Lambda Calculus and Combinatory Logic — WS 2017/18

Exercise sheet 7 — due 8.12. — T. Piecha

Exercise 1 (1 point)
Show $\mathbf{I}X \rhd_w X$ for arbitrary CL-terms X, where $\mathbf{I} :\equiv \mathbf{SKK}$.

Exercise 2 (10 points)
Give complete reduction series for the following CL-terms:

(a) $\mathbf{K}(\mathbf{K}xy)z$ (2 points)

(b) $\mathbf{S}(\mathbf{K}(\mathbf{SKK}))\mathbf{S}(\mathbf{KK})$ (2 points)

(c) $\mathbf{S}(\mathbf{K}x)(\mathbf{K}y)(\mathbf{SKK})$ (2 points)

(d) $\mathbf{S}(\mathbf{S}(\mathbf{KSS}))\mathbf{K}x$ (2 points)

(e) $\mathbf{SSSSS}$ (2 points)

Exercise 3 (5 points)

(a) Find a combinator $\mathbf{M}$ such that $\mathbf{M}x =_w xx$. (1 point)

(b) Assume that for all CL-terms X and Y there exists some CL-term Z combining X and Y in the sense that $Zx =_w X(Yx)$.

Using (a), show that every CL-term has a fixed point. (4 points)

Remark: Do not make use of Exercise 4.

Exercise 4 (4 points)
Let $\mathbf{Y} :\equiv \mathbf{WS}(\mathbf{BWB})$, where $\mathbf{B} :\equiv \mathbf{S}(\mathbf{KS})\mathbf{K}$ and $\mathbf{W} :\equiv \mathbf{SS}(\mathbf{K}(\mathbf{SKK}))$.
Show that for all CL-terms X: $\mathbf{Y}X =_w X(\mathbf{Y}X)$.