



Department of Mathematics

Module Handbook

Mathematik

Master of Education Erweiterungsfach

Lehramt Gymnasium*

Winter Semester 2026

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*This is a secondary school teaching degree with a major in mathematics. The module handbook is valid for the 2026 study and examination regulations.

Contents

1	Description of the Study Programme	3
1.1	Qualification Objectives	3
1.2	Structure of the Study Programme	3
2	Study Plans	5
2.1	Overview by Modules	5
2.2	Overview by the Course of Studies	7
2.3	Overview of Programme Structure with Semester Assignment	9
3	Module Descriptions	13
	Section 1: Foundations of Mathematics	13
	Section 2: Compulsory Intermediate Modules	17
	Section 3: Knowledge Expansion Mathematics	28
	Section 4: Didactics of Mathematics	32
	Section 5: Master Thesis	37
4	Courses for the Module Specialisation	39
4.1	Catalogue of Courses	39

1 Description of the Study Programme

1.1 Qualification Objectives

As part of the teaching-related Master's study programme Master of Education Erweiterungsfach Lehramt Gymnasium with mathematics as a subject graduates acquire fundamental and in-depth subject-specific and subject-didactic knowledge and competences, as required necessary for science-based teaching at secondary schools. These enable them to organise targeted teaching, learning and educational processes in the subject of mathematics and to introduce new subject-specific and interdisciplinary developments into teaching and school development independently.

Graduates are familiar with the fundamental problems in linear algebra, analysis, numerical mathematics, stochastics, geometry and algebra and have mastered the central techniques for solving them. They acquire basic mathematical thinking patterns such as structuring problems, creating chains of reasoning and finally proving mathematical theorems. Graduates are able to communicate mathematical facts, use suitable media and establish links to school mathematics. Building on the fundamental issues mentioned above, they expand their material and methodological skills in one or more compulsory elective areas from the specialisations offered in Tübingen: Algebra and Geometry, Analysis and Differential Geometry, Mathematical Physics, Numerical Mathematics and Optimisation or Stochastics. Graduates are proficient in the theoretical approaches, principles and methods of mathematics. They are able to provide examples of the current state of research and can critically scrutinise it. Graduates can use their in-depth knowledge to develop and solve their own simple research ideas. They can derive, analyse, prove and interpret specific questions from general mathematical concepts. Graduates are able to present, explain and discuss the results of their research work in depth in front of a scientific audience, both in writing and orally.

Students combine their scientific knowledge with didactic methods, use suitable media and are able to use theoretical concepts and empirical findings from mathematics-related teaching-learning research to analyse thought processes and ideas of pupils in exercises and to guide individual learning processes. They know and evaluate the concepts for learning and teaching mathematics at school on the basis of subject-didactic theories and empirical findings. They are able to analyse, plan and exemplarily implement mathematics lessons with heterogeneous learning groups on the basis of subject-didactic concepts. Students can justify the educational value of mathematical content, convey the societal significance of mathematics and place it in the context of the objectives and content of mathematics lessons.

1.2 Structure of the Study Programme

The study of the extension subject in the teacher training programme is usually taken up in parallel to the study of the two main subjects. This requires individual counselling for each and every student with regard to the structure and design of their study programme. At the beginning of their studies, all

students on this study programme should therefore seek academic advice from the Faculty Course Advisor in the Department of Mathematics and discuss with them an individual study plan that takes into account the student's previous knowledge and special circumstances. In the module handbook, we can only describe the structure and organisation of the degree programme for the rather atypical case that the degree programme is completed full-time and not in parallel with another degree programme.

The first year of study is dominated by the large compulsory module Foundations of Mathematics, in which the subject-specific basics of analysis and linear algebra from an academic point of view are conveyed. The corresponding lectures are accompanied by exercise classes, where students are intensively supervised and taught basic mathematical thinking and working methods as well as the ability to present solutions. In addition, the department provides students with revision sessions as question times, and in the practical course Exploring Mathematics, students learn how mathematicians approach mathematical problems.

In the second, third and, in some cases, fourth semesters, the students deepen their theoretical knowledge. They expand their knowledge in the areas of algebra, geometry, numerical mathematics and stochastics and attend a introductory seminar. The content of the compulsory modules is taught in lectures and accompanying exercise classes. Each lecture is accompanied by weekly tasks that students have to complete in writing. In the exercise classes, the students present their solutions or create them under supervision. Through this system, which is common in mathematics study programmes, it is intended to teach students to systematically work on the tasks set for them and to practise analytical and structural thinking. Furthermore, they should be able to present complex mathematical facts in writing and orally. This requires students to be able to organise themselves and to do a lot of independent study which is expected and recognised in the course of study. At the same time, intensive supervision and individualised support options are provided. In the third and fourth semesters, students also gain their first insights into specialisation areas through a compulsory elective lecture and a seminar. The focus of the final semester is the Master's thesis.

In addition to the specialised modules, students take modules in the field of subject didactics from the second semester onwards. The Subject Didactics 1 and 2 modules are designed in such a way that for the didactic courses in the areas of stochastics and geometry it is recommended to take them parallel with the corresponding subject modules. In the subject modules, the subject-specific prerequisites for the subject didactics courses are taught. Different focal points can then be set in the Subject Didactics 3 module, although one part of the module has a strong professional focus.

It is not possible to specify a specific time frame for a study period at a foreign university when studying an extension subject, as experience has shown that the study structure is too individual and different. This can only be planned in a personal consultation with the Faculty Course Advisor. Essentially, from the perspective of mathematics every semester is eligible for this. The decision will depend on the student's previous achievements and the programme offered at the chosen foreign university.

The programme in the additional subject may also be completed without a Master's thesis. In that case, only a certificate of successful participation is issued; no academic degree is awarded. After successful completion of the preparatory service, however, the certificate entitles the holder to teach the subject in schools.

2 Study Plans

2.1 Overview by Modules

Here we provide an overview of the study plan as a table showing the modules to be taken.

ST	Module Number	Module Title	Type of Course	Type of Module	Course-work	Type of Exam	ECTS-Points
Section 1: Foundations of Mathematics							
1+2	MAT-10-10	Foundations of Mathematics		PM		or.	27
		- Foundations of Mathematics 1	L+E+T		EC		
		- Foundations of Mathematics 2	L+E+T		EC		
		- Exploring Mathematics	P		PC		
Section 2: Compulsory Intermediate Modules							
2-4	MAT-20-05	Compulsory Intermediate Modules Analysis		PM		or.	13.5
		- Measure and Integration Theory	L+E		EC		
		- Introduction to ODE	L+E		EC		
		- Introduction to Complex Analysis	L+E		EC		
1-4	MAT-20-06	Compulsory Intermediate Modules Algebra and Geometry		PM		or.	13.5
		- Algebraic Structures	L+E		EC		
		- Introduction to Galois Theory	L+E		EC		
		- Introduction to Geometry	L+E		EC		
2-4	MAT-20-12	Stochastics	L+E	PM	EC	wr. o. or.	9
2-4	MAT-20-13	Numerical Mathematics		PM		wr. o. or.	6
		- Introduction to Numerical Mathe-matics	L+E		EC		
		- Mathematical Software	P		PC		
2-4	MAT-20-20	Introductory Seminar: Presentations in Mathematics	PS	PMW	s.M.	Pr	3
Section 3: Knowledge Expansion Mathematics							
3-4	MAT-40-51	Specialisation 1	L+E	PMW	EC	wr. o. or.	9
3-4	MAT-40-52	Specialisation 2	L+E	PMW	EC	wr. o. or.	9
Section 4: Didactics of Mathematics							
2-3	MAT-80-01	Subject Didactics Mathematics 1	LIC	PM	s.M.	wr., or., P o. T	3

2-3	MAT-80-02	Subject Didactics Mathematics 2	SLIC+SLIC	PM	-	wr., or., E, Pr o. T	6
3-4	MAT-80-03	Subject Didactics Mathematics 3	S+SV	PMW	-	wr., or., E, Pr o. T	6
Section 5: Master Thesis							
4	MAT-40-54	Master Thesis	MT	PM	s.M.	MT	15
Abbreviations: Type of Module : PM=compulsory module, PMW=compulsory module with choice, WPM=elective module Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted class-room Course Work : EC=exercise certificate, PEC=practical exercise certificate, PC=practical certificate Other : h=hours, o.=or, s.M.=see module description, ST=suggested term							

Note at the request of the student representatives on the Study Commission: For modules extending over several semesters, it is strongly recommended to review the content of each submodule thoroughly during the lecture-free periods, thereby spreading the preparation for the oral examination over a longer period of time.

2.2 Overview by the Course of Studies

All students on this study programme should attend a study counselling session with the Faculty Course Advisor at the Department of Mathematics at the beginning of their studies to discuss an individual study plan that takes into account the student's previous knowledge and the particular circumstances of the respective student. This applies especially if the degree programme is taken up parallel to the study of the two first subjects. Here we provide an overview of the possible course of study in the form of a table both for entry in the winter semester and for entry in the summer semester in the case of regular full-time study.

Possible Study Plan for Students Starting in the Winter Semester – Modules							
FS	CP	Module Achievements					
1	26	Foundations of Mathematics (27 CP)	Compulsory Intermediate Modules Algebra and Geometry (13,5 CP)		Subject Didactics Mathematics 2 (6 CP)		
2	31			Compulsory Intermediate Modules Analysis (13,5 CP)		Subject Didactics Mathematics 1 (3 CP)	Subject Didactics Mathematics 3 (6 CP)
3	30	Numerical Mathematics (9 CP)	Introductory seminar (3 CP)		Specialisation 1 (9 CP)		
4	33	Stochastics (9 CP)	Specialisation 2 (9 CP)	Master Thesis (15 CP)			
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points)							

Possible Study Plan for Students Starting in the Winter Semester – Submodules			
FS	CP	Liste der Teilmodule	
1	26	Foundations of Mathematics 1 (13 CP) Exploring Mathematics (1 CP)	Subject Didactics Mathematics 2 - Teil 1 (3 CP)
			Algebraic Structures (4,5 CP) Introduction to Geometry (4,5 CP)
2	31	Foundations of Mathematics 2 (13 CP)	Introduction to Galois Theory (4,5 CP)
		Subject Didactics Mathematics 1 (3 CP)	Subject Didactics Mathematics 2 - Teil 2 (3 CP)
		Introduction to Complex Analysis (4,5 CP)	Subject Didactics Mathematics 3 - Teil 1 (3 CP)
3	30	Measure and Integration Theory (4,5 CP)	Subject Didactics Mathematics 3 - Teil 2 (3 CP)
		Introduction to ODE (4,5 CP)	Introductory seminar (3 CP)
		Introduction to Numerical Mathematics (4,5 CP)	Specialisation (9 CP)
		Mathematical Software (1,5 CP)	
4	33	Stochastics (9 CP)	Specialisation 2 (9 CP)
		Master Thesis (15 CP)	
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points)			

Possible Study Plan for Students Starting in the Summer Semester – Modules							
FS	CP	Module Achievements					
1	21,5	Foundations of Mathematics (27 CP)	Compulsory Intermediate Modules Algebra and Geometry (13,5 CP)		Subject Didactics Mathematics 1 (3 CP)		
2	35,5			Compulsory Intermediate Modules Analysis (13,5 CP)	Subject Didactics Mathematics 2 (6 CP)	Numerical Mathematics (9 CP)	
3	33	Stochastics (9 CP)	Introductory seminar (3 CP)		Master Thesis (15 CP)	Specialisation 1 (9 CP)	Subject Didactics Mathematics 3 (6 CP)
4	30	Specialisation 2 (9 CP)					
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points)							

Possible Study Plan for Students Starting in the Summer Semester – Submodules			
FS	CP	Liste der Teilmodule	
1	26	Foundations of Mathematics 1 (13 CP)	Subject Didactics Mathematics 1 (3 CP)
		Exploring Mathematics (1 CP)	Algebraic Structures (4,5 CP)
2	31	Foundations of Mathematics 2 (13 CP)	Introduction to Geometry (4,5 CP)
			Introduction to Numerical Mathematics (4,5 CP)
		Measure and Integration Theory (4,5 CP)	Mathematical Software (1,5 CP)
		Introduction to ODE (4,5 CP)	Subject Didactics Mathematics 2 - Teil 1 (3 CP)
3	30	Introduction to Complex Analysis (4,5 CP)	Subject Didactics Mathematics 2 - Teil 2 (3 CP)
		Stochastics (9 CP)	Introduction to Galois Theory (4,5 CP)
		Specialisation 1 (9 CP)	Subject Didactics Mathematics 3 - Teil 1 (3 CP)
4	33	Specialisation 2 (9 CP)	Subject Didactics Mathematics 3 - Teil 2 (3 CP)
		Introductory seminar (3 CP)	Master Thesis (15 CP)
Explanation of the Abbreviations: FS=semester, CP=credit points (ECTS points)			

2.3 Overview of Programme Structure with Semester Assignment

Overview of Programme Structure with Semester Assignment for Students Starting in the Winter Semester													
		Exam				Teaching				Semester			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points(CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
Section 1: Foundations of Mathematics									27				
Foundations of Mathematics								24	27				
1.	Lecture	or.	20-30	g	27	L	o	12		9	9		
2.	Exercise class					E	o	6		4	4		
3.	Revision course					R	o	4		0	0		
4.	Practical course					P	o	2		1			
Section 2: Compulsory Intermediate Modules									45				
Compulsory Intermediate Modules Algebra and Geometry								9	13,5				
1.	Lecture	or.	20-30	g	13,5	L	o	6		6	3		
2.	Exercise class					E	o	3		3	1,5		
Compulsory Intermediate Modules Analysis								9	13,5				
1.	Lecture	or.	20-30	g	13,5	L	o	6			3	6	
2.	Exercise class					E	o	3			1,5	3	
Stochastics								6	9				
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	9	L	o	4					6
2.	Exercise class					E	o	2					3
Numerical Mathematics								4	6				
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	6	L	o	2				3	
2.	Exercise class					E	o	1				1,5	
3.	Praktical course	-		ng		P	o	1				1,5	
Introductory seminar								2	3				
1.	Introductory seminar	R		g	3	PS	o	2				3	
Section 3: Knowledge Expansion Mathematics									9				
Specialisation 1								6	9				
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	9	L	o	4				6	
2.	Exercise class					E	o	2				3	

