		

Requirements for obtaining Credits / Grading:

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Aspects of Deep Learing 2	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-14 Mathematical Aspects of Deep Learning 1.
Responsible Persons	Christoph Bohle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-22	Module Title: Algebraic Topology 2		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Further development of singular homology theory. • Simplicial complexes and their simplicial homology. • CW spaces and their cellular homology. • Axiomatic homology. • Homological algebra. • Cohomology. • Homology and Cohomology with coefficients. • Product structures in homology and cohomology. • The Poincaré duality theorem for topological manifolds.		
Objectives	The students extend their ability to transfer concrete topological problems into algebraic constructions. They deepen their knowledge in abstract mathematical disciplines to accomplish even technically very challenging tasks. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Topology 2	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.
Transfer	The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-21 Algebraic topology 1.
Responsible Persons	Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-23	**Module Title:** Algebraic Topology 3		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	A selection of the following topics will be covered: • Basic concepts of homotopy theory; • Homotopy group of spheres; • Spectral sequences; • K-theory; • Characteristic classes.		
Objectives	With the in-depth knowledge of algebraic topology that they have acquired, students are now introduced to current areas of research and they tackle a small research project themselves, which can lead to a Master's thesis, for example. They will also lay the foundations for a possible doctorate in algebraic topology. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Algebraic Topology 3	L	f	2	3	no	P		g	100

Specifics on the portfolio will be explained by the examiner at the beginning of the course.

Literature

Possible References :

- Allen Hatcher: Algebraic topology. Cambridge University Press 2009.
- Allen Hatcher: Vector bundles and K-theory. Manuskript 2009.
- John W. Milnor, James D. Stasheff: Characteristic classes. Princeton University Press 1974.
- John W. Milnor: Lectures on the h-cobordism theorem. Princeton University Press 1965.

Transfer

The module belongs to the *Study Specialisations Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-21 Algebraic topology 1, • MAT-50-22 Algebraic Topology 2.
Responsible Persons	Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**		**Type of Module:**
MAT-50-24	Mathematical Aspects of Reinforcement Learning		Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Foundations of reinforcement learning and comparison with classical optimal control theory. • Fundamental algorithms. • Computer-based experiments (possibly using the Robot Operating System (ROS2)).		
Objectives	The students have acquired the mathematical foundations of reinforcement learning. They are familiar with modern architectures and the mathematical questions arising from them. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Aspects of Reinforcement Learning	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 5.

Literature

Possible References :

- Richard S. Sutton, Andrew G. Barto: Reinforcement Learning: An Introduction. MIT Press 2018.
- Aske Plaat: Deep Reinforcement Learning. Springer 2022.

Transfer

The module belongs to the *Study Specialisations Algebra and Geometry*, *Analysis and Differential Geometry* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics. In addition, prior knowledge of deep learning is required, such as that provided in module MAT-50-14 and MAT-50-19.
Responsible Persons	Christoph Bohle

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-27	**Module Title:** Topological Vector Spaces and Distributions		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	A selection of the following topics will be covered: • Locally convex topological vector spaces, Frechet spaces, LF spaces and LB spaces. • Duality: Hahn-Banach theorem, dual space, topologies on the dual space. • Generalised functions, Radon measures and distributions. • Properties of distributions and operations on the space of distributions. • Applications and examples.		
Objectives	Students master the basic principles and techniques of the theory of topological vector spaces and understand how to apply this to the theory of generalised functions according to L. Schwartz. Students are also able to name the main applications of the theory and show which classical questions of mathematical physics can be treated with it. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Topological Vector Spaces and Distributions	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Gerald Folland: Real Analysis. Wiley 1999. • Helmut H. Schäfer: Topological Vector Spaces. Springer 1999. • Laurant Schwartz: Theorie des Distributions. Hermann 1998. • Laurant Schwartz: Mathematics for the Physical Sciences. Dover 2008. • Francois Trèves: Topological Vector Spaces, Distributions and Kernel. Dover 1967.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics.* Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Ulrich Groh, Rainer Nagel

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-28	**Module Title:** Uniformisation of Riemann Surfaces	**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5	
Workload - Time in Class - Self-Study	Workload: 150 h; Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester	
Frequency	not regularly	
Term	1-3	
Language of Instruction	German or English	
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS	
Content	• Uniformisation of Riemann surfaces	
Objectives	The students have learnt how to determine the simply connected Riemann surfaces by successively solving suitable differential equations. They are then able to classify Riemann surfaces under suitable conditions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.	

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Uniformisation of Riemann Surfaces	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.

Literature	**Possible References :** • Hershel M. Farkas, Irwin Kra: Riemann Surfaces. Springer 1992.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry* and *Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-50-15 Introduction to Riemannian Surfaces.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-50-40	**Module Title:** Gromov-Witten Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 3 SWS + Ex.cl. 1 SWS		
Content	• Enumerative geometry, • Moduli spaces of stable curves, • Moduli spaces of stable maps, • Universal families, • Forgetful maps, • Gluing maps, • Gromov-Witten invariants, • Computation of Gromov-Witten invariants, • Divisor equations, • Kontsevich's formula.		
Objectives	Students are based on their knowledge in Algebraic geometry introduced into the current research field of Gromov-Witten theory and enumerative geometry. The students know and understand important example classes of enumerative invariants and know how to present them as cut products on moduli spaces. Students master the basic algorithms for calculating Gromov-Witten invariants. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Gromov-Witten Theory	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Joachim Kock, Israel Vainsencher: An invitation to quantum cohomology: Kontsevich's formula for rational plane curves. Birkhäuser 2007. • Ravi Vakil: The moduli space of curves and Gromov-Witten theory. Enumerative invariants in algebraic geometry and string theory. Lecture Notes in Mathematics, 1947. Springer 2008.
Transfer	The module belongs to the *Study Specialisation Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-45-01 Introduction to Commutative Algebra and Algebraic Geometry.
Responsible Persons	Hannah Markwig

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-02</td><td colspan="2">Module Title:
Non-Linear Functional Analysis</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Differentiation and integration in Banach spaces.
• Compact, coercive, proper mappings and gradient mappings.
• Fredholm mappings.
• Continuity method.
• Degree of mapping.
• Fixed point theorems.
• Variational inequalities.
• Monotone operators.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students master the differentiation and integration of non-linear functions and various functional analytical methods for solving non-linear equations in infinite-dimensional spaces. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes, students have acquired confidence in the technical handling of the methods they have learnt and can apply them independently to other problems. They are able to present their problem solutions and participate in discourses on problems in this field of research.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Non-Linear Functional Analysis</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Melvyn Berger: Nonlinearity in Functional Analysis. Elsevier 1977. • Klaus Deimling: Nonlinear Functional Analysis. Springer 1985. • Eberhard Zeidler: Nonlinear Functional Analysis and its Applications I. Fixed-Point Theorems. Springer 1986.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-03	**Module Title:** Operator Theory		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		

Content

- Operator semigroups and abstract Cauchy problems.
- Theorem of Hille-Yosida.
- Applications of concrete evolution equations.
- Spectral theory of semigroups and their generators.
- Asymptotic of semigroups.
- Applications:
  - Semigroups of ordinary and partial differential equations;
  - Semigroups of transport problems;
  - Semigroups of control theory.

Objectives

Students have understood the concept of the operator-valued exponential function. They are able to deal with concrete evolution equations in this abstract form. They are able to prove well-posedness using the Hille-Yosida theorem and discuss the qualitative behaviour of solutions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Operator Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Bruce Blackadar: Operator algebras. Springer 2006. • Klaus Jochen Engel, Rainer Nagel: One-parameter semigroups for linear evolution equations. Springer 2000. • Klaus Jochen Engel, Rainer Nagel: A short course on operator semigroups. Springer 2006. • Gert Pedersen: Analysis now. Springer 1995.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Rainer Nagel, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-04**	**Module Title:** Operator Algebras		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Geometry of Hilbert spaces. • Operators on Hilbert spaces and their spectral properties. • Spectral theory in Banach algebras. • Commutative Banach algebras and the representation theorem of Gelfand and Gelfand-Naimark. • The spectral theorem for normal operators of a Hilbert space. • Operator topologies and von Neumann's bicommutant theorem. • Kaplansky's density theorem. • Von Neumann algebras and their classification according to Murray-von Neumann, construction of examples. • The axiomatics of C^*- and W^*-algebras, the theorem of Gelfand-Naimark-Segal theorem for C^*-algebras and the representation theorem of Sakai for W^*-algebras. • Applications and outlook.		
Objectives	The students have familiarised themselves with the central concepts, results and methods of the theory of operator algebras. They have learnt the interplay between algebra and topology using the example of von Neumann algebras and their classification. The students also recognise how taking a higher point of view, i.e. the axiomatic nature of the problem, allows different questions to be dealt with and solved simultaneously. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Operator Algebras	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	f	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									

