▸ THOMAS PIECHA, PETER SCHROEDER-HEISTER, *Intuitionistic logic is not complete for standard proof-theoretic semantics.*

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Prawitz conjectured that intuitionistic first-order logic is complete with respect to a notion of proof-theoretic validity [1, 2, 3]. We show that this conjecture is false. The notion of validity obeys the following standard conditions, where S refers to atomic bases (systems of production rules):

1. $\vDash_S A \wedge B \iff \vDash_S A$ and $\vDash_S B$.
2. $\vDash_S A \vee B \iff \vDash_S A$ or $\vDash_S B$.
3. $\vDash_S A \to B \iff A \vDash_S B$.
4. $\Gamma \vDash A \iff$ For all S: $(\vDash_S \Gamma \Longrightarrow \vDash_S A)$.
5. If $\Gamma \vDash A$ and $\Gamma, A \vDash_S B$, then $\Gamma \vDash_S B$.

Any semantics obeying these conditions satisfies the generalized disjunction property:

For every S: if $\Gamma \vDash_S A \vee B$, where $\vee$ does not occur positively in Γ, then either $\Gamma \vDash_S A$ or $\Gamma \vDash_S B$.

This implies the validity ($\vDash$) of Harrop's rule $\neg A \to (B \vee C)/(\neg A \to B) \vee (\neg A \to C)$, which is admissible but not derivable in intuitionistic logic.

[1] Dag Prawitz, *Towards a foundation of a general proof theory*, ***Logic, Methodology and Philosophy of Science IV*** (P. Suppes et al., editors), North-Holland, 1973, pp. 225–250.

[2] Dag Prawitz, *An approach to general proof theory and a conjecture of a kind of completeness of intuitionistic logic revisited*, ***Advances in Natural Deduction*** (L. C. Pereira, E. H. Haeusler and V. de Paiva, editors), Springer, Berlin, 2014, pp. 269–279.

[3] Peter Schroeder-Heister, *Validity concepts in proof-theoretic semantics*, ***Synthese***, vol. 148 (2006), pp. 525–571.