Overview of Programme Structure with Semester Assignment for Students Starting in the Winter Semester													
		Exam				Teaching				Semester			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points(CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
Specialisation 2								6	9				
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	9	L	o	4					6
2.	Exercise class					E	o	2				3	
Section 4: Subject Didactics Mathematics									15				
Subject Didactics Mathematics 1								2	3				
1.	Subject Didactics Mathematics 1	wr., or., P o. T	90-180 o. 20-30	g	3	SV	o	2			3		
Subject Didactics Mathematics 2								4	6				
1.	Subject Didactics Mathematics 2 - Teil 1	wr., or., Pr, E, P o. T	90-180 o. 20-30	g	3	SV	o	2		3			
2.	Subject Didactics Mathematics 2 - Teil 2	wr., or., Pr, E, P o. T	90-180 o. 20-30	g	3	SV	o	2			3		
Subject Didactics Mathematics 3								4	6				
1.	Seminar	wr., or., Pr, E, P o. T	90-180 o. 20-30	g	3	S	o	2			3		
2.	Seminar / Lecture	wr., or., Pr, E, P o. T	90-180 o. 20-30	g	3	SV	o	2				3	
Section 4: Master Thesis									15				
Master Thesis									15				
1.	Master Thesis	MA		g		MA	o						15
Explanation of the abbreviations:													
Marking system : g=graded, ng=non graded													
Form of examination : MA=master thesis, Or.=oral exam, Wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests													
Form of teaching : L=lecture, SL=seminar or lecture, E= exercise class, R=revision course, P=practical course, PS=introductory seminar													
Status : o=obligatory, f=facultative													
Other : o.=or, SWS=hours in class per week, CP=credit points=ECTS points													

Overview of Programme Structure with Semester Assignment for Students Starting in the Winter Semester													
		Exam				Teaching				Semester			
		Type of Exam	Duration (min)	Grading	Weight in the final grade	Type of Course	Status	SWS	ECTS Points(CP)	The allocation of examinations / ECTS points to semesters is of a recommendatory nature. The allocation of ECTS points to courses are of an informative nature. Credits are only awarded upon completion of the module.			
										1. CP	2. CP	3. CP	4. CP
Section 1: Foundations of Mathematics									27				
Foundations of Mathematics								24	27				
1.	Lecture	or.	20-30	g	27	L	o	12		9	9		
2.	Exercise class					E	o	6		4	4		
3.	Revision course					R	o	4		0	0		
4.	Practical course					P	o	2		1			
Section 2: Compulsory Intermediate Modules									45				
Compulsory Intermediate Modules Algebra and Geometry								9	13,5				
1.	Lecture	or.	20-30	g	13,5	L	o	6		3	3	3	
2.	Exercise class					E	o	3		1,5	1,5	1,5	
Compulsory Intermediate Modules Analysis								9	13,5				
1.	Lecture	or.	20-30	g	13,5	L	o	6			6	3	
2.	Exercise class					E	o	3			3	1,5	
Stochastics								6	9				
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	9	L	o	4				6	
2.	Exercise class					E	o	2				3	
Numerical Mathematics								4	6				
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	6	L	o	2			3		
2.	Exercise class					E	o	1			1,5		
3.	Praktical course	-		ng		P	o	1			1,5		
Introductory seminar								2	3				
1.	Introductory seminar	R		g	3	PS	o	2					3
Section 3: Knowledge Expansion Mathematics									9				
Specialisation 1								6	9				
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	9	L	o	4				6	
2.	Exercise class					E	o	2				3	
Specialisation 2								6	9				
1.	Lecture	wr. o. or.	90-180 o. 20-30	g	9	L	o	4					6
2.	Exercise class					E	o	2					3

3 Module Descriptions

Section 1: Foundations of Mathematics

Module Number: MAT-10-10	Module Title: Foundations of Mathematics		Type of Module: Compulsory Module
ECTS-Points	27		
Workload - Time in Class - Self-Study	Workload: 810 h	Time in Class: 300 h	Self-Study: 510 h
Duration	2 Semester		
Frequency	every Semester		
Term	1+2		
Language of Instruction	German		
Forms of Teaching and Learning	1. Semester: Foundations of Mathematics 1, Lecture 6 SWS + Ex.cl. 3 SWS + Rev.c. 2 SWS 2. Semester: Foundations of Mathematics 2, Lecture 6 SWS + Ex.cl. 3 SWS + Rev.c. 2 SWS 1. Semester: Exploring Mathematics, Practical course 2 SWS		
Higher Objectives	In the Foundations of Mathematics module, students learn the essential conceptual and methodological foundations of linear algebra as well as single-variable and multivariable calculus, exploring their interconnections with particular emphasis on the similarities and differences in their approaches. In the oral exam, students demonstrate that they have recognised these relationships and are capable of contextualising the core results of the lectures within these frameworks. The duration of the module supports these objectives while also accounting for the acquisition of a new language - the language of mathematics - and the development of a precise and rigorously logical working methodology. This provides students with the necessary time to make the significant transition from school-level mathematics to university-level mathematics. By demonstrating a deeper and more integrated understanding in the oral exams, students establish a strong foundation for successful participation in all subsequent modules in their academic programme.		

Content	• Basic logic and sets. • Structure of real and complex numbers. • Sequences, convergence and series; criteria for convergence; power series, sequences of functions; pointwise and uniform convergence. • Continuous functions in one dimension and between metric spaces and their properties. • One- and multidimensional differential calculus (especially: intermediate value theorem, Taylor expansion, implicit function theorem, inverse function theorem, extrema under constraints). • Onedimensional Riemann integral. • Basic concepts of topology in metric and normed spaces. • Vector spaces and linear maps. • Matrices and systems of linear equations. • Determinants, eigenvalues and diagonalisability. • Jordan canonical form. • Euclidean and unitary vector spaces, spectral theorems. • Basics of analytical geometry. • The lecture Foundations of Mathematics 1 focuses primarily on topics in one-dimensional analysis and introduces basic concepts of linear algebra up to systems of linear equations. The lecture Foundations of Mathematics 2 covers further aspects of linear algebra as well as multivariable analysis. • The submodule Exploring Mathematics serves as an introduction to mathematical working methods, mathematical modelling, and abstraction.
Objectives	The students are familiar with and understand the fundamental concepts, statements, and methods of single-variable and multivariable calculus as well as linear algebra. Their capacity for abstraction has been enhanced, they have been trained in analytical thinking, and their mathematical imagination has been stimulated. Through a proof- and structure-oriented approach, they have learned to comprehend mathematical proofs in calculus and linear algebra and to independently prove or disprove mathematical statements in simple examples. They have recognised the essential relationships within the theory of single-variable and multivariable calculus, their similarities and differences, as well as their connections to linear algebra, and are able to contextualise the core results of the lectures within these frameworks. Students are in principle able to derive a mathematical problem and approaches to its solution from a question that may not be formulated in mathematical terms by means of mathematical modelling, and to relate these back to the original question. In the exercises, they have developed a confident, precise, and independent approach to the concepts, statements, and methods covered in the lectures. Additionally, their presentation and communication skills have been cultivated through written assignments and presenting their own solutions. The students are capable of acquiring knowledge through self-study, while their teamwork abilities have been fostered through collaboration in small groups.

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade				
	Foundations of Mathematics 1	L	o	6	9	yes	or.	30-40	g	100				
		E	o	3	4									
		T	o	2	0									
	Foundations of Mathematics 2	L	o	6	9	yes								
		E	o	3	4									
		T	o	2	0									
	Exploring Mathematics	P	o	2	1	yes								
	In each of the two submodules Foundations of Mathematics 1 and 2, an exercise certificate is required as course work. The exercise certificate is awarded on the basis of regular participation in the exercises together with successful completion of a test related to the exercises. Both components must be completed within the same semester. In the submodule Exploring Mathematics, a practical certificate is required as course work and is obtained through regular active participation. The assessment of the module takes the form of an oral examination covering the submodules Foundations of Mathematics 1 and 2. Admission to the oral examination requires at least one of the two exercise certificates in the submodules Foundations of Mathematics 1 or 2. The module is deemed completed only once both exercise certificates and the practical certificate have been obtained and the oral examination has been passed. For modules extending over several semesters, it is strongly recommended to review the content of each submodule thoroughly during the lecture-free periods, thereby spreading the preparation for the oral examination over a longer period of time.													
Literature	Possible References : • Anton Deitmar: Analysis. Springer 2016. • Otto Forster: Analysis 1. Springer Spektrum 2013. • Otto Forster: Analysis 2. Vieweg+Teubner 2011. • Theodor Bröcker: Lineare Algebra und analytische Geometrie. Birkhäuser 2013. • Gerd Fischer: Lineare Algebra. Springer Spektrum 2014. • Daniel Grieser: Mathematisches Problemlösen und Beweisen. Springer Spektrum 2017.													
Transfer	The successful participation in the module Foundations of Mathematics is a prerequisite for the participation in all modules of Section 3 extension of knowledge mathematics and for the module Master Thesis. The exercise certificates of the module are prerequisite for all modules of the study programme but the module Consolidation of the Foundations of Mathematics.													
Prerequisites	There are no prerequisites.													
Responsible Persons	Daniele Agostini, Anton Deitmar, Frank Loose, Hannah Markwig, Thomas Markwig, Stefan Teufel													

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio,
T=continuous assessment tests

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted class-
room

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 2: Compulsory Intermediate Modules

Module Number: MAT-20-05	Module Title: Compulsory Intermediate Modules Analysis		Type of Module: Compulsory Module
ECTS-Points	13.5		
Workload - Time in Class - Self-Study	Workload: 405 h	Time in Class: 135 h	Self-Study: 270 h
Duration	2 Semester		
Frequency	• Measure and Integration Theory, regularly in Winter Semester • Introduction to ODE, regularly in Winter Semester • Introduction to Complex Analysis, regularly in Summer Semester		
Term	2-4		
Language of Instruction	German		
Forms of Teaching and Learning	• Measure and Integration Theory, Lecture 2 SWS + Ex.cl. 1 SWS • Introduction to ODE, Lecture 2 SWS + Ex.cl. 1 SWS • Introduction to Complex Analysis, Lecture 2 SWS + Ex.cl. 1 SWS		
Content	• Measure and Integration Theory: – Measures, in particular the Lebesgue measure. – Lebesgue integral, L^p-spaces, Fubini's theorem, convergence theorems. – Integration on areas of $\mathbb{R}^n$, transformation formula. – Convergence theorems. • Ordinary differential equations, a choice of the following: – Picard-Lindelöf existence and uniqueness theorem. – Linear ordinary differential equations, Gronwall's lemma. – Continuous and differential dependence on initial conditions. – Basics of dynamical systems, stability of equilibrium positions. – Ordinary differential equations over the complex numbers. • Complex Analysis: – Holomorphic functions, Cauchy-Riemann equations. – Antiderivatives, Cauchy's integral formula, Cauchy's integral theorem. – Compact convergence of families of functions, power series, identity theorem. – Liouville's theorem, inverse function theorem for holomorphic functions, open mapping theorem, maximum principle. – Laurent series. – Residue theorem and applications.		

Literature	Possible References : • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 1978. • Anton Deitmar: Analysis. Springer Spektrum 2017. • Jürgen Elstrodt: Maß- und Integrationstheorie. Springer 2011. • Otto Forster: Analysis 3. Friedr. Vieweg+Teubner 2011. • Lars Valerian Ahlfors: Complex analysis. McGraw-Hill 1979. • John B. Conway: Functions of one complex variable. Springer 1996. • Wolfgang Fischer, Ingo Lieb: Einführung in die Komplexe Analysis. Springer 2010. • Walter Rudin: Reelle und komplexe Analysis. Oldenbourg 2009. • Earl A. Coddington, Norman Levinson: Theory of ordinary differential equations. McGraw-Hill 1955. • William T. Reid: Ordinary differential equations. John Wiley & Sons 1971. • Hille, Einar: Ordinary differential equations in the complex domain. Dover Publications 1997. • Wasow, Wolfgang: Asymptotic expansions for ordinary differential equations. John Wiley 1965.
Transfer	If applicable the module is a prerequisite for the module Master Thesis.
Prerequisites	There are no further prerequisites.
Responsible Persons	Anton Deitmar, Peter Pickl, Reiner Schätzle, Stefan Teufel
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-20-06	Module Title: Compulsory Intermediate Modules Algebra and Geometry		Type of Module: Compulsory Module
ECTS-Points	13.5		
Workload - Time in Class - Self-Study	Workload: 405 h	Time in Class: 135 h	Self-Study: 270 h
Duration	2 Semester		
Frequency	- Algebraic Structures, regularly every Semester - Introduction to Galois Theory, regularly in Summer Semester - Introduction to Geometry, regularly in Winter Semester		
Term	1-4		
Language of Instruction	German		
Forms of Teaching and Learning	- Algebraic Structures, Lecture 2 SWS + Ex.cl. 1 SWS - Introduction to Galois Theory, Lecture 2 SWS + Ex.cl. 1 SWS - Introduction to Geometry, Lecture 2 SWS + Ex.cl. 1 SWS		
Content	- Algebraic Structures - Groups, subgroups, group homomorphisms, normal subgroups, quotient group. - Cyclic groups and the symmetric group. - Commutative rings with one, divisibility. - Euclidean rings, principal ideal domains, factorial rings. - The ring of integers and the polynomial ring. - Introduction to Galois Theory - Fields and field extensions. - Fundamental theorem of Galois theory and applications, in particular: - Constructions with ruler and compass. - Solvability of equations. - Introduction to Geometry - Selected topics in geometry from the following canon are covered: - Axiomatic foundation of planar geometry. - Euclidean geometry. - Non-Euclidean geometry. - Analytic geometry. - Parametrised curves and surfaces.		