Literature	**Possible References :** • Bruce Blackadar: Operator algebras. Springer 2006. • Ola Bratelli, Derek Robinson: Operator Algebras and Quantum Physics. Springer 1997. • Richard Kadison, John Ringrose: Fundamentals of the Theory of Operator Algebras I - IV. AMS 1997. • Gert Pedersen: Analysis now. Springer 1995. • Shoichiro Sakai: C^*- and W^*-Algebras. Springer 1998. • Masamichi Takesaki: Theory of Operator Algebras I - II. Springer 2002.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Ulrich Groh, Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-05	Module Title: Ergodic Theory	Type of Module: Compulsory Module with Choice
ECTS-Points	9	
Workload - Time in Class - Self-Study	Workload: 270 h Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester	
Frequency	not regularly	
Term	1-3	
Language of Instruction	German or English	
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS	
Content	• Topological and measure-theoretical dynamical systems. • Recurrence and mixing properties. • Ergodic theorems of von Neumann and Birkhoff. • Spectral theory of the Koopman operator. • Operators with discrete spectrum (Halmos-von Neumann) • Applications in stochastics and number theory.	
Objectives	The students have familiarised themselves with the central concepts, results and methods of ergodic theory. They have experienced the profound interplay between measure theory and topology using the example of dynamic systems and their classification. The functional-analytical perspective makes it possible to deal with and solve various problems simultaneously. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.	
Requirements for obtaining Credits / Grading (Weighting if applicable)	see table below	

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Ergodic Theory	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Manfred Einsiedler, Thomas Ward: Ergodic Theory with a View Towards Number Theory. Springer 2011. • Tanja Eisner, Balint Farkas, Markus Haase, Rainer Nagel: Operator Theoretic Aspects of Ergodic Theory. Springer 2015. • Paul Halmos: Lectures on Ergodic Theory. Martino Fine Books 2013. • Marcelo Viana, Krerley Oliveira: Foundations of Ergodic Theory. CUP 2016.
Transfer	The module belongs to the *Study specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-08	**Module Title:** Spectral Theory of Positive Operators		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	Starting from the classical theorems of Perron and Frobenius on the spectrum of positive matrices, positive linear mappings to C^*- and W^*-algebras and their spectral and algebraic properties are analysed. The ergodic properties of these operators, i.e. the convergence of powers and means, can then be derived from these. We then discuss the generalisation to operator semigroups. Applications of the theory can be found in mathematical physics, among others.		
Objectives	Students learn the basic spectral properties of positive operators on C^*- and W^*-algebras and the connections with non-commutative ergodic theory. In the seminar following the lecture, students can work on topics that lead to a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Spectral Theory of Positive Operators	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.

Literature

Possible References :

- Tanja Eisner, Markus Haase, Rainer Nagel : Operator Theoretic Aspects of Ergodic Theory. Springer 2015.
- Ulrich Groh: Spectral Theory of Completely Positive Maps on C^*- and W^*-Algebras. Preprint.

Transfer

The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Ulrich Groh, Rainer Nagel

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-12	**Module Title:** Harmonic Analysis in Euclidean Space		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Fourier transformation. • Covering-, decomposition- and interpolation theorems. • Singular integrals, Poisson integrals. • Hardy- and BMO-spaces, multiplier theorems, Littlewood-Paley theory.		
Objectives	The students got to know the central terms, results and methods of harmonic analysis in euclidean space. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see below		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Harmonic Analysis in Euclidean Space	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Charles L. Feffermann, Elias M. Stein: H^p spaces of several variables. Acta Mathematica 129, pp. 137-193, 1972. • Christopher D. Sogge: Fourier integrals in classical analysis. Cambridge University Press 2017. • Elias M. Stein: Singular integrals and differentiability properties of functions. Princeton University Press 1970. • Elias M. Stein: Harmonic analysis. Princeton University Press 1993. • Elias M. Stein, Guido Weiss: Introduction to Fourier analysis on Euclidean spaces. Princeton University Press 1971.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-13</td><td colspan="2">Module Title:
Harmonic Analysis on Abelian Groups</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Locally compact groups, existence and uniqueness of Haar measures.
• Convolution algebras, Banach algebras, the Gelfand-Neumark theorem.
• LCA groups, Pontryagin duality, Plancherel theorem, structure theory of LCA groups.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have become familiar with the central concepts and methods of abstract harmonic analysis and know how to use them. They have understood the connection between topological/analytical/geometric concepts such as LCA groups and their expression in algebraic structures such as C^*-algebras and are able to apply this way of thinking to other theories. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Harmonic Analysis on Abelian Groups</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Anton Deitmar: A first course in Harmonic Analysis. Springer 2005.
• Anton Deitmar, Siegfried Echterhoff: Principles of Harmonic Analysis. Springer 2008.
• Edwin Hewitt, Kenneth Ross: Abstract harmonic analysis. Vol. I. Springer 1979.
• Walter Rudin: Fourier analysis on groups. John Wiley 1990.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-14	**Module Title:** Harmonic Analysis on General Groups		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Representation theory of compact groups, Peter-Weyl theorem. • Representation theory of general groups. • trace formula and applications to the Heisenberg group and $SL_2(\mathbb{R})$.		
Objectives	The students have familiarised themselves with the deeper concepts and methods of abstract harmonic analysis and know how to use them. They have mastered the trace formula and understand its far-reaching implications, also for other areas of mathematics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Harmonic Analysis on General Groups	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Anton Deitmar, Siegfried Echterhoff: Principles of Harmonic Analysis. Springer 2008. • Gerald B. Folland: A course in abstract harmonic analysis. Studies in Advanced Mathematics. Boca Raton 1995. • Michael E. Taylor: Noncommutative Harmonic Analysis. AMS 1986.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Anton Deitmar

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-22</td><td colspan="2">Module Title:
Partial Differential Equations</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Schauder estimates.
• Calderon-Zygmund estimates.
• Harnack’s inequality.
• Hölder regularity.
• Viscosity solutions.
• Existence of solutions according to Perron.
• Evans-Krylov theorem.</td></tr>
<tr><td>Objectives</td><td colspan="3">After the students have learnt the basic concepts and methods in Introduction to Partial Differential Equations, this knowledge is deepened. Students are prepared for and introduced to current research questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Partial Differential Equations</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Luis Angel Caffarelli, Xavier Cabre: Fully nonlinear elliptic equations. American Mathematical Society 1995. • Michael G. Crandall, Hitoshi Ishii, Pierre-Louis Lions: User's Guide to Viscosity Solutions of second Order Partial Differential Equations. Bulletin of the American Mathematical Society 27, No. 1, pp. 1-67, 1992. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear elliptic equations.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Stationary Differential Equations or Numerics of Instationary Differential Equations, it can be included in the specialisation *Numerical Mathematics and Optimisation*.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-24	**Module Title:** Nonlinear Elliptic Partial Differential Equations illustrated by the Example of Minimal Surfaces		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	The course will consider PDE aspects of minimal surfaces, beginning with the existence theory for minimal graphs with prescribed boundary data. Emphasis will be placed on the De Giorgi-Nash estimate, which is one of the key achievements of 20th Century mathematics, and is foundational in the study of quasilinear elliptic and parabolic equations. We will also explore connections between minimal surfaces and the Allen-Cahn equation, which is a semilinear equation arising in the theory of phase transitions. Here the focus will be on rigidity results for entire solutions (namely the Bernstein problem and closely related De Giorgi conjecture) and their use in proving regularity via rescaling.		
Objectives	The students obtain an advanced working knowledge of nonlinear elliptic PDE theory, and an understanding of connections between this theory and profound problems in geometry. They will acquire an array of new techniques for establishing quantitative and qualitative control on objects governed by nonlinear differential equations. These techniques include advanced applications of Sobolev theory, rescaling and compactness arguments, Stampacchia iteration, Moser iteration, and the use of monotonicity formulae. Students will be able to assess if and when these techniques are applicable to a given problem. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Nonlinear Elliptic Partial Differential Equations illustrated by the Example of Minimal Surfaces	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Lawrence C. Evans: Partial Differential Equations. AMS 2010. • David Gilbarg, Neil Trudinger: Elliptic partial differential equations of second order. Springer 1083. • David Kinderlehrer, Guido Stampacchia: An introduction to variational inequalities and their applications. Siam 2000.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is helpful but not strictly required: • MAT-55-21 Introduction to Partial Differential Equations.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-26	**Module Title:** Partial Differential Equations in Conformal Geometry: the Yamabe Problem		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The uniformisation theorem states that a closed two-dimensional Riemannian manifold is conformally equivalent to one with constant sectional curvature. The appropriate higher-dimensional analogue problem is finding a conformal metric with constant scalar curvature. This is the so-called Yamabe problem, introduced by Yamabe in 1960. The flaws in his proof were fixed only in 1984, by combining the works of Trudinger, Aubin, Schoen, and others. This course provides an overview of the complete proof of the Yamabe problem, which is extremely rich in techniques of calculus of variation, geometric analysis, and elliptic partial differential equations. It also employs crucial results of mathematical relativity, such as the Positive Mass Theorem, which is motivated by physics but possesses powerful analytical consequences. Moreover, basic results for non-compact manifolds or manifolds with boundary and the parabolic analogue, the Yamabe flow, will be discussed.		
Objectives	The students can state the Yamabe problem and explain the structure of the proof. The students are familiar with basic methods of the calculus of variation, particularly concerning the (sub-critical) Yamabe energy functionals and the associated elliptic semi-linear PDEs. They can study the conformal Green's function and relate it to the solvability of the Yamabe problem. The students are also familiar with the notion of mass in mathematical relativity and the renowned Positive Mass Theorem, still linking it to the solution of the Yamabe problem; they can provide the main ideas of the proof of the Positive Mass Theorem and, in particular, master elements of the minimal surface theory. They are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Partial Differential Equations in Conformal Geometry: the Yamabe Problem	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • John M. Lee, Thomas H. Parker: The Yamabe problem. Bull. Am. Math. Soc., New Ser. 17, 37–91, 1987. • Richard Schoen, Shing-Tung Yau: Lectures on differential geometry. International Press 1994. • Thierry Aubin: Some nonlinear problems in Riemannian geometry. Springer 1998. • Michael Struwe: Variational methods. Applications to nonlinear partial differential equations and Hamiltonian systems. Springer 2008.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations.
Responsible Persons	Carla Cederbaum

