Lambda Calculus and Combinatory Logic — SS 2016

Exercise sheet 3 — due 6.5. — T. Piecha

Exercise 1 (4 points)
Show that $(\lambda xyz.xzy)(\lambda xy.x) =_\beta (\lambda xy.x)(\lambda x.x)$.

Exercise 2 (3 + 3 points)
Find two pairs of terms M_1, N_1 and M_2, N_2 such that, for $i = 1, 2$, $M_i =_\beta N_i$, but neither $M_i \triangleright_\beta N_i$ nor $N_i \triangleright_\beta M_i$.

Exercise 3 (4 points)
We extend $=_\beta$ to a relation $=_{\beta\phi}$ by allowing for steps of the form $P\left[\lambda xy.x\right] =_{1\phi} P\left[\lambda xy.y\right]$.
Prove that for all λ-terms M, N: $M =_{\beta\phi} N$.

Hint: The proof can be given by simply showing this $\beta\phi$-equality.

Exercise 4 (6 points)
Find two terms P and Q such that neither P nor Q has a β-nf, but PQ has a β-nf.