[illegible]

Literature	Possible References : • Serge Lang: Algebraische Strukturen. Vandenhoeck & Ruprecht 1979. • Siegfried Bosch: Algebra. Springer 2009. • Gerd Fischer, Reinhard Sacher: Einführung in die Algebra. Teubner 1983. • Christian Karpfinger, Kurt Meyberg: Algebra: Gruppen-Ringe-Körper. Springer Spektrum 2010. • Hans-Jörg Reiffen, Günter Scheja, Udo Vetter: Algebra. Bibliographisches Institut 1984. • Michele Audin: Geometry. Springer 2003. • Marcel Berger: Geometry Revealed: A Jacob's Ladder to Modern Higher Geometry. Springer 2010. • David A. Brannan, Matthew F. Esplen, Jeremy J. Gray: Geometry. Cambridge University Press 2012. • John Stillwell: The four pillars of geometry. Springer 2005.
Transfer	If applicable the module is a prerequisite for the module Master Thesis.
Prerequisites	There are no further prerequisites.
Responsible Persons	Daniele Agostini, Carla Cederbaum, Jürgen Hausen, Hannah Markwig, Thomas Markwig, Walther Paravicini
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-20-12	Module Title: Stochastics				Type of Module: Compulsory Module					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	regularly in Summer Semester									
Term	2-4									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Content	• Introduction to probability theory and statistics. • Topics from probability theory: Probability spaces, simple conditional probabilities, urn models, random variables, distribution functions, discret and continous distributions, expectation and variance, inequalities, independence, joint probability distribution, notions of convergence, laws of lagre numbers, central limit theorem. • Topics from statistics: Point estimators, hypothesis testing, standard testing methods.									
Objectives	The students know the basic principles of stochastics. They have the ability to abstract stochastic questions and are capable of using their knowledge on specific problems. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. Furthermore the presentation and communication skills of the students were trained by written assignments and presenting their own solutions. The students are capable of adopting knowledge by self-study and at the same time their capacity for teamwork was enhanced by working in small groups.									
Requirements for obtaining Credits / Grading (Weighting if applicable)										
	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Stochastics	L E	o o	4 2	6 3	yes	wr. o. or.	90-180 o. 20-30	g	100
In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.										
Literature	Possible References : • Hans-Otto Georgii: Stochastik. De Gruyter 2015. • Ulrich Krengel: Einführung in die Wahrscheinlichkeitstheorie und Statistik. Vieweg 2005.									
Transfer	If applicable, the module is prerequisite for the module Master Thesis.									
Prerequisites	At least one of the exercise certificates from the module Foundations in Mathematics must have been acquired. Prior knowledge of the submodule Measure and Integration Theory is assumed.									

Responsible Persons	Martin Möhle, Elmar Teufl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted class-room Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

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Prerequisites	At least one of the exercise certificates from the module Foundations in Mathematics must have been acquired. Furthermore, before admission to the examination, the practical certificate for the submodule Mathematical Software must have been obtained.
Responsible Persons	Christian Lubich, Andreas Prohl
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Module Number: MAT-20-20	Module Title: Introductory Seminar: Presentations in Mathematics						Type of Module: Compulsory Module with Choice			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	every Semester									
Term	2-4									
Language of Instruction	German									
Forms of Teaching and Learning	Introductory seminar, talk, presentation, e-learning, blended learning									
Content	Various topics from the foundations of mathematics.									
Objectives	The students independently work on a coherent mathematical topic and prepare it in a didactical appealing form. They learn how to present their work to a group, how to be responsive to questions regarding the content and how to lead a professional discussion.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Introductory Seminar	PS	o	2	3	yes	Pr	60-90	g	100
	The acquisition of the credit points requires alongside with a successful presentation the regular active participation in the course, like by asking questions, contributing to a discussion or working on problem tasks. Additionally a written elaboration of the own talk or the issue of a handout for the participants may be required. This further work constitutes the coursework of the module.									
Transfer	-									
Prerequisites	At least two of the exercise certificates from the module Foundations in Mathematics must have been acquired. One of these must be the certificate for linear algebra 1.									
Responsible Persons	The dean of studies at the Department of Mathematics									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Section 3: Knowledge Expansion Mathematics

Module Number: MAT-40-51	Module Title: Specialisation 1				Type of Module: Compulsory Module with Choice					
ECTS-Points	9									
Workload - Time in Class - Self-Study	Workload: 270 h			Time in Class: 90 h			Self-Study: 180 h			
Duration	1 Semester									
Frequency	every Semester									
Term	3-4									
Language of Instruction	German or English									
Forms of Teaching and Learning	Lecture 4 SWS + Ex.cl. 2 SWS									
Comment	A course must be selected from the catalogue of courses in Section 4.1 of the module handbook, comprising 4 hours of lectures and 2 hours of exercises per week. The approval of additional courses or alternative course formats (e.g., two courses with 2 hours of lectures and 1 hour of exercises each) is at the discretion of the head of the examination board, upon a written request by the student.									
Content	The content is determined by the choice of a course.									
Objectives	The students have acquired in-depth knowledge in a specific area of mathematics and gained further experience in presenting and communicating mathematical topics. They are capable of identifying the key statements of the lecture, reproducing the techniques used for their derivation and proof, and critically evaluating them. Additionally, they can integrate the methodological and theoretical foundations of the chosen mathematical subfield and place them within the broader mathematical context. In the exercise classes they have acquired a confident, precise and independent handling of the terms, statements and methods of the lecture. They have learned to transfer the methods onto new problems, to analyse them and to work on solution strategies on their own or in a team.									
Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	see Comment	L	o	4	6	yes	wr. o. or.	90-180 o. 20-30	g	100
		E	o	2	3					
	In this module an exercise certificate is to be acquired as coursework. For participation in the examination the coursework must have been acquired. Whether the examination is written or oral is decided by the instructor with approval by the head of the examination board.									
Transfer	If applicable the module is a prerequisite for the modules Seminar Specialisation Mathematics and Master Thesis.									
Prerequisites	The modules of Section 1 Foundations of Mathematics must have been completed successfully.									
Responsible Persons	The dean of studies at the Department of Mathematics									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio,
T=continuous assessment tests

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted class-
room

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

[illegible]

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio,
T=continuous assessment tests

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted class-
room

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-80-01	Module Title: Subject Didactics Mathematics 1						Type of Module: Compulsory Module			
ECTS-Points	3									
Workload - Time in Class - Self-Study	Workload: 90 h			Time in Class: 30 h			Self-Study: 60 h			
Duration	1 Semester									
Frequency	regularly in Summer Semester									
Term	2-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture, exercise class, seminar, talk, presentation, e-learning, blended learning, project work, case studies									
Content	Didactics of Algebra and Arithmetic: This course deals with the foundations of the didactics of mathematics in the educational plans and in particular the didactic reduction of important basic concepts of algebra and arithmetic to school level, various ways of introducing important concepts of algebra and arithmetic at school and ways of motivating basic algebraic and arithmetic ideas.									
Objectives	Students know the basic didactic principles of teaching concepts and can orientate themselves in the educational plans. They are able to compare and evaluate subject-specific approaches to central concepts in algebra and arithmetic. They have the ability to convey algebraic and arithmetic content in a way that is both student- and subject-orientated.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Subject Didactics Mathematics 1	LIC	o	2	3	no	wr., or., P o. T	90-180 o. 20-30	g	100
	Whether the form of the examination is decided by the instructor with approval by the head of the examination board.									
Transfer	-									
Prerequisites	At least two of the exercise certificates from the module Foundations of Mathematics must have been acquired. One of these must be the exercise certificate for Linear Algebra 1.									
Responsible Persons	Frank Loose, Walther Paravicini									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continuous assessment tests

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted classroom

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-80-02	Module Title: Subject Didactics Mathematics 2					Type of Module: Compulsory Module				
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	2 Semester									
Frequency	every Semester									
Term	2-3									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture, exercise, proseminar, talk, presentation, e-learning, blended learning, project work, case studies									
Content	The module consists of the two parts • didactics of geometry and linear algebra, • didactics of analysis and stochastics. It deals with the didactic reduction of important basic concepts of analysis, linear algebra, geometry or stochastics at school level, various options for introducing important terms in analysis, linear algebra, geometry or stochastics at school as well as motivational options for analytic, geometric and stochastic basic ideas.									
Objectives	Students are familiar with the basic didactic principles of teaching concepts. They are able to compare and evaluate subject-specific approaches to central concepts in analysis, linear algebra, geometry and stochastics. They have the ability to convey geometric and algebraic content in a way that is both student- and subject-orientated.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Subject Didactics Mathematics 2 – Part 1	SLIC	o	2	3	yes	wr., or., E, Pr o. T	90-180 o. 20-30	g	50
	Subject Didactics Mathematics 2 – Part 2	SLIC	o	2	3	yes	wr., or., E, Pr o. T	90-180 o. 20-30	g	50
	The module consists of two parts, for which the teaching learning methods (lecture, exercise class or seminar) as well as the type of examination (written or oral exam, presentation or paper, portfolio or tests) are usually different. This is taken into account by the fact that, the examination in this module consists of two equally weighted parts.									
Transfer	-									
Prerequisites	The module Foundations of Mathematics must be completed. The module Geometry should be taken parallel to the Didactics of Geometry or should have been taken beforehand, as knowledge from the module Geometry is required in the Didactics of Geometry module.									
Responsible Persons	Frank Loose, Walther Paravicini									

Abbreviations:

Grading System : g=graded, ng=not graded

Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio,
T=continuous assessment tests

Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class,
T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted class-
room

Status : o=obligatory, f=facultative

Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week

Module Number: MAT-80-03	Module Title: Subject Didactics Mathematics 3					Type of Module: Compulsory Module with Choice				
ECTS-Points	6									
Workload - Time in Class - Self-Study	Workload: 180 h			Time in Class: 60 h			Self-Study: 120 h			
Duration	2 Semester									
Frequency	every Semester									
Term	3-4									
Language of Instruction	German									
Forms of Teaching and Learning	Lecture, exercise, seminar, talk, presentation, e-learning, blended learning, project work, case studies									
Content	In the first part varying topics are covered, who are usually related to the teaching profession. However, also other advanced topics in didactics can be covered. In the second part varying topics from didactics will be studied, which can lead up to recent research in didactics.									
Objectives	The students • know subject didactical principles and educational concepts and can evaluate and scrutinise them, • can compare and evaluate subject related approaches to central terms and theorems of the treated fields, • can plan, execute, analyse and evaluate competence oriented mathematical education on the basis of subject didactical concepts, • can motivate the general educating value of mathematical contents and methods and the social importance of mathematics and put it in context with the goals and contents of mathematical education, • can specifically use subject specific media, • can create a portfolio and document important experiences, findings and insights in a structured manner.									
Requirements for obtaining Credits / Grading (Weighting if applicable)		Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Title									
	Subject didactics 3: professional knowledge	S	o	2	3	yes	wr., or., E, Pr o. T	90-180 o. 20-30	g	50
	Subject didactics 3: elective specialisation	SV	o	2	3	yes	wr., or., E, Pr o. T	90-180 o. 20-30	g	50
	The module consists of two parts (Professional Knowledge and Elective Specialisation), for which the teaching learning methods (lecture, exercise or seminar) as well as the type of examination (written or oral exam, presentation or paper, portfolio or tests) are usually different. This is taken into account by the fact that, the examination in this module consists of two equally weighted parts.									
Transfer	-									

Prerequisites	The participation in the module requires the successful completion of the modules from Section 1 Foundations of Mathematics.
Responsible Persons	Frank Loose, Walther Paravicini
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week	

Section 5: Master Thesis

Module Number: MAT-40-54	Module Title: Master Thesis		Type of Module: Compulsory Module
ECTS-Points	15		
Workload - Time in Class - Self-Study	Workload: 450 h	Time in Class: regularly, as required	Self-Study: 450 h
Duration	1 Semester		
Frequency	every Semester		
Term	4		
Language of Instruction	German		
Forms of Teaching and Learning	Master thesis		
Content	The master's thesis represents the culmination of the master's degree programme. Under the guidance of a supervisor, students are required to work on a well-defined problem in mathematics (including mathematics education), which may extend to the frontiers of current research. This involves using scientific methods and presenting the results in written form. Specifically, this includes: • formulating a scientific research question in consultation with the supervisor; • independently searching for and studying relevant scientific literature; • formulating appropriate research questions and methodological approaches for their solution; • independently carrying out the project, as well as presenting the project and its results in writing and, if applicable, orally, within the context of the current state of research. The results should contribute to scientific knowledge.		
Objectives	The students: • are capable of familiarising themselves with a problem, which may extend to the frontiers of current research, within a given timeframe and independently developing a solution approach, • can increasingly apply appropriate scientific methods independently and present the results in a scientifically appropriate manner, • are able to independently work on a scientific topic while applying their mathematical methodological knowledge, • deepen their problem-solving skills and can transfer their methodological knowledge, • are able to present the results of their project to a professional audience.		

Requirements for obtaining Credits / Grading (Weighting if applicable)	Title	Type of Course	Status	SWS	ECTS	Coursework	Type of Exam	Dur. of Exam (min)	Grading	Weight for Grade
	Master Thesis	MT	o	-	15	yes	MT	-	g	100
	The coursework consists of an examination interview on the contents of the master thesis. The coursework is graded as pass or fail. The module grade is the grade for the master thesis. The module is only passed if the examination and coursework have been successfully completed.									
Transfer	-									
Prerequisites	Subject specific prerequisite for admission to the module Master Thesis is besides the general part of the examination regulations the successful completion of all modules from Section 1 Foundations of Mathematics as well as the acquisition of at least further 42 credit points.									
Responsible Persons	The dean of studies at the Department of Mathematics									
Abbreviations: Grading System : g=graded, ng=not graded Examination Type : MT=master's thesis, or.=oral exam, wr.=written exam, Pr=presentation, E=essay, P=portfolio, T=continous assessment tests Teaching Format : L=lecture, LE=lecture with integrated exercises, SL=seminar or lecture, E=exercise class, T=revision course, P=practical course, PS=introductory seminar, S=seminar, IC=inverted classroom Status : o=obligatory, f=facultative Other : h=hours, o.=or, s.M.=see module description, SWS=contact hours per week										

4 Courses for the Module Specialisation

4.1 Catalogue of Courses

The following lists the courses that can be included in the modules Specialisation 1 and 2. Other courses may be approved upon written request to the head of the examination board.