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-27**	**Module Title:** Fully Non-Linear Elliptic Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Solution of general fully non-linear equations with elliptic equations. • Solution of the Monge-Ampere equation.		
Objectives	The students learn the techniques to successively estimate the supremum, the gradient and the second derivatives of a given solution of a fully non-linear elliptic equation. The students learn how the modulus of continuity of the second derivatives is then estimated using the Evans-Krylov theorem and learn the continuity method, which leads to the existence of a solution. In particular, they are able to apply the methods to general uniformly elliptic equations and to the special, non-uniformly elliptic Monge-Ampere equation. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Fully Non-Linear Elliptic Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.

Literature

Possible References :

- Luis A. Caffarelli, Joseph Kohn, Joel Spruck: The Dirichlet problem for nonlinear second-order elliptic equations. I. Monge-Ampere equation. In: Communications on Pure and Applied Mathematics 37,3 pp. 369-402.
- Luis A. Caffarelli, Joseph Kohn, Luis Nirenberg, Joel Spruck: The Dirichlet problem for nonlinear second-order elliptic equations. II. Complex Monge-Ampere, and uniformly elliptic, equations. In: Communications on Pure and Applied Mathematics 38,2 pp. 209-252.
- David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order. Springer 1998.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module MAT-60-36 'Fully nonlinear elliptic and parabolic partial differential equations' due to the large overlap in content.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-55-22 Partial Differential Equations.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-28</td><td colspan="2">Module Title:
Morse Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">3</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
90 h</td><td>Time in Class:
30 h</td><td>Self-Study:
60 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• Topology of differentiable manifolds.
• Riemannian metrics on differentiable manifolds.
• Dynamic systems on differentiable manifolds.
• Homotopy type of differentiable manifolds.
• Main approaches of Morse theory.
• Outlook on Morse homology.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students learn how to analyse problems in the differential topology of differentiable manifolds using the tools of analysis, in particular the theory of dynamical systems. In particular, they learn how the level surfaces of non-degenerate functions, so-called Morse functions, can be used to obtain statements about the homotopy type of manifolds. They also build a bridge to algebraic topology, which analyses the topology (of manifolds) using algebraic methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td>Morse Theory</td><td>L</td><td>f</td><td>2</td><td>3</td><td>no</td><td>wr. o. or.</td><td>90-180
o. 20-30</td><td>g</td><td>100</td></tr>
</table>
Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• John Milnor: Morse Theory. Annals of Math. Studies, Number 51. Princeton University Press 1961.
• Morris W. Hirsch: Differential Topology. Graduate Texts in Mathematics 33. Springer 1988.</td></tr>
</table>

Transfer	The module belongs to the *Study Specialisation Studienschwerpunkt Analysis and Differential Geometry* and *Algebra and Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is helpful but not strictly required: • MAT-65-11 Geometry in Physics.
Responsible Persons	Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-55-42</td><td colspan="2">Module Title:
Geometric Measure Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">First and second variation for varifolds.
Monotonicity formula.
Allard’s integral compactness theorem.
Lipschitz approximation.
tilt-excess descent.
Allard’s regularity theorem.
General and rectifible flows.
Deformation theorem.
Surface minimizing flows.</td></tr>
<tr><td>Objectives</td><td colspan="3">After having learned the basic terms and methods in introduction to geometric measure theory, this knowledge is deepened. The students will be prepared for and guided to problems of recent research. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if necessary to argue for it in a critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Geometric Measure Theory</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Herbert Federer: Geometric measure theory. Springer 1969. • Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-41 Introduction to Geometric Measure Theory.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-55-43**	**Module Title:** Area Minimising Flows		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Compactness theorem for integral flows. • Regularity of area minimising flows.		
Objectives	After having learned the essential notions and methods of geometric measure theory, this knowledge is deepened. The students will be prepared for and guided to problems of recent research. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Area Minimising Flows	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Herbert Federer: Geometric measure theory. Springer 1969. • Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-41 Introduction to Geometric Measure Theory, • MAT-55-42 Geometric measure theory.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-46	**Module Title:** Elastic Curves		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Classification of elastic curves according to Langer and Singer. • Order reduction of the Euler-Lagrange equation of the elastic energy of a curve. • Qualitative behaviour of an elastic curve. • Solving the Willmore equation under axial symmetry with variational methods.		
Objectives	Students learn how to deal with a geometrically relevant functional and its critical points using the example of the elastic energy of a curve. This gives them an insight into the theory of fourth-order elliptic differential equations where familiar techniques, such as the maximum principle, can no longer be used. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		
Requirements for obtaining Credits / Grading (Weighting if applicable)	see table below		

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Elastic Curves	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Filippo Gazzola, Hans-Christoph Grunau, Guido Sweers: Polyharmonic Boundary Value Problems, Springer 2010.
- David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 1998.
- Joel Langer, David A. Singer: The total squared curvature of closed curves, J. Differential Geom. Band 20, Nummer 1, Seiten 1-22, 1984.
- John M. Lee: Introduction to smooth manifolds. Springer 2013.
- Michael Struwe: Variational Methods. Springer 2008.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-47	Module Title: Geometric Measure Theory – Varifolds		Type of Module: Compulsory Module with Choice
ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• First and second variation for varifolds. • Monotonicity formula. • Allard's integral compactness theorem. • Lipschitz approximation. • tilt-excess descent. • Allard's regularity theorem.		
Objectives	After the students have learnt the basic concepts and methods in Introduction to Geometric Measure Theory, this knowledge is deepened with a view to variabilities. Students will be prepared for and introduced to current research questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Geometric Measure Theory – Varifolds	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Herbert Federer: Geometric measure theory. Springer 1969.
- Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984.
- Leon Simon: Lectures on geometric measure theory. Australian National University 1984.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is part of the module Geometric Measure Theory and cannot be taken together with it.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-41 Introduction to Geometric Measure Theory.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-48	**Module Title:** Geometric Measure Theory – Flows		**Type of Module:** Compulsory Module with Choice
ECTS-Points	4.5		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• General and rectifiable flows. • Deformation theorem. • Surface minimizing flows.		
Objectives	After the students have learnt the basic concepts and methods in Introduction to Geometric Measure Theory, this knowledge is deepened with a view to variabilities. Students will be prepared for and introduced to current research questions. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Geometric Measure Theory – Flows	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Herbert Federer: Geometric measure theory. Springer 1969.
- Enrico Giusti: Minimal surfaces and functions of bounded variation. Birkhäuser 1984.
- Leon Simon: Lectures on geometric measure theory. Australian National University 1984.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module is part of the module Geometric Measure Theory and cannot be taken together with it.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-41 Introduction to Geometric Measure Theory.
Responsible Persons	Reiner Schätzle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-55-71	Module Title: Operator Algebras and their Applications to Statistical Mechanics	Type of Module: Compulsory Module with Choice
ECTS-Points	6	
Workload - Time in Class - Self-Study	Workload: 180 h; Time in Class: 60 h; Self-Study: 120 h	
Duration	1 Semester	
Frequency	not regularly	
Term	1-3	
Language of Instruction	English	
Forms of Teaching and Learning	Lecture 3 SWS + Ex.cl. 1 SWS	

Content

- Basics on operator algebras (C*-algebras, algebraic states, inductive limits);
- Introduction to algebraic deformation quantization (general set-up, coherent states, examples);
- Applications to the classical limit of quantum mechanics and statistical mechanics including asymptotic emergence (phase transitions, large deviations (entropy), spontaneous symmetry breaking).

