Lambda Calculus and Combinatory Logic — SS 2016

Exercise sheet 2 — due 29.4. — T. Piecha

Exercise 1 $(5 + 2 + 2$ points)

We consider the following λ-terms:

(i) $\lambda y.z$

(ii) $(\lambda x.xxy)(\lambda y.xyy)$

(iii) $(\lambda y.yy)(\lambda x.xx)$

(iv) $(\lambda yx.xy)((\lambda z.z)y)(\lambda xz.x)$

(v) $(\lambda x.xyy)(\lambda x.xxy)$

(vi) $(\lambda x.y)x$

(vii) $(\lambda xyz.xz)((\lambda zy.yy)z)((zz)(zz))(\lambda x.xx)$

(viii) $(\lambda x.x(xy))z$

(ix) $(\lambda x.(\lambda y.yx)z)v$

(a) Determine by successive β-contractions, which terms have a β-normal form.

(b) Which terms are strongly normalisable?

(c) Which terms are β-equal?

Exercise 2 (8 points)

Give β-reduction series for the following λ-terms (which are formed by applications of the combinators $\mathbf{S} :\equiv \lambda xyz.xz(yz)$, $\mathbf{K} :\equiv \lambda xy.x$ and $\mathbf{\Omega} :\equiv (\lambda x.xx)(\lambda x.xx)$):

(a) $\mathbf{SSS}$

(b) $\mathbf{KK}(\mathbf{KK})$

(c) $\mathbf{K\Omega}(\mathbf{K\Omega})$

(d) $\mathbf{\Omega K}(\mathbf{\Omega K})$

Exercise 3 $(1 + 1 + 1$ points)

Which of the following statements holds for arbitrary λ-terms M and N?

(a) If $M[N/x]$ is in β-normal form, then M is in β-normal form.

(b) If $M[N/x]$ has a β-normal form, then M has a β-normal form.

(c) If M has a β-normal form, then $M[N/x]$ has a β-normal form.