• Algebraic Curves and Riemann Surfaces	42
• Algebraic Number Theory	44
• Algebraic Topology 1	43
• Algorithms of Numerical Mathematics	44
• Automorphic Forms	45
• Bayesian Networks and Causality	46
• Calculus of Variations	94
• Commutative Algebra	72
• Consistency Proofs	96
• Convex Geometry	73
• Cryptography	74
• Elementary Number Theory	59
• Elliptic Curves and Cryptography	60
• Explicit Mathematics	61
• Financial Mathematics and Numerics	62
• Functional Analysis	63
• Game Theory	88
• Geometric Group Theory	65
• Geometry in Physics	66
• Geometry of Manifolds 1	64
• Graph Theory	67

• Groups and Representations	68
• Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic	69
• Information Geometry	70
• Information Theory, Pattern Recognition, and Neural Networks	70
• Interacting Many-Body Quantum Systems	71
• Introduction to Analytic Number Theory	48
• Introduction to Berkovich Geometry	49
• Introduction to Commutative Algebra and Algebraic Geometry	55
• Introduction to Dynamical Systems	52
• Introduction to Geometric Measure Theory	53
• Introduction to Geometric Measure Theory – Measure Theoretic Methods	54
• Introduction to Geometric Measure Theory – Varifolds	54
• Introduction to Mathematical Logic	50
• Introduction to Optimisation	51
• Introduction to Partial Differential Equations	56
• Introduction to Partial Differential Equations – Part 1	57
• Introduction to Riemann Surfaces	58
• Introduction to Stochastic Differential Equations - Part 1	59
• Introduction to set theory	51
• Lie Groups	75
• Linear Control Theory	76
• Mathematical Aspects of 3d Computer Vision	77
• Mathematical Aspects of Deep Learning 1	78
• Mathematical Introduction to Data Science	79
• Mathematical Statistics	80
• Nonparametric Estimation of Distributions	81
• Number Theory and Cryptography	97
• Numerics of Stationary Differential Equations	82
• Numerics of Stochastic Differential Equations	82

• Optimal Control Theory with Ordinary Differential Equations	83
• Optimal Transport	84
• Optimisation with Differential Equations	85
• Ordinary Differential Equations - Analysis and Numerics	66
• Probability Theory	95
• Problem Solving Strategies	85
• Propagation of Chaos	86
• Representation Theory of Finite Groups	47
• Software Obfuscation: Mixed Boolean-Arithmetic Expressions	87
• Statistical Learning 1	89
• Stochastic Differential Equations	89
• Theoretical Aspects of Machine Learning	90
• Theory and Numerics for Constrained Optimisation Problems	91
• Theory of Mathematical Proofs	78
• Topology	92
• Tropical Enumerative Geometry	92
• Tropical Enumerative Geometry - Part 1	93

Course Title:	Algebraic Curves and Riemann Surfaces		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Compact Riemann surfaces. • Normalisation of plane curves. • Topological genus. • Coverings. • Forms and integration. • Sheaves and cohomology. • Hodge theory. • Arithmetic and geometric genus. • Abel's theorem. • Riemann-Roch theorem. • Serre duality. • Jacobian and Abelian varieties. • Riemann bilinear relations. • Jacobi inverse problem. • Elliptic curves and functions. • j-Invariant. • Uniformisation. • Topology of non-compact Riemann surfaces.		
Special Objectives	Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. By example of the concept of sheaves, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	Possible References : • Frederice Mangolte: Real Algebraic Varieties. Springer 2020. • Robert Silhol: Real Algebraic Surfaces. Springer 1989. • Riccardo Benedetti, Jean-Jacques Risler: Real Algebraic and Semi-algebraic Sets. Editions Hermann 1990. • Alex Degtyarev, Viatcheslav Kharlamov: Topological properties of real algebraic varieties: du côté de chez Rokhlin. arXiv:math/0004134.
Responsible Persons	Daniele Agostini, Ivo Radloff

Course Title:	Algebraic Topology 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Set theoretical topology. • Basic concepts of category theory. • The fundamental group of a punctured topological space. • Theory of covering spaces. • Basic concepts of singular homology theory. • Applications.		
Special Objectives	The students learn how to realise ideas in topology, e.g. the detection of holes in topological spaces, into a precise theory, even with a sophisticated technique. In particular, they recognise how abstract concepts, e.g. from category theory and homological algebra, provide effective ways of speaking that enable the formation of ideas to be adequately implemented.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	Possible References : • Allen Hatcher: Algebraic topology. Cambridge University Press 2009. • Horst Schubert: Topologie. Teubner 1971. • Edwin H. Spanier: Algebraic topology. McGraw-Hill 1966. • Ralph Stöcker, Heiner Zieschang: Algebraische Topologie. Teubner 1994.		
Responsible Persons	Anton Deitmar, Frank Loose		

Course Title:	Algebraic Number Theory		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Rings of integers. • Class numbers. • Dirichlet's unit theorem. • Extension of Dedekind rings. • Valuation theory. • Local fields. • Adeles and ideles.		
Special Objectives	The students have learned the central terms, results and methods of algebraic number theory.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.		
Literature	Possible References : • Jürgen Neukirch: Algebraische Zahlentheorie. Springer 2007. • Alexander Schmidt: Einführung in die algebraische Zahlentheorie. Springer 2007. • Andre Weil: Basic number theory. Springer 1995.		
Responsible Persons	Daniele Agostini, Anton Deitmar		

Course Title:	Algorithms of Numerical Mathematics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		

Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS
Content	Advanced, important algorithms of numerics (without differential equations) such as: • Fast Fourier transformation; • QR algorithms for the calculation of eigenvalues; • Method of conjugated gradients and more general Krylov space methods as iterative methods in numeric linear algebra and in non-linear optimisation; • Simplex method and interior point methods in linear optimisation.
Special Objectives	The students have learned the key concepts, results, and methods of algorithmic numerical mathematics.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-11 / MAT-20-13 Numerical Mathematics.
Literature	Possible References : • Peter Deufhard, Andreas Hohmann: Numerische Mathematik 1. De Gruyter 2008. • Martin Hanke-Bourgeois: Grundlagen der Numerischen Mathematik und des Wissenschaftlichen Rechnens. Vieweg 2009.
Responsible Persons	Christian Lubich, Andreas Prohl

Course Title:	Automorphic Forms		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Modular forms for the module group and its congruence subgroups. • Examples: Eisenstein series, Ramanujan delta function, theta series. • Modular curves. • Arithmetic applications and conjectures. • Hecke operators. • The L-function of a modular form and its connections with elliptic curves.		
Special Objectives	Students have familiarised themselves with the central concepts and methods of the theory of automorphic forms in examples and are able to use them. They have understood the connection between modular, real representation theory and adelic L-functions.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Literature	Possible References : • Deitmar, Anton: Automorphic Forms. Springer 2012. • Goldfeld, Dorian: Automorphic forms and L-functions for the group $GL(n, \mathbb{R})$. Cambridge University Press 2015. • Serre, Jean-Pierre: A course in arithmetic. Springer 1973.
Responsible Persons	Anton Deitmar

Course Title:	Bayesian Networks and Causality		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	Uncertainty is a fact of life, and robust artificial intelligence for real life application has to be able to deal with it. Therefore, the development of mathematical representations that effectively incorporate probabilities was a key step in the development of artificial intelligence. However, human understanding goes further than that: We go beyond observing events, such as <i>Whenever the sprinkler is on in my greenhouse, the plants are wet</i> or <i>Whenever the plants are wet, the sprinkler is on</i>, to postulating a relationship of cause and effect: <i>If I turn on the sprinkler, the plants will be wet, but if I water the plants, that will certainly not activate the sprinkler.</i> In this course, Bayesian networks are studied, which are a widely used representation for probability distributions. In particular, commonly used inference and learning algorithms are discussed. Moreover, it will be shown that Bayesian networks do not only represent probability distributions, but are also able to express causal relationships. Finally the causal expressivity of Bayesian networks will be looked at, aiming to learn causal structure from observational data. • Part I: Bayesian Networks as an Efficient Representation of Probability Distributions: – Computing probabilities using Bayesian networks. – d-Separation: A graphical criterion for probabilistic independence. – Parameter and structure learning in Bayesian networks. • Part II: Bayesian Networks as a Representation for Causal Knowledge: – Functional causal models: A representation of causal knowledge. – Pearl's causal ladder: Predicting the effects of external interventions and reasoning with counterfactuals. – Causal Bayesian networks. – Causal structure discovery: Learning causal relationships from data. – Counterfactual identifiability: Answering counterfactual. questions using causal Bayesian networks.		
Special Objectives	In the first part of the course, students have learned how Bayesian networks represent probability distributions and how this representation can be used to efficiently compute probabilities or determine whether two random variables are independent. In the second part, students have learned to distinguish inference tasks in artificial intelligence according to Pearl's causal ladder: probabilistic, interventional, and counterfactual reasoning, which generally require increasingly detailed knowledge. They are familiar with identifiability results, which provide assumptions under which certain queries on the causal ladder can be answered using only knowledge from lower levels. For instance, they know how to answer queries about the effects of external interventions using only knowledge of the correct probability distribution, which can often be estimated from observational data.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.		
Literature	Possible References : • Judea Pearl: Causality: Models, Reasoning and Inference. Cambridge University Press 2009.		
Responsible Persons	Stephan Eckstein		

Course Title:	Representation Theory of Finite Groups		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h

Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	• Groups and group actions. • Representations, irreducibility, Schursch's lemma. • Semisimplicity, Maschke's theorem. • Characters, orthogonality relations. • Isotypical decomposition, character tables. • Representations of the symmetric group. • Semi-simple Artinian algebras.		
Special Objectives	In the lecture, students learn the basic principles of of representation theory and develop an understanding for the interaction of geometric and algebraic methods. methods.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		
Literature	Possible References : • William Fulton, Joe Harris: Representation theory. Springer 1991. • Bertram Huppert: Character theory of finite groups. De Gruyter 1998. • Serge Lang: Algebra. Springer 2002. • Jean-Pierre Serre: Linear representations of finite groups. Springer 1977.		
Responsible Persons	Jürgen Hausen, Thomas Markwig		

Course Title:	Introduction to Analytic Number Theory		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		

Content	• Arithmetic functions and Dirichlet series, • Prime number theorem and Dirichlet's prime number theorem, • Zeros of the Riemannian zeta function, Riemann hypothesis and the explicit formula.
Special Objectives	The students understand the interplay between analysis and number theory. They can apply analytical methods to number theoretic problems. They understand the mechanism of analytical continuation through integral representation and have learned to independently transfer it to other cases, such as automorphic L-functions. They have gained an understanding of the Riemann hypothesis, which is considered the most difficult problem of all math, and understand its depth.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.
Literature	Possible References : • Komaravolu Chandrasekharan: Introduction to Analytic Number Theory. Springer 1968.
Responsible Persons	Anton Deitmar

Course Title:	Introduction to Berkovich Geometry		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Non-Archimedean fields, valuations, and absolute value functions. • Ultrametric triangle inequality and induced topology. • Affinoid domains. • Berkovich affine and projective line. • Analytification of algebraic varieties.		
Special Objectives	The students have become familiar with the most important examples of non-Archimedean fields and their induced topology. They have gained an understanding of the fundamental challenges in developing a theory of analytic geometry over these fields and have studied Berkovich's approach to addressing these issues. The students have examined the projective line in Berkovich's framework in detail, both set-theoretically and topologically. In doing so, they have encountered a type of geometric space fundamentally different from other examples encountered in their studies (such as vector spaces, varieties, or manifolds). They are also familiar with the connections to algebraic geometry through the analytification functor.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Literature	Possible References : • Annette Werner: Nichtarchimedische Geometrie. Vorlesungsskript.
Responsible Persons	Hannah Markwig

Course Title:	Introduction to Mathematical Logic		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Propositional logic. • Languages of the first order: – Completeness and compactness. • Theory of computations: – Register machines; – Gödelisation. • Incompleteness of arithmetic: – First and second incompleteness theorem. • Set theory: – Ordinal- and cardinal numbers; – Incompleteness of set theory.		
Special Objectives	Students are able to understand mathematical theorems and theories in the context of mathematical logic. They understand the limits of possible mathematical knowledge, recognise the difference between truth and provability and can apply basic theoretical model thinking to mathematical content.		
Prerequisites	There are no prerequisites.		
Literature	Possible References : • Rautenberg, Wolfgang: Einführung in die Mathematische Logik. Vieweg+Teubner 2008. • Ziegler, Martin: Mathematische Logik. Birkhäuser 2016.		

Responsible Persons	Anton Deitmar
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Course Title:	Introduction to set theory		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	Content: •		
Special Objectives	-		
Prerequisites	There are no further prerequisites.		
Literature	Possible References : •		
Responsible Persons	Frank Loose		

Course Title:	Introduction to Optimisation		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 3 SWS + Exercise class 1 SWS		
Content	• Optimality theory for smooth, convex and linear optimisation problems optimisation problems with constraints. • Foundations of the theory of convex sets and functions. • Duality theory for convex and linear optimisation problems. • Solution methods for linear optimisation problems.		

Special Objectives	Students know and understand methods and algorithms for solving convex and linear optimisation problems. They have learnt to apply the methods to simple problems related to economics, technology or physics. They will be able to critically assess the possibilities and limitations of using the methods.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	Possible References : • Florian Jarre, Joseph Stoer: Optimierung: Einführung in mathematische Theorie und Methoden. Springer 2019. • Jorge Nocedal, Stephen J. Wright: Numerical optimization. Springer 2006.
Responsible Persons	Christian Lubich

Course Title:	Introduction to Dynamical Systems		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Kepler's laws. • Equilibrium positions. • Stability. • Predator-prey model. • Poincaré-Bendixson theorem. • Limit sets. • Periodic orbits. • Celestial mechanics.		
Special Objectives	The students can ask and examine qualitative questions about the solutions of ordinary differential equations, like e.g.: How long do mathematical solutions exist? Are there equilibrium states or periodic orbits? When are trajectories stable?		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations.		