Objectives

The students have obtained deepend knowledge in selections questions in algebraic quantum theory with an emphasis on algebraic deformation quantisation and their applications to the classical limit of quantum mechanics and statistical mechanics. They have learned algebraic techniques in order to develop abstract structures encoding the features of a physical theory. They are familiar with techniques to prove existence results of limits of sequences/nets encoded by algebraic states, examine these, and put them into a general perspective. Moreover, they understand the physical relevance of the results and are able to relate them to features of equilibrium thermodynamics, such as phase transitions and spontaneous symmetry breaking. They are able to describe the current state of research in the specific area. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Operator Algebras and their Applications to Statistical Mechanics	L	f	3	4,5	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Klaas Landsman: Foundations of Quantum Theory, From Classical Concepts to Operator Algebras. Springer 2017.

Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry*, *Mathematical Physics* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-01 Functional Analysis.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-01	**Module Title:** Geometric Evolution Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Examples of geometric evolution equations such as Mean curvature flow, Ricci flow, Inverse mean curvature flow. • Parabolic maximum principles. • Regularity theory for parabolic evolution equations. • Rescaling techniques and description of singularities. • Asymptotic behavior of solutions.		
Objectives	Students learn to combine their knowledge of differential geometry and partial differential equations and apply it to specific problems in selected geometric evolution equations. They learn techniques for checking solutions of non-linear evolution equations, which enables them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Geometric Evolution Equations	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. In combination with one of the modules Numerics of Nonstationary Differential Equations or Numerics of Differential Equations on Surfaces, it can be included in the subject specialisation *Numerical Mathematics and Optimisation.*
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.

Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-60-02</td><td colspan="2">Module Title:
Geometric Variation Problems</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">3</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
90 h</td><td>Time in Class:
30 h</td><td>Self-Study:
60 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Examples of geometric variation problems such as minimal surfaces, capillary surfaces, harmonic mappings and associated boundary value problems.
Direct methods of calculating variations.
Regularity theory for solutions of variational problems.
Relationship between variational problems and partial differential equations.
Stability properties of solutions.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students learn to combine their knowledge of differential geometry and analysis and apply it to specific problems in selected geometric variational problems. They learn techniques for proving solutions to various variational problems and for analysing the properties of solutions, which provide a basis for independent scientific work, for example in a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3"><table><tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr><tr><td>Geometric Variation Problems</td><td>L</td><td>f</td><td>2</td><td>3</td><td>no</td><td>wr. o. or.</td><td>90-180
o. 20-30</td><td>g</td><td>100</td></tr></table>Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisation Analysis and Differential Geometry. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">Knowledge from the following modules is required; please also note the prerequisites specified for these:
MAT-55-21 Introduction to Partial Differential Equations,
MAT-65-11 Geometry in Physics.</td></tr>
<tr><td>Responsible Persons</td><td colspan="3">Carla Cederbaum, Gerhard Huisken</td></tr>
</table>

Abbreviations:

Grading System	:	g=graded, ng=not graded
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Teaching Format	:	L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	:	o=obligatory, f=facultative
Other	:	h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-03	**Module Title:** Topics in Mathematical Relativity		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• A selection of concrete models of Mathematical Relativity, such as black holes, static metrics, physical invariants of isolated systems, positivity estimates for energy and mass. • Geometric and analytical structure of the models, existence and properties of concrete models as solutions to Einstein's equations.		
Objectives	Students acquire in-depth knowledge of selected issues in the mathematical theory of relativity. They learn analytical and geometric techniques for proving and investigating solutions to Einstein's equations and are able to categorise the physical relevance of the mathematical results. The lecture introduces students to their first independent scientific work, for example in a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Topics in Mathematical Relativity	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classess; in this case, only 3 credit points will be awarded for the module instead of 6.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.		
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-13 Mathematical Relativity.		
Responsible Persons	Carla Cederbaum, Gerhard Huisken		

Abbreviations:

Grading System	: g=graded, ng=not graded
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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-60-04</td><td colspan="2">Module Title:
Space-Like Hypersurfaces in Lorentzian Manifolds</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Space-like hypersurfaces of a Lorentzian manifold play a crucial role in the study of solutions to Einstein's equations modelling some phenomenon in General Relativity. The course explores how geometric choices of space-like hypersurfaces such as maximal surfaces, constant mean curvature surfaces or solutions of mean curvature flow and inverse mean curvature flow can be used to achieve a splitting of space and time that is suitable for the study of both isolated gravitating systems and cosmological spacetimes.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students obtain deepend knowledge on selected questions in mathematical relativity. They learn analytic and geometric techniques in order to prove existence of solutions of Einstein equations and to examine these. Moreover, they do understand the physical relevance of the mathematical solutions. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area.
Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Space-Like Hypersurfaces in Lorentzian Manifolds</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Barrett O'Neill: Semi-Riemannian Geometry - With applications to Relativity. Academic Press 1983. • Andrejs E. Treibergs: Entire space-like hypersurfaces of constant mean curvature in Minkowski space. Inventiones Math. 66, (1982) 39–56. • Klaus Ecker, Gerhard Huisken: Parabolic methods for the construction of spacelike slices of prescribed mean curvature in cosmological spacetimes. Comm. Math. Phys. 135 (1991), 595–613. • Helmut Friedrich, Alan Rendall: The Cauchy Problem for the Einstein Equations. In: Schmidt B.G. (eds) Einstein's Field Equations and Their Physical Implications. Lecture Notes in Physics, vol 540. Springer 1999. • Hans Ringström: The Cauchy Problem in General Relativity. European Math. Society 2009.
Transfer	The module belongs to the *Study Specialisation Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-13 Mathematical Relativity.
Responsible Persons	Gerhard Huisken

Abbreviations:

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Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-60-06</td><td colspan="2">Module Title:
The Ricci Flow of Riemannian Metrics</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">The lecture introduces the basic properties of the Ricci flow and develops the necessary techniques, e.g. tensor maximum principles and regularity estimation. The long-term existence of solutions and resulting classifications for metrics of positive curvature are presented. Finally, the monotonicity of functionals according to Perelman is derived and used for the classification of possible singularities, with an outlook on the surgery methods of Hamilton and Perelman, which have led to the proof of the Poincaré and geometrisation conjectures.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have learnt basic methods for the treatment of geometric evolution equations in Riemannian geometry. At the same time, they have experienced the interplay of local geometric assumptions on the curvature properties of a metric with analytic techniques for the study of parabolic equations and have learnt and understood how the local assumptions have global consequences for the geometry and topology of the underlying manifolds. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">The Ricci Flow of Riemannian Metrics</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">no</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>o</td><td>2</td><td>3</td></tr>
</table>
Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classess; in this case, only 3 credit points will be awarded for the module instead of 6.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Simon Brendle: Ricci-flow and the sphere theorem. AMS 2010.
• Peter Topping: Lectures on the Ricci-Flow. Lecture Notes 2006.
• Richard Hamilton: Riemannian 3-manifolds with positive Ricci curvature. J. Diff. Geom. 17, 1982.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisation Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
</table>

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Abbreviations:

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Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-07	**Module Title:** Special Relativity		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Derivation of the Minkowski metric from basic physical assumptions. • Physical consequences of relativity such as length contraction, time dilation and some popular paradoxes.		
Objectives	Students have learnt and understood the derivation of the special theory of relativity and important concepts such as length contraction and time dilation. They are familiar with important paradoxes that arise. Students have developed an intuition for various aspects of the theory of relativity. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Special Relativity	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Albert Einstein: Relativity: the special and general theory. Public domain 1920. • Thomas A. Moore: Six ideas that shaped physics: unit R. McGraw-Hill 2003. • Robert Resnick: Introduction to Special Relativity. Wiley 2007. • Bernard Schutz: A First Course in General Relativity. Cambridge University Press 2009.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Carla Cederbaum, Gerhard Huisken