Literature	Possible References : • Morris W. Hirsch, Stephen Smale: Differential equations, dynamical systems, and linear algebra. Academic Press 1974. • Vladimir I. Arnold: Mathematical methods of classical mechanics. Springer 2010. • Carl Ludwig Siegel, Jürgen Moser: Lectures on celestial mechanics. Springer 1995.
Responsible Persons	Frank Loose

Course Title:	Introduction to Geometric Measure Theory		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem. • Surface- and cosurface formula. • Countable rectifiable sets, rectifiable varifolds.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	Possible References : • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.		
Responsible Persons	Reiner Schätzle		

Course Title:	Introduction to Geometric Measure Theory – Measure Theoretic Methods		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Measures, covering theorems, differentiation of measures, Hausdorff measures and densities. • Isodiametric inequality. • Rademacher's theorem and Whitney's embedding theorem.		
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with the basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	Possible References : • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.		
Responsible Persons	Reiner Schätzle		

Course Title:	Introduction to Geometric Measure Theory – Varifolds		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	• Surface- and cosurface formula. • Countable rectifiable sets, rectifiable varifolds.
Special Objectives	Students have familiarised themselves with an important mathematical field that combines analysis and geometry and whose concepts and methods can be successfully applied to various problems. They have familiarised themselves with basic concepts, results and methods of geometric measure theory and can successfully apply these methods in further courses.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Literature	Possible References : • Lawrence C. Evans, Ronald F. Gariepy: Measure theory and fine properties of functions. CRC Press 1992. • Herbert Federer: Geometric measure theory. Springer 1969. • Leon Simon: Lectures on geometric measure theory. Australian National University 1984.
Responsible Persons	Reiner Schätzle

Course Title:	Introduction to Commutative Algebra and Algebraic Geometry		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Rings and ideals. • Gröbner bases. • Localization. • Noetherian rings and modules. • Integral ring extensions. • Krull's principal ideal theorem and dimension theory. • Hilbert's Nullstellensatz and Noether normalisation. • Affine varieties, Zariski topology, morphisms.		

Special Objectives	The students have become familiar with the central concepts, results, and methods of commutative algebra and affine algebraic geometry. They have experienced the profound interplay between algebra and geometry through the example of affine varieties. Furthermore, the students understand how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.
Literature	Possible References : • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.
Responsible Persons	Daniele Agostini, Jürgen Hausen

Course Title:	Introduction to Partial Differential Equations		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces. • L^2 theory. • Important examples (Laplace equation, wave equation, heat equation). • Fundamental solutions (elliptic situation). • Weak solutions of elliptic equations.		

Special Objectives	The students got to know a central branch of analysis, whose terms and methods are fundamental for many fields, like numerics or stochastics. Also evolutionary equations, who have strong connections to geometry, are issue of the lecture. The students are acquainted with central terms, results and methods of linear partial differential equations and are able to use these methods in advanced courses.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Literature	Possible References : • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Course Title:	Introduction to Partial Differential Equations – Part 1		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Harmonic functions. • Maximum principles. • Sobolev spaces.		
Special Objectives	The students have familiarised themselves with the first basic features of a central area of analysis, the concepts and methods of which are fundamental for many other areas, such as numerics and stochastics. Students are familiar with the central concepts, results and methods of linear partial differential equations and can successfully apply these methods in the more advanced courses.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	Possible References : • Lawrence C. Evans: Partial differential equations. American Mathematical Society 2010. • David Gilbarg, Neil S. Trudinger: Elliptic partial differential equations of second order. Springer 2001. • Olga A. Ladyzenskaja, Vsevolod A. Solonnikov, Nina N. Uralceva: Linear and quasilinear equations of parabolic type. AMS 1968.
Responsible Persons	Gerhard Huisken, Reiner Schätzle

Course Title:	Introduction to Riemann Surfaces		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Coverings and fundamental groups. • Topological classification of the surfaces. • Theorem of Riemann-Hurwitz. • Differential forms and integration. • Sheaves and cohomology. • Theorem of Riemann-Roch. • Serre duality. • Kobayashi metric. • Theorem of Picard.		
Special Objectives	Students develop an approach to abstract surfaces and understand classification techniques based on local-to-global reasoning. In the concept of holomorphy, they grasp the principles of rigidity resulting from analytical properties. Using the sheaf concept, students see how fundamental questions naturally lead to increasingly abstract conceptualisations and how these can ultimately be used to answer questions. They learn how geometry and analysis are interrelated and in many cases mutually dependent.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.		

Literature	Possible References : • Hershel M. Farkas, Irwin Kra: Riemann Surfaces. Springer 1992. • Otto Forster: Riemannsche Flächen. Springer 1977. • Klaus Lamotke: Riemannsche Flächen. Springer 2009. • Jürgen Jost: Compact Riemann surfaces. Springer 2006.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Course Title:	Introduction to Stochastic Differential Equations - Part 1		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Introduction to Brownian motion and stochastic integration. • Solution concepts for stochastic differential equations. • Stability of stochastic differential equations. • Numerical approximation of stochastic differential equations.		
Special Objectives	Students master the basic principles and techniques for constructing solutions of stochastic differential equations.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.		
Literature	Possible References : • Bernt Oksendal: Stochastic differential equations. Springer 2000.		
Responsible Persons	Andreas Prohl		

Course Title:	Elementary Number Theory		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h

Frequency	not regularly
Language of Instruction	German
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS
Content	• Divisibility in the integers. • Prime numbers. • Congruences. • Quadratic residues. • Arithmetic functions. • Multiplicative functions. • Classical theorems. • Applications.
Special Objectives	Students deepen their basic knowledge of integers and experience applying this knowledge to mathematical problems of various kinds.
Prerequisites	Knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	Possible References : • Friedhelm Padberg: Elementare Zahlentheorie. Spektrum Akademischer Verlag 2001. • Stefan Mueller-Stach, J. Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg 2006.
Responsible Persons	Thomas Markwig

Course Title:	Elliptic Curves and Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Basic concepts of cryptography. • Symmetric cryptosystems, public key systems, discrete logarithm, RSA. • Factorisation into primes, attacks on cryptosystems. • Basic concepts of plane projective geometry. • Elliptic curves as Abelian groups. • Curves over finite fields, Frobenius morphism, endomorphism ring. • Counting points, Hasse bound, Weil conjectures, Schoof's algorithm. • Cryptosystems on elliptic curves, algorithms and attacks.		
Special Objectives	Students are familiar with the basic tasks and concepts of cryptography. They understand the cryptographically motivated questions relating to elliptic curves and have an insight into the advanced algebraic and geometric techniques for answering them. They understand the challenges of algorithmic implementation and are familiar with standard algorithms.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory, • MAT-20-04 / MAT-20-05 Introduction to Complex Analysis.		
Literature	Possible References : • Albrecht Beutelspacher, Jörg Schwenk, Klaus-Dieter Wolfenstetter: Moderne Verfahren in der Kryptographie. Springer 2015. • Joseph H. Silverman: The arithmetic of elliptic curves. Springer 2009. • Ian Blake, Gadiel Seroussi, Nigel Smart: Elliptic curves in cryptography. CUP 1999.		
Responsible Persons	Thomas Markwig		

Course Title:	Explicit Mathematics		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		

Content	• Applicative theories. • Explicit mathematics. • Universes in explicit mathematics. • Applications in proof theory.
Special Objectives	Students are familiar with an alternative logical theoretical framework for arithmetic and subsystems of analysis and are familiar with their function in proof-theoretical investigations.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	Possible References : • Solomon Feferman: A language and axioms for explicit mathematics, in Algebra and Logic. Lecture Notes in Mathematics, 450, pp. 87-139, Springer-Verlag, Berlin, 1975. • Solomon Feferman: Constructive theories of functions and classes. In Logic Colloquium '78, (Proc. Mons Colloq.), pp. 159-224, North-Holland, Amsterdam, 1979. • Gerhard Jäger, Reinhard Kahle, Thomas Strahm: On applicative theories. In Andrea Cantini, Ettore Casari, and Pierluigi Minari, editors, Logic and Foundations of Mathematics, pages 83–92, Kluwer, 1999. • Reinhard Kahle: The applicative realm. Textos de Matemática, 40, Departamento de Matemática, Universidade de Coimbra, 2007. • Gerhard Jäger, Reinhard Kahle, Thomas Studer: Universes in explicit mathematics. Annals of Pure and Applied Logic, 109(3), 141-162, 2001.
Responsible Persons	Reinhard Kahle

Course Title:	Financial Mathematics and Numerics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	The course is designed to provide a comprehensive introduction to the mathematical models and numerical techniques that are essential for understanding and solving problems in modern finance. The course will delve into stochastic processes, modeling random phenomena in financial markets, and the numerical methods necessary for approximating solutions to complex financial equations. Key topics include the mathematics behind derivative pricing models, such as the Black-Scholes framework, and the use of stochastic calculus (SDE theory) in risk management and portfolio optimisation. By integrating theory with numerical practice, this course bridges the gap between financial mathematics and real-world applications, equipping students with the quantitative skills required for careers in quantitative finance, risk analysis, and financial engineering.		
Special Objectives	The students know important mathematical models for the description of problems in financial mathematics and can apply numerical approaches to their solutions in a targeted manner.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-11 / MAT-20-13 Numerical Mathematics, • MAT-20-12 Stochastics.
Literature	Possible References : • Steven Shreve: Stochastic Calculus for Finance. Springer 2005.
Responsible Persons	Andreas Prohl

Course Title:	Functional Analysis		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Normed spaces, Banach spaces, dual spaces. • Hahn-Banach theorem, uniform boundedness principle. • Closed graph theorem, open mapping theorem, Banach-Alaoglu theorem. • Compact operators, normal operators, spectral theorems.		
Special Objectives	The students are acquainted with the basic principles and techniques of the theory of infinite dimensional spaces and can apply them to problems in analysis and geometry. They understand the complexity of problems of spectral theory and can use its results for the solution of analytical problems.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	Possible References : • Nicolas Bourbaki: Topological vector spaces. Springer 1987. • Adam Bowers, Nigel Dalton: An introductory course in functional analysis. Springer 2014. • Harro Heuser: Funktionalanalysis. Teubner 2006. • Markus Haase: Functional analysis. American Mathematical Society 2014. • Peter D. Lax: Functional analysis. Wiley 2002. • Gert Kjaergaard Pedersen: Analysis now. Springer 1995. • Walter Rudin: Functional analysis. McGraw-Hill 1991. • Dirk Werner: Funktionalanalysis. Springer 2011. • Kosaku Yosida: Functional analysis. Springer 1995. • Hans Wilhelm Alt: Lineare Funktionalanalysis. Springer 2012.
Responsible Persons	Anton Deitmar, Reiner Schätzle

Course Title:	Geometry of Manifolds 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Manifolds and submanifolds. • Vector fields and flows. • Metrics, foundations of Riemannian geometry. • Complex structures. • Theorem of Gauß-Bonnet on surfaces.		
Special Objectives	The students know and understand the fundamental concepts of real and complex differential geometry and the basic techniques for handling them. Especially they have deepened their understanding of differential and integral calculus and have exemplarily experienced how mathematical concepts are used in a natural way in geometry.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		

Literature	Possible References : • Sylvestre Gallot, Dominique Hulin, Jacques Lafontaine: Riemannian Geometry. Springer 2004. • John M. Lee: Introduction to Smooth Manifolds. Springer 2012. • Liviu I. Nicolaescu: Lectures On The Geometry Of Manifolds. World Scientific 1996. • Clifford Henry Taubes: Differential Geometry: Bundles, Connections, Metrics and Curvature. Oxford University Press 2011.
Responsible Persons	Christoph Bohle, Frank Loose

Course Title:	Geometric Group Theory		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Group actions on graphs, free groups. • Quasi isometries. • Growth types. • Hyperbolic groups. • Ends.		
Special Objectives	Students learn to explore properties of finitely generated groups using geometric tools, starting from the Cayley graph of the group. They are able to investigate the geometric properties of the Cayley graphs with the help of analytical methods and to work out their connections to the underlying group. Students understand how algebra and analysis can work together to develop a new theory at the interface of algebra and geometry that leads to interesting statements about groups.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		
Literature	Possible References : • Clara Löh: Geometric Group Theory - an Introduction. Springer 2017. • Thorsten Camps, Volkmar Große Rebel, Gerhard Rosenberger: Einführung in die kombinatorische und die geometrische Gruppentheorie. Heldermann Verlag 2008.		
Responsible Persons	Hannah Markwig		

Course Title:	Geometry in Physics		
Specialisation	Mathematical Physics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	The module provides an introduction to fundamental methods of differential geometry and their relevance for physics. Particular topics are manifolds, differential forms, Riemannian metrics and associated notions of curvature, Riemannian geometry of submanifolds, real vector bundles, and connections. Applications of these concepts in Physics are discussed.		
Special Objectives	Students obtain knowledge, understanding, and acquaintance with the use of the listed notions of differential geometry. They develop, in particular, a deeper understanding of differential and integral calculus and experience through examples how the mathematical notions are naturally applied within physical theories.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	Possible References : • John Lee: Introduction to smooth manifolds. Springer 2012. • John Lee: Riemannian manifolds: An introduction. Springer 1997. • Chris Isham: Modern differential geometry for physicists. World Scientific 1999. • Mikio Nakahara: Geometry, Topology and Physics. IOP Publishing 2003.		
Responsible Persons	Christoph Bohle, Carla Cederbaum, Stefan Teufel		

Course Title:	Ordinary Differential Equations - Analysis and Numerics		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Non-linear ordinary differential equations: Theorems of Hartman-Grobman and Poincare-Bendixson, bifurcation theory. • Numerical approximation: linear multi-step processes, adaptive processes, geometric integration.
Special Objectives	Students are familiar with the basic methods for studying qualitative behavior and for simulating solutions of non-linear ordinary differential equations. They have learned constructive methods for solving them and are in principle able to implement these with the help of the computer.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics. Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.
Literature	Possible References : • Lawrence Perko: Differential equations and dynamical systems. Springer 1993. • David Griffiths, Desmond J. Higham: Numerical methods for ordinary differential equations. Springer 2010.
Responsible Persons	Andreas Prohl

Course Title:	Graph Theory		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Basic concepts in graph theory, • Basic graph theory algorithms, • Flows, cuts, connectedness, matchings, • Cycle and cut space (cohomology theory), • Spectral graph theory, matrix tree theorem, • Planar graphs, theorem of Kuratowski and Wagner, • Planar embeddings, • Graph colorings, • Theory of minors.		

Special Objectives	Students know the basic concepts of graph theory, can analyse the structure of graphs and use graph theory methods in practice. They will also recognise connections to geometry and algebra and be able to benefit from them.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	Possible References : • Bela Bollobas: Modern graph theory, Springer, 1998. • John Adrian Bondy, Uppaluri Siva Ramachandra Murty: Graph theory, Springer, 2008. • Reinhard Diestel: Graph theory, Springer, 2018. • Jonathan L. Gross, Jay Yellen, Mark Anderson: Graph theory and its applications, CRC Press, 2019.
Responsible Persons	Elmar Teufl

Course Title:	Groups and Representations		
Specialisation	Mathematical Physics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Groups: subgroups, homomorphisms, isomorphisms, group actions, orbits, stabilisers, equivalence classes, normal subgroups, cosets, factor groups. • Representations: faithful, unitary and irreducible representations, reducibility, characters, Schur's lemma(s), orthogonality of irreducible representations. • Applications: symmetries and degeneracies in quantum mechanics, selection rules. • Representations of finite groups: group algebra, regular representation, ideals, idempotents. • Symmetric groups: Young tableaux, Young operators, dimensions and characters. • Applications: identical particles in quantum theories. • Lie groups: Haar measure, representations, Lie algebras. • Tensor representations of classical groups: symmetry classes, Young tableaux. • Applications: SU(2) and SU(3) in particle physics (spin, isospin, flavour) • Moreover a selection of the following: – Irreducible representations of the Lorentz and Poincaré groups. – Applications: notion of particles in quantum theories. – Roots and weights; Killing-Cartan classification of semi-simple Lie algebras		

Special Objectives	The students know the basic concepts of group and representation theory. They are able to apply these abstract algebraic concepts in the context of theoretical physics and have, thus, developed a deepened understanding for the connections between mathematics and physics. The students are familiar with a number of complex examples of applications of the representation theory of groups in physics.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	Possible References : • Irene Verona Schensted: A course on the Application of Group Theory to Quantum Mechanics. NEO Press 1976. • Barry Simon: Representations of Finite and Compact Groups. AMS 1996. • Wu-Ki Tung: Group Theory and Physics. World Scientific 1985.
Responsible Persons	Stefan Keppeler

Course Title:	Hyperbolic Geometry: Axiomatic, Reflection Geometric, Algebraic		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	Starting from a system of axioms for plane absolute geometry with the basic concepts of incidence and congruence, the associated Bachmann reflection geometry is developed. After the introduction of the hyperbolic axiom, this is continued with reflection-geometric end theory. A Euclidean field is created from the rotations around an end and the translations along a straight line, with the help of which the hyperbolic plane under consideration is described algebraically.		
Special Objectives	The students have learnt to look at one and the same mathematical object (in this case absolute and hyperbolic planes) from completely different perspectives and to link them together. In particular, they have learnt about Bachmann's group-theoretically oriented reflection geometry, which rarely appears in the curriculum, and thus deepen their knowledge of groups. They also deepened their knowledge about the interweaving of geometry and algebra.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures. Knowledge from the following modules is helpful but not strictly required: • MAT-20-06 Introduction to Geometry.		

Literature	Possible References : • Friedrich Bachmann: Aufbau der Geometrie aus dem Spiegelungsbegriff. Springer 1959. • Robin Hartshorne: Geometry: Euclid and beyond. Springer 2000. • Helmut Karzel, Kay Sörensen, Dirk Windelberg: Einführung in die Geometrie. Vandenhoeck und Ruprecht 1973.
Responsible Persons	Hermann Hähl, Hannah Markwig

Course Title:	Information Geometry		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Foundations of Information Geometry (e.g. Fisher information metrics and dual relationships for parametric statistical models, Kullback-Leibler divergence, natural gradient). • Application to neural data processing (in particular supervised learning in artificial neural networks).		
Special Objectives	Students have an elementary understanding of how to apply concepts of differential geometry to problems in information theory and statistics.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	Possible References : • Shun-Ichi Amari, Hiroshi Nagaoka: Methods of Information Geometry. AMS 2001. • Anthony C. C. Coolen, Reimer Kuehn, Peter Sollich: Theory of Neural Information Processing Systems. OUP 2005. • Shun-Ichi Amari: Natural Gradient works Efficiently in Learning. Neural Computation 1998. • Yann Ollivier: Riemannian Metrics for Neural Networks I - Feedforward Networks. Information and Inference, IMA 2015.		
Responsible Persons	Christoph Bohle		

Course Title:	Information Theory, Pattern Recognition, and Neural Networks
Specialisation	Stochastics

Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Introduction to information theory: – The possibility of reliable communication over unreliable channels. The Hamming code and repetition codes. • Entropy and data compression: – Entropy, conditional entropy, mutual information, Shannon information content. The idea of typicality and the use of typical sets for source coding. Shannon's source coding theorem. Codes for data compression. Uniquely decodeable codes and the Kraft-MacMillan inequality. Completeness of a symbol code. Prefix codes. Huffman codes. Arithmetic coding. • Communication over noisy channels: – Definition of channel capacity. Capacity of binary symmetric channel; of binary erasure channel; of Z channel. Joint typicality, random codes, and Shannon's noisy channel coding theorem. Real channels and practical error-correcting codes. Hash codes. • Statistical inference, data modelling and pattern recognition: – The likelihood function and Bayes' theorem. Clustering as an example. • Approximation of probability distributions: – Laplace's method. (Approximation of probability distributions by Gaussian distributions.) Monte Carlo methods: Importance sampling, rejection sampling, Gibbs sampling, Metropolis method. (Slice sampling, Hybrid Monte Carlo, Overrelaxation, exact sampling.) Variational methods and mean field theory. Ising models. • Neural networks and content-addressable memories: – The Hopfield network.		
Special Objectives	Students are familiar with fundamental concepts of information theory – such as entropy and noisy channel coding – and how these are applied in computing and approximating probabilities.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.		
Literature	Possible References : • David J.C. MacKay: Information Theory, Inference, and Learning Algorithms. CUP 2003.		
Responsible Persons	Stephan Eckstein		

Course Title:	Interacting Many-Body Quantum Systems
Specialisation	Mathematical Physics

Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Bosons and fermions. • Bose-Einstein condensation. • Mean-field theory. • Nonlinear Schrödinger equation. • Hartree- and Hartree-Fock theory. • Bogoliubov approximations.		
Special Objectives	Students have learned about the difference between Fermions and Boson and its impact on many body systems. They understand that, at cold temperatures, the fluctuations of the particle number in a certain area is significantly suppressed for Fermions. They understand how the Bosonic nature leads to the famous effect of Bose Einstein condensation and that, in certain scaling regimes, interacting many body systems can in good approximation be described by nonlinear evolution equations. Finally they are able to give higher order corrections and to calculate the leading order correlations between the particles.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.		
Literature	Possible References : • Elliott H. Lieb, Robert Seiringer, Jan Philip Solovej, Jakob Yngvason: The Mathematics of the Bose Gas and its Condensation. Birkhäuser 2000. • Niels Benedikter , Marcello Porta , Benjamin Schlein: Effective Evolution Equations from Quantum Dynamics. Springer 2016.		
Responsible Persons	Peter Pickl		

Course Title:	Commutative Algebra		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Rings and Ideals. • Localisation and local rings. • Noetherian and Artinian rings and modules. • Integral ring extensions and Cohen-Seidenberg theorems. • Krull's principal ideal theorem and dimension theory. • Primary decomposition. • Normality, regularity and discrete valuation rings. • Hilbert's Nullstellensatz and Noether normalisation.		
Special Objectives	The students are familiar with and understand the language and methods of commutative algebra, which are essential for studying the fields of algebra, geometry, and number theory. They recognise how adopting a higher perspective - namely, abstracting the problem - enables the simultaneous treatment and resolution of seemingly unrelated questions.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-03 / MAT-20-06 Introduction to Galois Theory.		
Literature	Possible References : • Michael Francis Atiyah, Ian G. Macdonald: Introduction to commutative algebra. Addison Wesley 1969. • David A. Cox, John B. Little, Donal O'Shea: Ideals, varieties, and algorithms. Springer 2008. • David Eisenbud: Commutative algebra with a view toward algebraic geometry. Springer 1995. • Ernst Kunz: Einführung in die kommutative Algebra und algebraische Geometrie. Vieweg 1980. • Miles Reid: Undergraduate Commutative Algebra. Cambridge University Press 1997.		
Responsible Persons	Daniele Agostini, Thomas Markwig		

Course Title:	Convex Geometry		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		

Content	• Cones, polytopes, polyhedra, fans, polyedral complexes. • Normal fans of polygons. • Triangulations, subdivisions, secondary fans, discriminants.
Special Objectives	In the lecture the students learn basic terms, results and methods of convex geometry. They develop a deepened understanding for the concept of duality of mathematical objects on the example of polytopes and fans. Furthermore they enhance their geometric view and their spatial sense.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	Possible References : • Günter M. Ziegler: Lectures on Polytopes. Springer 1998.
Responsible Persons	Hannah Markwig

Course Title:	Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Brief review of key concepts and results from algebra and number theory. • Historical ciphers and their cryptanalysis (Caesar, Vigenere, substitution); encryption schemes. • Diffie-Hellman protocol and fast exponentiation. • Discrete logarithms: Shanks' algorithm and Pollard's rho method. • RSA: correctness, security, and attacks. • Signature schemes.		

Special Objectives	Students are familiar with the fundamental concepts and results of elementary number theory and algebra, as well as their application in cryptography. They can implement the methods covered in Python or SageMath in an exemplary manner and know what to pay attention to. Using classical ciphers, they understand typical strengths and weaknesses; they master the Diffie-Hellman protocol and are familiar with the man-in-the-middle attack. They can compute discrete logarithms in cyclic groups, understand the RSA scheme, and are able to interpret the recommendations of the Federal Office for Information Security (BSI). In various attack scenarios, they can identify weaknesses of RSA when the requisite conditions are not met. By engaging with numerous open problems in cryptography – whose solution approaches can, perhaps surprisingly, stem from very different areas of mathematics – students practise critical thinking. The exercises are central and support students in working independently and in a practice-oriented way, especially with CAS systems such as SageMath.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	Possible References : • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Christian Karpfinger, Hubert Kiechle: Kryptologie, Algebraische Methoden und Algorithmen, Vieweg 2010. • Dan Boneh, Victor Shoup: A Graduate Course in Applied Cryptography. 2023 (online Version: https://toc.cryptobook.us/). • Jonathan Katz, Yehuda Lindell: Introduction to Modern Cryptography. Chapman and Hall/CRC 2020.
Responsible Persons	Thomas Markwig

Course Title:	Lie Groups		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Manifolds and Lie groups, • Lie algebras and exponential map, • Covering spaces and classification of Lie groups by their Lie algebras, • Classical Lie groups, • Operations of Lie groups and homogeneous spaces.		

Special Objectives	Lie groups lie at the interface between geometry, algebra and analysis. They are suitable for describing the symmetries of geometric objects, but also algebraic equations or solutions of differential equations, in particular if these symmetries form a continuous set. The students learn from a prominent example how different disciplines of mathematics can work together very successfully and how a convincing formalism is developed that can precisely describe a variety of symmetry phenomena.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures, • MAT-20-01 Measure and Integration Theory, • MAT-20-04 Integration on Submanifolds of $\mathbb{R}^n$.
Literature	Possible References : • Joachim Hilgert, Karl-Hermann Neeb: Liegruppen und Lie-Algebren. Vieweg 1991. • Gerhard P. Hochschild: The structure of Lie groups. Holden-Day 1965. • Frank W. Warner: Foundations of differentiable manifolds and Lie groups. Springer 1983.
Responsible Persons	Anton Deitmar, Frank Loose

Course Title:	Linear Control Theory		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	Mathematical methods are indispensable for the management and control of complex systems and processes. The underlying theory is not only fascinating due to its diverse applications, but also, in its abstract form, due to the clarity and elegance of its methods and results. In this lecture, finite-dimensional systems are dealt with first, for which a good knowledge of analysis and linear algebra is sufficient. The aims are Kalman's controllability criterion and the resulting criteria for stabilisability. If there is enough time, we will extend the theory to infinite-dimensional systems. In the exercise classes we will apply the theory to concrete examples.		
Special Objectives	Students have learnt basic methods of linear control theory. At the same time, they have experienced and understood the interaction of various theoretical concepts from linear algebra and analysis and their benefits for specific applications.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		

Literature	Possible References : • Hans Wilhelm Knobloch, Huibert Kwakernaak: Lineare Kontrolltheorie. Springer 1985. • Jerzy Zabczyk: Mathematical Control Theory. Birkhäuser 1992. • Ruth F. Curtain, Hans Zwart: An Introduction to Infinite-Dimensional Systems Theory. Springer 1995.
Responsible Persons	Rainer Nagel