Abbreviations:

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Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number:	**Module Title:**	**Type of Module:**
MAT-60-08	Null Geometry in General Relativity	Compulsory Module with Choice

ECTS-Points	4.5
Workload - Time in Class - Self-Study	Workload: 150 h; Time in Class: 45 h; Self-Study: 105 h
Duration	1 Semester
Frequency	not regularly
Term	1-3
Language of Instruction	English
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 1 SWS
Content	This module provides an introduction to null geometry. Topics include the properties of light-like vector fields and curves, as well as the geometry of light-like hypersurfaces that carry a degenerate induced metric. Another major topic is the extrinsic curvature of space-like surfaces in higher codimension, which are considered in particular along light-like hypersurfaces. Optionally, geometric flows along light-like hypersurfaces can also be treated.
Objectives	Students know and understand the concepts and methods mentioned and can use them to analyse known and new questions from null geometry. Furthermore, they link physical problems in cosmology and astrophysics and their mathematical modelling using differential geometric methods and are able to question the relevance and adequacy of mathematical modelling and the mathematical results derived from it. In particular, they expand on the methods learnt in the MAT-65-11 module and connect their methodological and specialist knowledge. They are able to name and prove the main statements of the lecture and to categorise and explain the relationships presented. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Null Geometry in General Relativity	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	1	1,5					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classes; in this case, only 3 credit points will be awarded for the module instead of 5.

Literature

Possible References :

- Barrett O'Neill: Semi-Riemannian Geometry. Academic Press 1983.
- Johannes Sauter: Foliations of Null hypersurfaces and the Penrose Inequality. Dissertation (ETH Zürich), url: https://www.research-collection.ethz.ch/handle/20.500.11850/150826.

Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-11 Geometry in Physics.
Responsible Persons	Carla Cederbaum

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Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-09	**Module Title:** The Einstein Constraint Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Introduction to GR and the Einstein equations: – The Einstein equations, special solutions and geometric constructions; – The Cauchy problem for the Einstein equations. • The constraint equations and the conformal method: – The conformal method; – Overview of the elliptic theory on closed manifolds; – Classification of constant mean curvature on closed manifolds. • Asymptotically Euclidean (AE) initial data: – AE manifolds and elliptic operators; – Constructions of AE initial data sets.		
Objectives	Students can use conformal methods to transform Einstein's constraints into a system of elliptic partial differential equations and thus describe parts of the solution spaces of Einstein's equations and analyse properties of the associated solutions. They have learnt about connections between the theory and questions of geometric analysis such as the scalar curvature problem and the Yambe problem and are familiar with the interplay of methods of Riemannian geometry, geometric analysis and physics for answering questions of general relativity. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

<table>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td>
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">The Einstein Constraint Equations</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercises; in this case, only 3 credit points will be awarded for the module instead of 5.</td></tr>
<tr><td>Transfer</td><td>The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
<tr><td>Prerequisites</td><td>Knowledge from the following modules is required; please also note the prerequisites specified for these:
• MAT-65-11 Geometry in Physics,
• MAT-50-16 Riemannian Geometry.
Knowledge from the following modules is helpful but not strictly required:
• MAT-55-21 Introduction to Partial Differential Equations.</td></tr>
<tr><td>Responsible Persons</td><td>Carla Cederbaum</td></tr>
</table>

Abbreviations:

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Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
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Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-10	**Module Title:** Special Topics in Evolution Equations for Submanifolds (with Exercise Class)		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload **- Time in Class** **- Self-Study**	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	Students learn about recent results from the theory of geometric evolution equations that deform curves, hypersurfaces and other submanifolds of an ambient space. Examples are the flow of hypersurfaces along the mean curvature or flows with other geometrically defined velocities.		
Objectives	The students have learnt techniques for controlling solutions of non-linear parabolic evolution equations, which will enable them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Special Topics in Evolution Equations for Submanifolds	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Klaus Ecker: Regularity theory for mean curvature flow. Birkhäuser 2004.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* und *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-11	**Module Title:** Special Topics in Evolution Equations for Submanifolds (without Exercise Classes)		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	Students learn about recent results from the theory of geometric evolution equations that deform curves, hypersurfaces and other submanifolds of an ambient space. Examples are the flow of hypersurfaces along the mean curvature or flows with other geometrically defined velocities.		
Objectives	The students have learnt techniques for controlling solutions of non-linear parabolic evolution equations, which will enable them to start their first own research project, for example as part of a Master's thesis or with a view to a doctorate. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Special Topics in Evolution Equations for Submanifolds	L	f	2	3	no	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Klaus Ecker: Regularity theory for mean curvature flow. Birkhäuser 2004.
Transfer	The module belongs to the *Study Specialisation Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-60-12</td><td colspan="2">Module Title:
The Positive Mass Theorem</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">This course revolves around fundamental theorems in Geometric Analysis, which have applications in Mathematical Relativity. The center of gravity is the so called Positive Mass Theorem, which basically says that non-negative matter density implies non-negative total mass. We discuss the (by now) classical proofs using minimal surfaces and more recent proofs based on level sets of carefully chosen functions, which arise as solutions to an elliptic partial differential equation. Interesting follow-up questions such as rigidity and stability of the case of zero or near zero total mass will be a substantial part of the lecture.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students acquire advanced knowledge of selected topics in mathematical relativity. They learn analytical and geometrical techniques that are used in the Positive Mass Theorem and related results. Through the lecture, students are guided towards their first independent academic research, for example in the context of a Master's thesis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">The Positive Mass Theorem</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">no</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board. – The module may exceptionally be offered by the lecturer without exercise classess; in this case, only 3 credit points will be awarded for the module instead of 6.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisation Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">Knowledge from the following modules is required; please also note the prerequisites specified for these:
• MAT-55-21 Introduction to Partial Differential Equations,
• MAT-65-11 Geometry in Physics.</td></tr>
<tr><td>Responsible Persons</td><td colspan="3">Carla Cederbaum</td></tr>
</table>

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-60-35	**Module Title:** Non-Linear Elliptic and Parabolic Partial Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Semilinear and quasilinear elliptic and parabolic partial differential equations; • Minimum surface operator and surfaces of prescribed mean curvature; • Parabolic geometric equations, e.g. flow along the mean curvature; • Hölder continuity according to De Giorgi and Nash; • Inner regularity and boundary regularity of solutions.		
Objectives	Students have learnt analytical methods that are central to the treatment of non-linear partial differential equations of second order of the elliptic and parabolic type. Using concrete examples of partial differential equations from mathematical physics and differential geometry, techniques were learnt to prove the existence and regularity of solutions to such equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes, students have acquired confidence in the technical handling of the methods they have learnt and can apply them independently to other problems. They are able to present their problem solutions and participate in discourses on problems in this field of research.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Non-Linear Elliptic and Parabolic Partial Differential Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Lawrence C. Evans, Partial Differential Equations: Chapters on Sobolev Spaces and elliptic PDEs. AMS 1998. • Gary Lieberman: Second order parabolic differential equations. World Scientific 1996. • Fritz John: Introduction to Partial Differential Equations. Springer 1982. • Jürgen Jost: Partielle Differentialgleichungen. Springer 1998. • David Kinderlehrer, Guido Stampacchia: An introduction to variational inequalities and their applications, Pure and Applied Mathematics, Vol. 88. Academic Press 1980.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: **MAT-60-36**	**Module Title:** Fully Non-Linear Elliptic and Parabolic Partial Differential Equations		**Type of Module:** Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The lecture investigates fully non-linear elliptic and parabolic partial differential equations of second order. Classical examples are the Monge-Ampère equation, the equation of prescribed Gaussian curvature or, more general, equations of prescribed other scalar invariants of curvature together with their parabolic analogues. They also arise in problems of stochastic control and optimal transport. The course establishes basic techniques for solving Dirichlet- and Neumann boundary value problems for such equations, in particular techniques for deriving the necessary a priori estimates for solutions.		
Objectives	The students have learnt analytical methods that are central for the treatment of strongly non-linear partial differential equations of the elliptic and parabolic type. Using concrete examples of such differential equations, techniques were learnt to prove the existence and regularity of solutions of such equations and the associated boundary value problems. Students can apply the methods they have learnt to other problems and related equations independently. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Fully Non-Linear Elliptic and Parabolic Partial Differential Equations	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100

Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Lawrence C. Evans, Partial Differential Equations: Chapters on Sobolev Spaces and elliptc PDEs. AMS 1998. • Gary Lieberman: Second order parabolic differential equations. World Scientific 1996. • Ilya J. Bakelman: Convex functions and nonlinear geometric elliptic equations. Springer 1994. • Luis Caffarelli, Xavier Cabrè: Fully nonlinear elliptic equations. AMS 1995.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section. The module cannot be taken together with the module 'MAT-55-27 Fully Nonlinear Elliptic Equations' due to the large overlap in content.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-55-21 Introduction to Partial Differential Equations, • MAT-65-11 Geometry in Physics.
Responsible Persons	Gerhard Huisken

Abbreviations:

Grading System	: g=graded, ng=not graded
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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-13	Module Title: Mathematical Relativity		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly in Summer Semester		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	The module provides an introduction to the mathematical theory of relativity. Particular topics are Newton's theory of gravity, special theory of relativity, relativistic effects, Einstein's equations, the Schwarzschild spacetime. Optionally, other topics such as cosmological models, matter models, black holes, Cauchy problem and ADM decomposition, singularity theorems or gravitational waves can be discussed.		
Objectives	Students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from the theory of relativity. They are able to interrelate physical problems in cosmology and astrophysics and their mathematical models through methods from differential geometry and to question the relevance and adequacy of the mathematical model and the results derived from it. Thereby, they enhance their knowledge on methods and subjects gained throughout the first semester, in particular in module MAT-65-11. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Relativity	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	o	2	3					

In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.

Transfer	Successful completion of module Mathematical Relativity is a prerequisite for the participation in the module Advanced Topics in Mathematical Relativity. Successful completion of one of the modules Mathematical Relativity or Mathematical Quantum Theory is a prerequisite for the participation in the module Scientific Project.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-11 Geometry in Physics.

Responsible Persons	Carla Cederbaum, Gerhard Huisken, Frank Loose

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-65-21</td><td colspan="2">Module Title:
Advanced Topics in Mathematical Quantum Theory</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">The module provides an introduction to an advanced topic of mathematical quantum theory, like Hartree and Hartree-Fock theory, BCS theory, adiabate theory, renormalisation group, mathematical models in quantum field theory and transport in interdependent ferminon systems. It will present both the fundamental mathematical results and physical notions of the particular area, as well as provide an insight into the current state of research and the existing open problems.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students obtain knowledge and understanding of the acquired notions and methods and are able to apply them in the analysis of known and new problems from the specific area of Mathematical Quantum Theory. Students are able to name and prove the essential statements and concepts from the lecture as well as to explain the context developed in the lecture and to put it into a larger framework. They are able to describe and critically challenge the current state of research in the specific area.
Through homework assignments and exercise classes students develop a confident, precise, and independent acquaintance with the notions, statements, and methods explained in the lectures. They learn how to transfer these methods to new problems, to analyse them and to develop solution strategies on their own and within a group. They are able to present their solutions and to stand for them in a critical discourse if necessary.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Advanced Topics in Mathematical Quantum Theory</td><td>L</td><td>o</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180
o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>o</td><td>2</td><td>3</td></tr>
</table>
In this module students need to successfully complete assignments in order to be admitted to the exam. The type of examination is set by the instructor.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module may be a prerequisite for the master thesis.</td></tr>
<tr><td>Prerequisites</td><td colspan="3">Knowledge from the following modules is required; please also note the prerequisites specified for these:
• MAT-65-12 Mathematical Quantum Theory.</td></tr>
<tr><td>Responsible Persons</td><td colspan="3">Stefan Teufel, Marius Lemm</td></tr>
</table>

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-65-31</td><td colspan="2">Module Title:
Operator Algebras in Mathematical Physics</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">The module provides an introduction to operator algebras and their roles in mathematical physics. Particular topics are C^*-algebras and the GNS construction, von Neumann algebras and the double commutant theorem, and elements of Tomita-Takesaki theory. The mathematical theory is developed in parallel with applications in mathematical physics, most importantly the dynamics and statistical physics of extended many-body quantum systems.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students obtain knowledge and understanding of the listed notions and methods and can use them to analyse known and new problems from operator algebras. They are able to interrelate fundamental physical concepts with their mathematical modelling via operator algebra methods and to question the relevance and adequacy of the mathematical model and of the results derived from it. Thereby, they enhance their knowledge on methods and subjects gained throughout the earlier semesters, in particular on functional analysis. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Operator Algebras in Mathematical Physics</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :

Kenneth R. Davidson: Functional Analysis and Operator Algebras. Springer 2025.
Ola Bratelli, Derek W. Robinson: Operator Algebras and Quantum Statistical Mechanics 1. Springer 1987.
Ola Bratelli, Derek Robinson: Operator Algebras and Quantum Statistical Mechanics 2: Springer 1997.
Masamichi Takesaki: Operator Algebras I. Springer 1979.
</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisations Analysis and Differential Geometry and Mathematical Physics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
</table>

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's and Master's programmes is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-55-01 Functional Analysis or MAT-65-12 Mathematical Quantum Theory.
Responsible Persons	Stefan Teufel

Abbreviations:

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Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-65-38</td><td colspan="2">Module Title:
Hamiltonian Systems</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German or English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">The module provides an introduction to the theory of Hamiltonian systems as used in classical mechanics. This builds a bridge between the fields of differential geometry, symplectic geometry and dynamical systems as well as theoretical physics. The main points of the lecture are:
Symplectic manifolds and the canonical 1-form of the cotangent bundle.
Darboux-Moser theorem.
Lagrangian and Hamiltonian systems.
Integrable systems and Arnold-Liouville theorem.
Moment mappings.
Symplectic reduction.
Symplectic manifolds and toric effects.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students are familiar with the theory of Hamiltonian systems and their investigation using methods of symplectic geometry. They are familiar with the interplay of methods and questions of different areas of mathematics (differential geometry, geometry, dynamical systems) and theoretical physics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3"><table><tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr><tr><td rowspan="2">Hamiltonian Systems</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr><tr><td>E</td><td>f</td><td>2</td><td>3</td></tr></table>In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 1989. • Ana Cannas da Silva: Lectures on symplectic geometry. Springer 2001.
Transfer	The module belongs to the *Study Specialisations Analysis and Differential Geometry* and *Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-11 Geometry in Physics.
Responsible Persons	Carla Cederbaum

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-65-41	Module Title: Fredholm Index Theory for Condensed Matter and Introduction to Spin Systems		Type of Module: Compulsory Module with Choice
ECTS-Points	3		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Reminder on compact and trace class operators in infinite dimensional Hilbert spaces. • Fredholm index theory in Hilbert spaces: definitions and proofs of stability of the index and homotopy invariance. • The Fredholm index as a difference of traces. • Introduction to the physics of the Integer Quantum Hall Effect and application of Fredholm index theory. • Definition and properties of the Bott index of two unitary matrices and application to the index theory of the Integer quantum Hall effect on a two-dimensional torus. • The bulk-boundary correspondence for chiral topological system in one dimension. • Tensor products of finitely many finite dimensional Hilbert spaces and introduction to the main models of spin systems. • Lieb-Robinson bounds.		
Objectives	Students have learned about operators in infinite dimensional Hilbert spaces with finite dimensional kernel and finite codimension (Fredholm operators), their index and its stability (topological invariance). They know the meaning of an index theorem in the context of condensed matter physics, that relates a physically measurable quantity to an integer (in certain units), and are able to prove it for the Integer Quantum Hall Effect in two dimensions. The students have learned about the equality of the topological indexes defined at the boundary and in the bulk of a one-dimensional systems of non-interacting particles with chiral. They recognize the main models of spin 1/2 system: Ising model, XY and XXZ models and can sketch their phase diagrams. The students can discuss the meaning of the dynamical spread of information in a spin system and prove the associated Lieb-Robinson bounds. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	<table><tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr><tr><td>Fredholm Index Theory for Condensed Matter and Introduction to Spin Systems</td><td>L</td><td>f</td><td>2</td><td>3</td><td>no</td><td>wr. o. or.</td><td>90-180 o. 20-30</td><td>g</td><td>100</td></tr></table> Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.
Literature	**Possible References :** • Gerard J. Murphy: Fredholm Index Theory and the Trace. Proceedings of the Royal Irish Academy. Section A: Mathematical and Physical Sciences 94A, 161 (1994). • Daniele Toniolo: On the Bott index of unitary matrices on a finite torus. Letters in Mathematical Physics 112, 126 (2022). • Loic Jezequel, Clement Tauber, Pierre Delplace: Estimating bulk and edge topological indices in finite open chiral chains. Journal of Mathematical Physics 63, 121901 (2022). • Peter D. Lax: Functional Analysis. Wiley 2002.
Transfer	The module belongs to the *Study Specialisation Mathematical Physics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-65-12 Mathematical Quantum Theory.
Responsible Persons	Daniele Tonolio

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-03	Module Title: Numerics of Instationary Differential Equations		Type of Module: Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	Numerical treatment of transient (i.e. time-dependent) differential equations, such as: stiff ordinary differential equations, stochastic differential equations, parabolic and hyperbolic partial differential equations.		
Objectives	The students have learned the central terms, results and methods of the numerical treatment of boundary value problems of instationary differential equations. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Numerics of Instationary Differential Equations	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Ernst Hairer, Gerhard Wanner: Solving Ordinary Differential Equations II. Stiff Problems. Springer 1996.
- Vidar Thomee: Galerkin Finite Element Methods for Parabolic Problems. Springer 1997.