Course Title:	Mathematical Aspects of 3d Computer Vision		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	• Foundations of projective geometry. • Visual odometry. • Classical Simultaneous Localisation and Mapping (SLAM) methods and deep learning variants. • Computer-based experiments using the Robot Operating System (ROS2).		
Special Objectives	The students have acquired the mathematical foundations of 3D computer vision. They are familiar with modern architectures and the mathematical questions arising from them.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics. In addition, prior knowledge of deep learning is required, such as that provided in module MAT-50-14.		
Literature	Possible References : • Richard Hartley, Andrew Zisserman: Multiple View Geometry in Computer Vision. Cambridge University Press 2003. • Sebastian Thrun, Wolfram Burgard, Dieter Fox: Probabilistic Robotics. MIT Press 2005. • Luca Carlone, Ayoung Kim, Timothy D. Barfoot, Daniel Cremers, Frank Dellaert: SLAM Handbook: From Localization and Mapping to Spatial Intelligence. Cambridge University Press 2026.		
Responsible Persons	Christoph Bohle		

Course Title:	Mathematical Aspects of Deep Learning 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Artificial neural networks and their training using backpropagation/stochastic gradient descent. Probabilistic interpretation. Mathematical problems arising in neural network science. • Classical architectures for computer vision (CNNs) and sequential data (RNNs). (Self-)supervised learning methods. • Transformer models, a modern architecture based on the attention mechanism, and its applications in natural language processing (GPT) and computer vision. • Simple neuroscience models for neural networks. Biological plausibility of backpropagation and transformers.		
Special Objectives	The students have learned the fundamentals of information processing using artificial neural networks and biologically more plausible alternatives. They are familiar with contemporary architectures and mathematical questions arising in the framework.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.		
Literature	Possible References : • Ian Goodfellow, Yoshua Bengio, Aaron Courville: Deep Learning. MIT 2016. • Simon J. D. Price: Understanding Deep Learning. MIT Press 2023. • Christopher M. Bishop, Hugh Bishop: Deep Learning - Foundations and Concepts. Springer 2024.		
Responsible Persons	Christoph Bohle		

Course Title:	Theory of Mathematical Proofs		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		

Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS
Content	• Axiomatic theories, incompleteness. • Gentzen's proof of consistency for arithmetic. • Ordinal number analysis. • Provable recursive functions. • Predicative analysis. • Theories of inductive definitions.
Special Objectives	Students are familiar with the methods and results of modern proof theory, in particular with calculations for mathematical theories and their metamathematical properties.
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.
Literature	Possible References : • Wolfram Pohlers. Proof Theory. Springer 2009.
Responsible Persons	Reinhard Kahle

Course Title:	Mathematical Introduction to Data Science		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	The lecture provides a mathematically rigorous survey of core data-science techniques from regression and k-nearest neighbours to high-dimensional phenomena, dimensionality reduction, and modern classifiers (Perceptron, SVMs, Kernel Methods, Neural Networks). Emphasis is put on both theoretical foundations and hands-on Python implementation with real datasets.		
Special Objectives	The students can formulate and solve linear, polynomial and logistic regression models. They are able to implement k-nearest neighbours classification and build and evaluate classifiers (Perceptron, SVM (kernel methods), Neural Networks). Moreover, they know how to optimise convex objectives using gradient-descent techniques.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.		
Literature	Possible References : • Sven A. Wegner: Mathematical Introduction to Data Science. Springer 2024.		

Responsible Persons	Andreas Prohl
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Course Title:	Mathematical Statistics		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Statistical models, exponential families, sufficient statistics. • Rao-Blackwell theorem, Lehmann-Scheffe theorem, Cramer-Rao theorem. • Estimation methods, UMVU estimator, quality criteria, asymptotic behaviour of estimators. • Hypothesis testing, confidence interval, Neyman-Pearson lemma. • Testing methods, UMPU tests, 1- and 2-sample tests. • Models with growing density quotients, non parametric models. • Introduction in regression and variance analysis.		
Special Objectives	Students can model statistical relationships mathematically. They can mathematically construct, analyse, compare and apply statistical estimation and test methods and interpret their results.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics. Knowledge from the following modules is helpful but not strictly required: • MAT-75-01 Probability Theory.		
Literature	Possible References : • Peter J. Bickel, Kjell A. Doksum: Mathematical Statistics: Basic Ideas and Selected Topics. Chapman & Hall 2016. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Erich L. Lehmann, Joseph P. Romano: Testing statistical hypotheses. Springer 2005. • Erich L. Lehmann, George Casella: Theory of point estimation. Springer 1998. • Wiebe R. Pestman: Mathematical Statistics. De Gruyter 2009 • Helmut Pruscha: Vorlesungen über Mathematische Statistik. Springer Vieweg 2000. • Mark J. Schervish: Theory of Statistics. Springer 1995.		

Responsible Persons	Martin Möhle, Elmar Teufl		
Course Title:	Nonparametric Estimation of Distributions		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 115 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	This course explores a fundamental question in statistics: Given n data points from an unknown random vector, how well can we infer the true characteristics of the random vector, i.e., its distribution? In particular, we will focus on the role of the dimension of the vector, leading to a treatment of the curse of dimensionality. Further, we will explore how prior knowledge (e.g., smoothness or structural understanding of the random vector) can help overcome the curse of dimensionality. In contrast to parametric statistics, where the distribution is assumed to belong to a finite-dimensional family (e.g., estimating mean and covariance matrix of a normal distribution), the focus is purely on nonparametric settings, where no such parametric forms are given a priori. These settings require suitably chosen evaluation criteria based on distances between probability distributions, and we will treat both density estimation with L^p-type losses and estimation in Wasserstein distance.		
Special Objectives	The statistical learning problem for probability distributions can be formulated precisely. The importance of the chosen metric and of the dimension of the distribution is well understood. Students are familiar with the fundamental proof techniques for establishing both upper bounds, using explicit estimators, and lower bounds, using information-theoretic methods. They develop a structural understanding of why $\sqrt{\frac{\log n}{n}}$ (Bandwidth) assumptions are necessary to achieve convergence rates that are better than those dictated by the curse of dimensionality. Through the exercise classes, students demonstrate their ability to work independently on topics covered in the lectures.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.		
Literature	Possible References : • Alexandre B. Tsybakov: Introduction to Nonparametric Estimation. Springer 2009. • Jonathan Niles-Weed, Quentin Berthet: Minimax estimation of smooth densities in Wasserstein distance. The Annals of Statistics 50.3 (2022): 1519-1540. • Nicolas Fournier, Arnaud Guillin: On the rate of convergence in Wasserstein distance of the empirical measure. Probability Theory and Related Fields 162.3 (2015): 707-738. • Daniel Bartl, Stephan Eckstein: Fast Wasserstein rates for estimating probability distributions of probabilistic graphical models. arXiv:2510.09270 (2025).		
Responsible Persons	Stephan Eckstein		

Course Title:	Numerics of Stationary Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	Numerical covering of boundary value problems of stationary (i.e. time independent) ordinary and elliptic partial differential equations, with emphasis to the methods of finite elements.		
Special Objectives	The students have learned the central terms, results and methods of the numerical treatment of boundary value problems of stationary differential equations.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations, • MAT-20-11 / MAT-20-13 Numerical Mathematics. Knowledge from the following modules is helpful but not strictly required: • MAT-70-01 Algorithms of Numerical Mathematics.		
Literature	Possible References : • Dietrich Braess: Finite Elemente. Springer Spektrum 2013. • Wolfgang Hackbusch: Theorie und Numerik elliptischer Differentialgleichungen. Teubner 1986.		
Responsible Persons	Christian Lubich		

Course Title:	Numerics of Stochastic Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		

Content	• Random number generator, Ito-Taylor expansion. • Strong and weak approximation, consistency. • Euler-Maruyama method, Milstein method, stochastic Runge-Kutta method. • Approximation of stopped diffusion processes.
Special Objectives	Students master the basic principles and techniques for the numerical approximation of solutions of stochastic differential equations.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Literature	Possible References : • Peter E. Kloeden, Eckhard Platen: Numerical solution of stochastic differential equations. Springer 1999.
Responsible Persons	Andreas Prohl

Course Title:	Optimal Control Theory with Ordinary Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Brief overview of existence and uniqueness theory for ODEs. • Numerical solutions to ODEs. • Introduction to optimal control problems with ODEs. • Existence and uniqueness theory for linear quadratic optimal control problems (LQ problems). • Pontryagin's maximum principle. • Numerical approximation of LQ problems.		
Special Objectives	Students are familiar with the problems of optimal control using ordinary differential equations and various approaches to solving the problem. They are also familiar with qualitative statements on unambiguous solvability.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-05 Introduction to Ordinary Differential Equations.
Literature	Possible References : • Matthias Gerds: Optimal Control of ODEs and DAEs. De Gruyter 2012.
Responsible Persons	Andreas Prohl

Course Title:	Optimal Transport		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• The course introduces the theory of optimal transport at the interface of analysis, probability, and optimisation. Starting from the classical problems of Monge and Kantorovich, the fundamental measure-theoretic and analytical tools are developed. • Building on this, central results are presented, including the existence and duality of optimal transport plans, c-cyclical monotonicity, Brenier's theorem, and the geometric structure of Wasserstein spaces. • In the second part of the course, the algorithmic perspective of optimal transport is explored. Topics include entropic regularisation, the Sinkhorn algorithm, and selected applications, for example in image processing, statistics, and machine learning.		
Special Objectives	The students are familiar with the fundamental concepts, methods, and key results of the theory of optimal transport and are able to situate these within a broader mathematical context spanning analysis, probability, and optimisation. They can understand the classical formulations due to Monge and Kantorovich, as well as essential structural concepts such as duality, c-cyclical monotonicity, and the geometry of Wasserstein spaces, and apply them in simple settings. Furthermore, they are acquainted with basic algorithmic approaches to optimal transport and can assess their relevance for selected applications. In the exercise classes, they deepen their understanding of the lecture material, further develop their ability to construct precise mathematical arguments, and practise the written and oral presentation of their own solution methods.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 Submodules Measure and Integration Theory, and Integration on Submanifolds of $\mathbb{R}^n$. • MAT-20-12 Stochastics.		

Literature	Possible References : • Alessio Figalli, Federico Glaudo: An Invitation to Optimal Transport, Wasserstein Distances, and Gradient Flows. EMS 2023. • Gabriel Peyré, Marco Cuturi: Computational Optimal Transport. Now Publishers 2019.		
Responsible Persons	Nicolai Jork, Andreas Prohl		

Course Title:	Optimisation with Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Direct method in the calculus of variations, Euler-Lagrange equation. • Brouwer-Minty theorem, non-linear evolution equations. • Gateaux and Frechet differentiability. • Proof of existence of optimal controls, necessary optimality conditions. • Adjoint, convergent optimisation methods in Banach spaces. • Variational discretisation concepts.		
Special Objectives	Students master the basic principles and techniques for deriving optimality conditions for prototypical control problems with constraints in the form of partial differential equations.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-11 / MAT-20-13 Numerical Mathematics.		
Literature	Possible References : • Michael Hinze, Rene Pinnau, Michael Ulbrich, Stefan Ullrich: Optimization with PDE constraints. Springer 2009.		
Responsible Persons	Andreas Prohl		

Course Title:	Problem Solving Strategies
Specialisation	Algebra

Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Basic Techniques and Problem Reduction. • Proofs and Logic. • Sets and Functions. • Elementary Combinatorics. • Counting Techniques and Cardinality. • The Pigeonhole Principle (Box Principle). • Bounds, Estimation, and Inequalities. • The Extremal Principle. • The Invariance Principle: Invariants and Monovariants. • Proofs by Colouring. • Graph-Theoretic Modelling.		
Special Objectives	Students acquire a versatile toolkit of mathematical and heuristic problem-solving strategies that they can draw upon both in their subsequent studies and, where appropriate, in competition-oriented mathematical problem solving. This includes fundamental results and proof techniques, as well as the ability to recognise characteristic structural features of a problem that indicate suitable solution approaches or allow unsuitable strategies to be ruled out at an early stage.		
Prerequisites	Knowledge from Linear Algebra and the submodule Algebraic Structures are assumed.		
Literature	Possible References : • George Pólya: How to Solve It: A New Aspect of Mathematical Method. Princeton University Press 1973. • Arthur Engel: Problem-Solving Strategies. Springer 1998.		
Responsible Persons	Thomas Markwig		

Course Title:	Propagation of Chaos		
Specialisation	Mathematical Physics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		

Forms of Teaching and Learning	Lecture 4 SWS
Content	• Interacting many body systems (quantum and classical), importance of correlations. • Mean-field situations (e.g., Vlasov) and collisions (Boltzmann). • Explicit treatment of correlations. • Large deviations from the expected value.
Special Objectives	Students learn how different kinds of many-body systems can be described by effective, non-linear equations. They are able to distinguish and compare different types of convergence of microscopic many-body systems against the effective theory, both in classical and quantum mechanical situations. Based on an argument similar to the law of large numbers, they understand how the independence of particles leads to the effective equation. They learn to prove that independence is indeed preserved - at least approximately - under the evolution of time (propagation of chaos). Building on this, they understand various proof strategies adapted to the respective situation.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Literature	Possible References : • Louis-Pierre Chaintron, Antoine Diez: Propagation of chaos: a review of models, methods and applications. arXiv:2203.00446. • Francois Golse: Mean-Field Limits in Statistical Dynamics. arXiv:2201.02005.
Responsible Persons	Peter Pickl

Course Title:	Software Obfuscation: Mixed Boolean-Arithmetic Expressions		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	• What is software obfuscation and what is it used for? What techniques are there, and why is deobfuscation so difficult? What does a reverse engineer do? Introductory examples. • Mathematical foundations: ideals in $\mathbb{Z}$, greatest common divisor, Euclidean algorithm, time complexity estimates, rings and fields, computations in residue class rings, permutations, matrices and determinants, linear algebra over rings. • Boolean algebras and Boolean functions. • Fundamentals of mixed Boolean-arithmetic expressions (MBA). • Obfuscation using MBAs and permutation polynomials. • Deobfuscation using MBAs; analysis and comparison of state-of-the-art tools and their weaknesses. • Deobfuscation using program synthesis.
Special Objectives	The students are familiar with initial techniques in the field of software obfuscation and deobfuscation and understand the mathematical results required for this. They are able to apply these confidently.
Prerequisites	Knowledge from Linear Algebra and the submodule Algebraic Structures are assumed.
Literature	Possible References : • Yongxin Zhou, Andrew Main, Yuxin Gu, Harrison Johnson. Information Hiding in Software with Mixed Boolean-Arithmetic Transforms. In: International Workshop on Information Security Applications (WISA). Springer, 2007, pp. 6175. • Bernhard Reichenwallner, Peter Meerwald-Stadler. Efficient Deobfuscation of Linear Mixed Boolean-Arithmetic Expressions. In: Proceedings of the 15th ACM Workshop on Artificial Intelligence and Security (AISec 2022). ACM, 2022, pp. 119130.
Responsible Persons	Thomas Markwig

Course Title:	Game Theory		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	The focus is on Nash and generalised Nash equilibrium problems and their numerical solution.		
Special Objectives	Students are familiar with the fundamental issues of game theory. They are familiar with analytical and numerical approaches to analysing them.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	Possible References : • Christian Kanzow, Alexandra Schwartz: Spieltheorie. Birkhaeuser 2018.		