Transfer

The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites

Knowledge from the following modules is helpful but not strictly required:

- MAT-70-02 Numerics of Stationary Differential Equations.

Responsible Persons	Christian Lubich

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-06	**Module Title:** Numerics of Differential Equations of Surfaces		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload **- Time in Class** **- Self-Study**	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Numerical treatment of differential equations on moving (or stationary) surfaces. • Semi- and fully discretization of elliptic and parabolic equations on surfaces using surface finite elements and efficient time integrators. • Implementation of the algorithms.		
Objectives	Students have learned the basic methods and techniques of numerics for problems on (moving) surfaces. In particular, they are familiar with the discussed energy techniques, which are very strong, general and rich in application, even in surface-independent areas of numerics. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Numerics of Differential Equations of Surfaces	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Gerhard Dziuk: Finite elements for the Beltrami operator on arbitrary surfaces. 1988. • Gerhard Dziuk, Charles M. Elliott: Finite elements on evolving surfaces. 2007.
Transfer	The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.
Responsible Persons	Christian Lubich

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-70-21	Module Title: Non-Linear Optimisation	Type of Module: Compulsory Module with Choice
ECTS-Points	9	
Workload - Time in Class - Self-Study	Workload: 270 h / Time in Class: 90 h / Self-Study: 180 h	
Duration	1 Semester	
Frequency	regularly	
Term	1-3	
Language of Instruction	German	
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS	

Content

- Finite-dimensional optimisation, gradient method with Armijo's rule, globalised Newton method.
- Restricted optimisation, Farkas' lemma, tangent cone.
- Abadie CQ, KKT conditions, Slater conditions.
- Linear programme, duality, simplex method.
- Penalty and barrier methods, interior point method.
- Nonlinear programs, SQP methods, non-smooth optimisation.

Objectives

Students master the basic principles and techniques of analysis and numerics of constrained optimisation problems. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Non-Linear Optimisation	L	f	4	6	yes	or.	20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature

Possible References :

- Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.

Transfer

The module belongs to the *Study Specialisation Numerical Mathematics and Optimisation*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Responsible Persons	Andreas Prohl

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-70-32</td><td colspan="2">Module Title:
Statistical Learning 2</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">9</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
270 h</td><td>Time in Class:
90 h</td><td>Self-Study:
180 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 4 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">• The uniform law of large numbers on function classes (Vapnik-Chervonenkis theory).
• Abstract (strong) consistency theory for least-squares regression estimators on (approximating) function classes.
• Examples, in particular the data dependent partitioning estimator and the least squares neural networks estimator.
• Rate convergence for least-squares estimators.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students are familiar with in-depth methods of stochastic learning and their analysis, as required for machine learning applications. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><td>Title</td><td>Type of Course</td><td>Status</td><td>SWS</td><td>ECTS</td><td>Coursework</td><td>Type of Exam</td><td>Dur. of Exam (min)</td><td>Grading</td><td>Weight for Grade</td></tr>
<tr><td rowspan="2">Statistical Learning 2</td><td>L</td><td>f</td><td>4</td><td>6</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
<tr><td>Literature</td><td colspan="3">Possible References :
• Laslo Györfi, Michael Kohler, Adam Krzyzak, Harro Walk: A distribution-free theory of nonparametric regression. Springer 2002.</td></tr>
<tr><td>Transfer</td><td colspan="3">The module belongs to the Study Specialisation Numerical Mathematics and Optimisation and Stochastics. Taking into account the chosen personal Study Specialisation, it can be included in the Sections Study Focus, Advanced Knowledge in Mathematics or Elective Specialisation, in accordance with the restrictive requirements of the respective section.</td></tr>
</table>

Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-70-31 Statistical Learning 1.
Responsible Persons	Andreas Prohl

Abbreviations:

Grading System	: g=graded, ng=not graded
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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-04	**Module Title:** Stochastic Processes		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	Stochastic processes in continuous time, such as • Markov processes; • Martingale; • Brownian motion, Poisson processes and general Levy processes; • Gaussian processes. Among other things, existence and convergence statements as well as path properties of these processes are analysed.		
Objectives	The students have learnt the central concepts, results, methods and examples of the theory of stochastic processes in continuous time and can handle them mathematically. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Stochastic Processes	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Joseph L. Doob: Stochastic Processes. Wiley 1990. • Olav Kallenberg: Foundations of Modern Probability. Springer 2021. • Samuel Karlin, Howard Taylor: A First Course in Stochastic Processes. Academic Press 1975. • Samuel Karlin, Howard Taylor: A Second Course in Stochastic Processes. Academic Press 1981. • Götz Kersting, Anton Wakolbinger: Stochastische Prozesse. Birkhäuser 2014. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2020. • James R. Norris: Markov Chains. Cambridge University Press 1997.
Transfer	The module belongs to the *Study Specialisation ...*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-75-01 Probability Theory.
Responsible Persons	Martin Möhle, Elmar Teufl

Abbreviations:

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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-06	**Module Title:** Stochastic Analysis		**Type of Module:** Compulsory Module with Choice
ECTS-Points	9		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS		
Content	• Martingales and stopping times in continuous time. • Doléans measure, compensator, Doob-Meyer decomposition. • Stochastic integral for square integrable martingales (in particular for non-continuous martingales). • Semimartingales, transformation of stochastic integrals. • Itô formula (in particular for processes with jumps). • Stochastic differential equations .		
Objectives	The students know the main notions, results, methods and examples from stochastic analysis and they know how to handle them. The students can name and prove the central results of the lecture and they can explain their intrinsic connections. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods to new problems, to analyse them and to work on solution strategies on their own or in a team. They are capable of presenting their results and if applicable to argue for it in a critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Stochastic Analysis	L	f	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Fabrice Baudoin: Diffusion Processes and Stochastic Calculus. EMS 2014. • Kai Lai Chung and Ruth J. Williams: Introduction to Stochastic Integration. Birkhäuser 1990. • Richard Durrett: Stochastic Calculus. CRC Press 2006. • Albrecht Irle: Finanzmathematik. Teubner 2003. • Ioannis Karatzas, Steven Shreve: Brownian Motion and Stochastic Calculus. Springer 1991. • Michel Métivier: Semimartingales. De Gruyter 1982. • Bernt Oksendal: Stochastic Differential Equations. Springer 2007. • Nicolas Privault: Stochastic Analysis in Discrete and Continuous Settings. Springer 2009. • Daniel Revuz, Marc Yor: Continuous Martingales and Brownian Motion. Springer 1999. • Heinrich von Weizsäcker, Gerhard Winkler: Stochastic Integrals, Vieweg 1990.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-75-01 Probability Theory.
Responsible Persons	Martin Möhle

Abbreviations:

Grading System	: g=graded, ng=not graded
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Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-75-08	**Module Title:** Mathematical Population Genetics		**Type of Module:** Compulsory Module with Choice
ECTS-Points	6		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Duration	1 Semester		
Frequency	not regularly		
Term	1-3		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Ex.cl. 2 SWS		
Content	• Exchangeable population models. • Probability of extinction. • Descendants and ancestors. • Duality of Markoff processes. • Coalescent processes and associated convergence rates. • Simple mutation models, Ewens sampling formula. • Statistical applications, e.g. estimating the mutation rate.		
Objectives	In the lecture, students learn the basic principles of of representation theory and develop an understanding for the interaction of geometric and algebraic methods. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.		

Requirements for obtaining Credits / Grading (Weighting if applicable)

Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
Mathematical Population Genetics	L	f	2	3	yes	wr. o. or.	90-180 o. 20-30	g	100
	E	f	2	3					

In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.