Responsible Persons	Andreas Prohl
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Course Title:	Statistical Learning 1		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Non-parametric regression, regression estimator. • (Universal) consistency. • Rate convergence. • Stone's theorem. • Kernel estimator, k-NN estimator. • Slow rate convergence, minimax convergence rates.		
Special Objectives	Students are familiar with basic non-parametric regression estimators, in particular with their universal consistency and rate convergence. They are familiar with the basic principles and methods of stochastic learning as required for machine learning applications.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-12 Stochastics.		
Literature	Possible References : • Laslo Györfi, Michael Kohler, Adam Krzyzak, Harro Walk: A distribution-free theory of nonparametric regression. Springer 2002.		
Responsible Persons	Andreas Prohl		

Course Title:	Stochastic Differential Equations		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German		

Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS
Content	• Stochastic processes, filtrations, martingales. • Wiener process, random walk, Donsker's theorem. • Diffusion semigroup, Ito's integral. • Solution of a stochastic differential equation. • Markov property, Malliavin calculus, rough path theory.
Special Objectives	Students master the basic principles and techniques for constructing solutions of stochastic differential equations.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis, • MAT-20-12 Stochastics.
Literature	Possible References : • Bernt Oksendal: Stochastic differential equations. Springer 2000.
Responsible Persons	Andreas Prohl

Course Title:	Theoretical Aspects of Machine Learning		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	The lecture covers some recent aspects of theoretical machine learning such as: • The theory of Reproducing Kernel Hilbert Spaces (RKHS). • Applications of RKHS theory such as SVMs, kernel regression, kernel PCA, and kernel mean embeddings. • Approximation capabilities of neural networks. • Dynamics of neural networks and the neural tangent kernel. • Recent advances in high dimensional statistics, in particular overparametrisation and generalisation.		

Special Objectives	The students learn the mathematical foundations of supervised learning theory, neural networks, support vector machines and kernel methods. They are familiar with fundamental modern topics in machine learning and with their theoretical basis, mathematical approach and conceptual tools as needed for the discussion and justification of algorithms.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.
Literature	Possible References : • Mehryar Mohri, Afshin Rostamizadeh, Ameet Talwalkar: Foundations of Machine Learning. MIT Press 2012. • Shai Shalev-Shwartz, Shai Ben-David: Understanding Machine Learning: From Theory to Algorithms. CUP 2014. • Peter L. Bartlett, Andrea Montanari, Alexander Rakhlin: Deep learning: a statistical viewpoint. Acta Numerica 2021. • Daniel A. Roberts, Sho Yaida, Boris Hanin: The Principles of Deep Learning Theory: An Effective Theory Approach to Understanding Neural Networks. Cambridge University Press 2022.
Responsible Persons	Andreas Prohl

Course Title:	Theory and Numerics for Constrained Optimisation Problems		
Specialisation	Scientific Computing		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	We start with the unconstrained convex minimisation problem (on spaces), and the gradient method with step size control according to Armeijo for the approximate calculation of a minimum, as well as its variants. The simplex method solves linear programmes on polyhedra. Central to this is the convex (non-linear) minimisation task on sets, and the characterisation of a minimum with (necessary) optimality conditions (tangent cone, linearised tangent cone, Abadie condition, Karush-Kuhn-Tucker conditions). In addition, numerical solution methods based on these theoretical concepts (interior points method, penalty methods, SQP method) are presented and analysed.		
Special Objectives	The participants have become familiar with relevant current algorithms for the optimisation of constrained optimisation problems: these include gradient methods with step size control, the simplex method, interior point methods, penalisation methods and the SQP method. You will be able to analyse the algorithms and compare their complexity.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-11 / MAT-20-13 Numerical Mathematics.		

Literature	Possible References : • Carl Geiger, Christian Kanzow: Theorie und Numerik restringierter Optimierungsaufgaben. Springer 2002.
Responsible Persons	Andreas Prohl

Course Title:	Topology		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 90 h	Time in Class: 30 h	Self-Study: 60 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS		
Content	• Review of metric spaces: closed sets, environment, continuity, complete metric spaces, compactness in metric spaces metric spaces. • Set-theoretic topology: topological spaces, continuity convergence, compactness, separation axioms. • Spaces of continuous functions: Urysohn's lemma and applications, Stone-Cech compactification, the theorem of Stone-Weierstraß, notions of convergence in functions, compactness in spaces of functions. • Baire's spaces and application of Baire's theory: Baire's function classes, existence theorems. • Outlook on algebraic topology.		
Special Objectives	Students have familiarised themselves with the central concepts, results and methods of set-theoretical topology and have understood that this theory can be used to describe many phenomena in different areas of mathematics. In this way, they link their knowledge of very different areas of mathematics.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		
Literature	Possible References : • Felix Hausdorff: Grundzüge der Mengenlehre. Von Veit & Comp. 1914. • Boto von Querenburg: Mengentheoretische Topologie. Springer 2001. • Volker Runde: A Taste of Topology. Springer 2005.		
Responsible Persons	Ulrich Groh, Rainer Nagel		

Course Title:	Tropical Enumerative Geometry
Specialisation	Geometry

Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Enumerative geometry of algebraic curves, in particular in the plane. • Tropical enumerative problems and multiplicities. • Combinatorial methods, floor diagrams and lattice paths. • Correspondence theorems for curves in the plane through given points. • Tropical and classic Gromov-Witten theory in genus 0. • Real counts, Welschinger invariants and polynomial invariants. • Hurwitz numbers. • Tropical correspondences for Hurwitz numbers.		
Special Objectives	The students know basic terms, results and methods of enumerative geometry in the context of tropical geometry methods. They develop a deeper understanding of the possibilities and limitations of the tropical access in connection with more complex issues. Furthermore, they deepen their knowledge in the field of algebraic geometry towards modular spaces and Gromov-Witten theory.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		
Literature	Possible References : • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.		
Responsible Persons	Hannah Markwig		

Course Title:	Tropical Enumerative Geometry - Part 1		
Specialisation	Geometry		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		

Content	• Enumerative geometry of algebraic curves, in particular in the plane. • Tropical enumerative problems and multiplicities. • Combinatorial methods, floor diagrams and lattice paths. • Correspondence theorems for curves in the plane through given points. • Real counts, Welschinger invariants and polynomial invariants.
Special Objectives	The students know basic terms, results and methods of enumerative geometry in the context of tropical geometry methods. They develop a deeper understanding of the possibilities and limitations of the tropical access in connection with more complex issues. Furthermore, they deepen their knowledge in the field of algebraic geometry towards moduli spaces.
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.
Literature	Possible References : • Grigory Mikhalkin, Johannes Rau: Tropical geometry. Manuscript 2018.
Responsible Persons	Hannah Markwig

Course Title:	Calculus of Variations		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 150 h	Time in Class: 45 h	Self-Study: 105 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 1 SWS		
Content	• Direct method of calculus of variations. • Euler-Lagrange equations. • Palais-Smale condition. • Mountain-Pass Lemma according to Ambrosetti-Rabinowitz.		
Special Objectives	In the first part of the course, students have learnt the direct method of calculus of variations, which is primarily used to prove the existence of weak solutions of partial differential equations, but also has applications in e.g. differential geometry. They have also acquired the necessary basics from functional analysis and partial differential equations and can also use these in a different context, e.g. geometric analysis. In the second part of the course, students learnt about a so-called mountain-pass lemma. With its help, they can analyse non-uniqueness in the existence of solutions of partial differential equations.		

Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-04 / MAT-20-04 Compulsory Intermediate Modules Analysis.
Literature	Possible References : • Michael Struwe: Variational Methods, Springer 2008. • David Gilbarg, Neil S. Trudinger: Elliptic Partial Differential Equations of Second Order, Springer 1998. • Walter Rudin: Functional Analysis, Mc Graw Hill Education 1991.
Responsible Persons	Reiner Schätzle

Course Title:	Probability Theory		
Specialisation	Stochastics		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	regularly in Winter Semester		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• Characteristic functions and additions to the central limit theorem. • Conditional expectations and further measure-theoretic foundations. • Markov chains and martingales in discrete time, classification, asymptotic behaviour, stopping times, stationarity, ergodicity. • Introduction to processes in continuous time like Poisson processes and Brownian motion.		
Special Objectives	The students got to know the central terms results and methods of probability theory. They can model, analyse and interpret stochastic dependency structures of random quantities in a measure theoretically founded manner. The students are capable of naming and proving the central results of the lecture as well as assessing and explaining the presented connections.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-20-01 Measure and Integration Theory, • MAT-20-12 Stochastics.		

Literature	Possible References : • Heinz Bauer: Wahrscheinlichkeitstheorie und Grundzüge der Maßtheorie. De Gruyter 2010. • Richard Durrett: Probability, Theory and Examples. Cambridge University Press 2019. • Hans-Otto Georgii: Stochastik. De Gruyter 2009. • Jean Jacod, Philip E. Protter: Probability essentials. Springer 2004. • Olav Kallenberg. Foundations of Modern Probability. Springer 2021. • Achim Klenke: Wahrscheinlichkeitstheorie. Springer 2020. • David Meintrup, Stefan Schäffler: Stochastik. Springer 2005. • Albert N. Shiryaev: Probability-1. Springer 2016.
Responsible Persons	Martin Möhle, Elmar Teufl

Course Title:	Consistency Proofs		
Specialisation	Analysis		
Workload - Time in Class - Self-Study	Workload: 180 h	Time in Class: 60 h	Self-Study: 120 h
Frequency	not regularly		
Language of Instruction	German		
Forms of Teaching and Learning	Lecture 2 SWS + Exercise class 2 SWS		
Content	• Historical examples of the question of consistency (limits; parallel axiom; set-theoretic paradoxes). • Philosophical foundational programs (logicism; formalism; intuitionism). • The Hilbert program and Gödel's theorems. • Gentzen's transfinite consistency proof for number theory. • Alternative approaches to consistency (including Gödel's T). • Current situation of consistency proofs.		
Special Objectives	Students learn about the historical context from which the question of the non-contradiction of formal mathematical theories arose, as well as the relevant modern techniques for investigating this question mathematically. They are able to categorise the problem of non-contradiction in mathematics both historically and philosophically. In addition, they have acquired the mathematical tools to be able to comprehend the corresponding proofs of non-contradiction and, to a certain extent, to carry them out themselves.		
Prerequisites	Knowledge from the module Foundations of Mathematics (MAT-10-10) is required.		

Literature	Possible References : • Kurt Gödel: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatsh. f. Mathematik und Physik 38, 173-198 (1931). • Gerhard Gentzen: Die Widerspruchsfreiheit der reinen Zahlentheorie. Math. Ann. 112, 493-565 (1936). • Reinhard Kahle, Michael Rathjen (Hrsg.): Gentzen's Centenary: The quest for consistency. Springer 2015. • Reinhard Kahle, Michael Rathjen (Hrsg.): The Legacy of Kurt Schütte. Springer 2020.
Responsible Persons	Reinhard Kahle

Course Title:	Number Theory and Cryptography		
Specialisation	Algebra		
Workload - Time in Class - Self-Study	Workload: 270 h	Time in Class: 90 h	Self-Study: 180 h
Frequency	not regularly		
Language of Instruction	German or English		
Forms of Teaching and Learning	Lecture 4 SWS + Exercise class 2 SWS		
Content	• RSA cryptosystem, primality tests, AKS algorithm. • Factorisation methods, number field sieve. • Quadratic reciprocity in cryptography. • Evaluation of the discrete logarithm. • Dynamical systems and Pollard's rho algorithm. • Elliptic curve cryptography. • Lattices and post-quantum cryptography. • Zero-knowledge proofs, digital signatures and hash functions.		
Special Objectives	The students know the basic concepts of elementary number theory and their applications in cryptography. They have deepened and extended their knowledge about neighbouring disciplines: They encounter methods of the theory of dynamical systems and become acquainted with elliptic curves over finite fields. They understand how fundamental cryptographic protocols are working. Through studying many open problems of cryptography, whose solutions may surprisingly come from most distinct branches of mathematics, the students learn to think critically.		
Prerequisites	In addition to knowledge from the module Foundations of Mathematics (MAT-10-10), knowledge from the following (sub)modules of the Bachelor's programme is required: • MAT-10-12 / MAT-20-06 Algebraic Structures.		

Literature	Possible References : • Jeffrey Hoffstein, Jill Pipher, Joseph H. Silverman: An introduction to mathematical cryptography. Springer 2008. • Stefan Müller-Stach, Jens Piontkowski: Elementare und algebraische Zahlentheorie. Vieweg+Teubner 2011. • Joseph H. Silverman, John T. Tate: Rational points on elliptic curves. Springer 1992. • Nigel Smart: Cryptography: An introduction. McGraw-Hill 2003. (online version: https://www.cs.bris.ac.uk/~nigel/Crypto_Book/). • Lawrence C. Washington: Elliptic curves: Number theory and cryptography. Chaman & Hall/CRC 2008.
Responsible Persons	Thomas Markwig