Literature	**Possible References :** • Jean Bertoin: Random Fragmentation and Coagulation Processes. Cambridge 2006. • Stewart N. Ethier, Thomas G. Kurtz: Markov Processes. Wiley 1986. • Warren J. Ewens: Mathematical Population Genetics. Springer 2004. • Jim Pitman: Combinatorial Stochastic Processes. LNM 1875. Springer 2006. • John Wakeley: Coalescent Theory. Roberts & Company Publishers 2008.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-75-01 Probability Theory.
Responsible Persons	Martin Möhle

Abbreviations:

Grading System	: g=graded, ng=not graded
Examination Type	: MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio
Teaching Format	: L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=project, S=seminar, IC=inverted classroom
Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-75-09</td><td colspan="2">Module Title:
Point Processes</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">German</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">Random measures, point processes, Poisson processes.
Factorial measure, Mecke equation.
Transformation, labelling, thinning.
Characterisation of point processes.
Stationary Poisson processes.
Poisson integrals.
Cox processes.</td></tr>
<tr><td>Objectives</td><td colspan="3">The students have familiarised themselves with the central concepts, results, methods and examples of the theory of point processes and can handle them mathematically. The students are capable of naming and proving the essential results of the lecture as well as assessing and explaining the presented connections. Students will be able to reflect and critically analyse the current state of research in the subject area.
In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Point Processes</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Daryl John Daley, David Vere-Jones: An Introduction to the Theory of Point Processes. Springer 2008. • Martin Jacobsen: Point Process Theory and Applications. Birkhäuser 2006. • Olav Kallenberg: Foundations of Modern Probability. Springer 2002. • John F. C. Kingman: Poisson Processes. Clarendon Press 1993. • Günter Last, Mathew D. Penrose: Lectures on the Poisson Process. Cambridge 2016.
Transfer	The module belongs to the *Study Specialisation Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-75-01 Probability Theory.
Responsible Persons	Martin Möhle

Abbreviations:

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Status	: o=obligatory, f=facultative
Other	: h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

<table>
<tr><td>Module Number:
MAT-75-20</td><td colspan="2">Module Title:
Probability Distances for Data Science</td><td>Type of Module:
Compulsory Module with Choice</td></tr>
<tr><td>ECTS-Points</td><td colspan="3">6</td></tr>
<tr><td>Workload
- Time in Class
- Self-Study</td><td>Workload:
180 h</td><td>Time in Class:
60 h</td><td>Self-Study:
120 h</td></tr>
<tr><td>Duration</td><td colspan="3">1 Semester</td></tr>
<tr><td>Frequency</td><td colspan="3">not regularly</td></tr>
<tr><td>Term</td><td colspan="3">1-3</td></tr>
<tr><td>Language of Instruction</td><td colspan="3">English</td></tr>
<tr><td>Forms of Teaching and Learning</td><td colspan="3">Lecture 2 SWS + Ex.cl. 2 SWS</td></tr>
<tr><td>Content</td><td colspan="3">We study different concepts of distances between probability measures aimed at applications in data science. The classes of distances which are studied include optimal transport distances, f-divergences and integral probability metrics. The focus is on fundamental mathematical properties of these distances, like duality, famous inequalities, geometric aspects, and quantisation. Several applications in the area of data science and machine learning are illustrated throughout, for instance related to clustering, autoencoders, GANs, image processing, and compression.</td></tr>
<tr><td>Objectives</td><td colspan="3">Students are familiar with commonly used distances on the space of probability measures, particularly optimal transport distances, divergences, and integral probability metrics. They understand key mathematical results in this area, for instance related to duality, geometric aspects, and quantisation, as well as the interplay between different distances. They have further obtained an understanding of computational aspects and applicability in selected areas of data science. They are able to name and prove the main statements of the lecture as well as categorise and explain the relationships presented. Students will be able to reproduce and critically scrutinise the current state of research in the specialist area addressed.
In the exercises, they have developed a confident, precise and independent approach to the concepts, statements and methods from the lecture. They have learned to transfer the methods to new problems, to analyse them and to develop solution strategies alone or in a team. They are able to present their solutions and, if necessary, defend them in critical discourse.</td></tr>
<tr><td>Requirements for obtaining Credits / Grading (Weighting if applicable)</td><td colspan="3">
<table>
<tr><th>Title</th><th>Type of Course</th><th>Status</th><th>SWS</th><th>ECTS</th><th>Coursework</th><th>Type of Exam</th><th>Dur. of Exam (min)</th><th>Grading</th><th>Weight for Grade</th></tr>
<tr><td rowspan="2">Probability Distances for Data Science</td><td>L</td><td>f</td><td>2</td><td>3</td><td rowspan="2">yes</td><td rowspan="2">wr. o. or.</td><td rowspan="2">90-180 o. 20-30</td><td rowspan="2">g</td><td rowspan="2">100</td></tr>
<tr><td>E</td><td>f</td><td>2</td><td>3</td></tr>
</table>
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.</td></tr>
</table>

Literature	**Possible References :** • Gabriel Peyre, Marco Cuturi: Computational optimal transport: with ap- plications to data science. Foundations and Trends in Machine Learning 11.5-6 (2019): 355-607. • Alison L. Gibbs, Francis Edward Su: On choosing and bounding probability metrics. International Statistical Review 70.3 (2002): 419-435. • Cedric Villani: Topics in optimal transportation. American Mathematical Society, 2003. • Imre Csiszar, Paul C. Shields: Information theory and statistics: a tutorial. Foundations and Trends in Communications and Information Theory 1.4 (2004). 417-528. • Ily Tolstikhin et al.: Wasserstein auto-encoders. 6th International Conference on Learning Representations (ICLR 2018) • Siegfried Graf, Harald Luschgy: Foundations of quantization for probability distributions. Springer, 2007.
Transfer	The module belongs to the *Study Specialisations Numerical Mathematics and Optimisation* and *Stochastics*. Taking into account the chosen personal Study Specialisation, it can be included in the Sections *Study Focus*, *Advanced Knowledge in Mathematics* or *Elective Specialisation*, in accordance with the restrictive requirements of the respective section.
Prerequisites	Knowledge from the following modules is required; please also note the prerequisites specified for these: • MAT-75-01 Probability Theory.
Responsible Persons	Stephan Eckstein

Abbreviations:

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4 Exchange Programmes

4.1 Exchange Programme with the Università degli Studi di Trento

As part of the M.Sc. Mathematics programme, students can participate in the exchange programme with the Università degli Studi di Trento. After successful completion of the programme, students will receive the M.Sc. Mathematics degree from the University of Tübingen and the Laurea Magistrale in Mathematica degree from the Università degli Studi di Trento. The basis for this is the agreement on the exchange programme between the two universities, which can be downloaded from the websites of the department for the M.Sc. Mathematics programme.

The courses at the Università degli Studi di Trento are offered in English, while the courses at the University of Tübingen are offered in German or English. Therefore, students of the Università degli Studi di Trento must provide proof of German language skills (DSH-2 or DSH-3), and students of the University of Tübingen must provide proof of English language skills (GER B2). Students are only required to pay tuition fees, if applicable, at their home university.

Students of the University of Tübingen study the first academic year at the University of Tübingen and the second year at the Università degli Studi di Trento; for students of the Università degli Studi di Trento, it is the opposite. At both universities, students earn 60 credit points each; the thesis is to be written in the second academic year.

The achievements obtained during studies at both locations will be credited to both programmes, provided they are essentially equivalent to the achievements they are supposed to replace. The creditability of the intended achievements is ensured for students from Tübingen by the study and examination plan, which must be discussed with the personal mentor and approved by a joint commission responsible for the programme at both universities. Students of the University of Tübingen may replace the module *Introduction to Scientific Work* with other achievements that they complete at the Università degli Studi di Trento according to the requirements there in the second year of study. The conversion of grades for recognition purposes is carried out in accordance with the conversion table in the annex of the agreement on the exchange programme.

All modules listed in this module handbook can be included in the exchange programme at both locations. Information about the modules currently part of the study programme at the Università degli Studi di Trento can be found at:

`http://offertaformativa.unitn.it/en/lm/mathematics/course-content`

Current semester course offerings and module descriptions can be viewed in the course catalog at:

`https://www.esse3.unitn.it/Guide/PaginaRicercaInse.do?cod_lingua=eng`

A list of modules that were part of the regular offerings at the Università degli Studi di Trento at the time of signing the agreement is also included in the agreement on the exchange programme.